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BIOMETRIKA

A JOURNAL FOR THE STATISTICAL STUDY OF BIOLOGICAL PROBLEMS

FOUNDED BY
W. F. R. WELDON, FRANCIS GALTON AND KARL PEARSON

EDITED
IN CONSULTATION WITH SIR FRANCIS GALTON
BY
KARL PEARSON

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BIOMETRIKA.

CANARY BREEDING. A PARTIAL ANALYSIS OF RECORDS FROM 1891—1909.

BY A. RUDOLF GALLOWAY, M.B., C.M., M.A., Aberdeen.

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I. INTRODUCTORY AND HISTORICAL.

FROM the commencement of my canary and hybrid breeding experiments in 1891, it has been my custom to make notes of the different matings and their results. These refer mostly to fancy points for show purposes, but at the same time include questions of colour and quality of plumage, of sex, of eye colour, of cinnamon inheritance, and other subjects which have much scientific importance at the present time.

Inheritance in Canaries. The appearance of an interesting set of canary and hybrid breeding experiments entitled "Inheritance in Canaries" by Charles B. Davenport, and published by the Carnegie Institute of Washington in 1908, has induced me to look over and summarize my records, in order to see how far they agree with the conclusions stated in that paper. Before giving my own results and comparing them with those referred to, it is necessary to indicate certain points in the planning of Davenport's experiments, which may possibly account for considerable difference in our conclusions.

It would have been advisable for Davenport to have selected the original stock with much greater care. Most of the original birds used were of the Hartz mountain variety (page 8), which is bred purely for song quite regardless of colour and crest, the two points concerning which the author wished to test Mendel's theories, and also the practical rules of fanciers.

"A few of the Norwich type" (page 8) were used and one is represented (Plate I, fig. 2) which no crest breeder would have in his room, and which shows that the "Norwich type" used, so far as established crest properties are concerned, was quite as poor as the nondescript Hartz crests.

The author's sole criterion of a perfect crest (even though the requisite shape is given on page 8) seems to be absence of a bald occiput. Now this bald spot accompanies most of the perfect crests of the present day. See my figs. 3, 5. Plate IV, fig. 1.

The fancier's idea of an imperfect crest is well depicted in 118 ♂ (*loc. cit.* Plate I, fig. 1) which is called perfect (page 12). This bird has practically no front to its crest—one of the worst faults—and is deficient in radiation over the eye. It might well be called an intergrade form, neither crest nor plainhead, and such birds are well known to appear occasionally in the best bird rooms. This fact is so well known that no bird with a crest showing this imperfection in the slightest degree, is used for crest-breeding purposes. I have frequently seen, and occasionally bred, these intergrade forms, varying from short fronts to complete absence of frontal crest, the posterior half only being present, and spreading from a line on the top of the head, instead of radiating evenly from a central spot. Thus it would appear that "crest" is not strictly "alternate" in inheritance.

While the application of Mendelian principles to the inheritance of a tuft of feathers, or rudimentary crest, is interesting, it is unfortunate that the terms "perfect" and "imperfect" were used, and that the bald occiput was regarded as the sign of an imperfect crest (page 11).

Similarly the disagreement with Russ with regard to "green × yellow" mating always producing "mottled" progeny (page 15), may be explained; for Russ states that it is "important that two pure-bred birds be mated." Now, neither in the Hartz nor Norwich canaries had this been the case, for pure greens and pure "yellows" are not bred in these varieties (the only exception being green Norwich in very limited numbers).

Meaning of Terms. Before I show that results vary according to the pedigree of the birds used in spite of markings, it is necessary to point out certain misunderstandings of terms. Among fanciers, the term "yellow" is synonymous with "jonque," and "buff" with "mealy." Yellow and buff are the terms usually employed, and "yellow" does not mean merely absence of dark feathering, as in

Davenport's paper, but a particular quality of feather which may occur in a green, variegated, or clear bird. The term "clear" signifies complete absence of dark feathers (Plate III, figs. 1 and 4).

The plumage qualities "yellow" and "buff." One of the most important and interesting points in plumage is the difference between "yellow" and "buff" in the technical sense. In Davenport's paper no experiments are recorded concerning the inheritance of these primary and essential qualities, nor do they seem to be understood, for "jonque" (yellow) is not merely the "dense bright shade" of a colour, nor "mealy" (buff) the "dilute, dull tint" of a colour (*vid.* page 14). There may be numerous shades of yellow, also of buff, and the richest buff may be denser and brighter than the poorest yellow. The buff feathering is thicker, has more substance, and shows pale or buff tips. The yellow feathering is thinner, has less bulk, is more silky in quality, and shows no buff tips or mealiness (Plate III, figs. 1 and 2). In addition to having less bulk of feather, the yellow bird is usually smaller in body and more lanky than the buff *.

Davenport and Russ. No. 82 of my muling strain, a yellow variegated cock now in his seventh year, has green feathers on forehead, cheeks, on a back band $\frac{3}{4}''$ broad, on both wings except three primaries on one side, and four on the other: the two outer tail feathers on each side are also green. Mated with clear hens bred from clear birds, 82's record to date is :—clears 15, even-marked 3, variegated 2, the even-marked being green on wings only (secondaries) in one case; on eyes, wings and tail (six pointed) in the second; and marked on four innermost wing quills on one side, and one on the other, in the third case. The two variegated young are very similar to 82, one, however, showing a stronger tendency to even-marking. This record of 82 would seem to support Russ in his statement that the young of such a cross resemble either parent, and that any mixture of colours shows itself in the shape of even-marking †.

---

* Bateson (*Mendel's Principles of Heredity*, 1909, p. 298) writes as follows: "The colours of Canaries are mostly of this class (unfixable because the result of the meeting of dissimilar gametes) and, in order to obtain the requisite shades of yellow, various crosses between *pure-coloured* varieties are made, scarcely any being bred pure for exhibition."

It is difficult for a fancier to understand the meaning of this sentence, as the requisite shade of yellow is obtained from variegated birds, which cannot be called "pure-coloured."

The only way I can catch any meaning in it, is to suppose that the technical qualities of "yellow" and "buff" are not understood by the author, and that he refers to the universal rule in the fancy to mate a "yellow" with a "buff," but this is not done for the purpose of producing the requisite shade of yellow, but to maintain the structure and quality of feather desirable, and to prevent deterioration in stamina and size.

† Bateson (*loc. cit.* p. 43) gives: "Presence of black, as in green and pied types, dominant to absence of black, as in the various yellows and cinnamons."

This cannot be universally true, for if so, my variegated yellow cock, No. 82, would have given very different results.

Moreover, this season I have two yellow variegated cocks similar in appearance to No. 82, and similarly bred, paired to cinnamon-variegated hens, and, so far, out of seven young reared, only two show any sign of dark plumage; in one of these merely a tick on the head, and in the other small wing-marks, and a small head mark.

We certainly have not in this case the equal numbers of "mottled and yellow" (variegated and clear) which the Mendelian theory, as interpreted by Davenport, demands.

II. CANARY HYBRIDS.

Davenport and Canary Hybrids. With some statements concerning hybrid breeding, I must also express disagreement, and the use of Hartz canaries is again the probable explanation of our differences. These canaries are known to have a greater tendency to produce variegated hybrids than the general run of other sorts.

In the experience of all hybrid breeders, hybrids with the "yellow" canary do *not* "frequently show more or less of the canary yellow" (page 23), the great bulk of them being completely dark, the lightly variegated being very rare, and the clear ones in Britain being scarcely more numerous than the fingers of one hand.

The same criticism applies to the following statement on page 23: "Consequently when the yellow canary is crossed with a pigmented canary, or with a finch, the hybrids are mottled." With regard to the relative frequency of the different light hybrids, the goldfinch crosses are commonest because most are bred—the goldfinch being a favourite cage bird. Quite as light ones may be obtained from the linnet, redpoll, twite (Plate III, fig. 3), greenfinch (Plate III, fig. 4), and siskin (Plate III, fig. 1). Since 1891 I have bred a great number of all the different sorts of canary hybrids. The great bulk of these have been quite dark (Plate III, fig. 2): several almost clear of different kinds have appeared, but not until this season (1908) have I been able to say that a perfectly clear mule has fallen to my lot (Plate III, fig. 1). The following table gives the relative proportions of hybrids bred from my strain, which has been carefully built up since 1891, for the purpose of producing "light mules."

Table of Canary Hybrids bred since 1891, *arranged to show Plumage Colour.*

D=Dark plumage, with no white or clear feathers.
S. V.=Slightly variegated, a few small white or clear spots in an otherwise dark plumage.
V=Variegated, from ¼ clear to large marks (eyes, wings, and tail).
L. V.=Lightly variegated, from even marks to small ticks of dark on a clear plumage.
Cl.=Clear, complete absence of dark feathers.

	D	*S. V.*	*V*	*L. V.*	*Cl.*	Totals
Goldfinch (*C. elegans*) ...	172	74	75	19	0	340
Siskin (*C. spinus*)	35	8	4	1	1	49
Linnet (*L. cannabina*) ...	61	17	17	0	0	95
Greenfinch (*L. chloris*) ...	19	4	4	1	0	28
Redpoll (*L. rufescens*) ...	6	7	1	0	0	14
Totals	293	110	101	21	1	526

With five or six exceptions these hybrids have all been bred from male wild bird and female canary. From 1904 to 1908 the sex was noted as far as possible. Some mules are difficult to sex, and all are difficult to keep in numbers because of their pugnacity. The results recorded were: males 110, females 25, unknown 48. After making allowance for calling some females males, these results will still show that there is an excess of males in canary hybrids.

Clear Hybrids. Besides an almost clear goldfinch-canary hybrid* which was bred by myself, I have at present in my bird-room a clear yellow siskin-canary mule† also bred by myself, and which is the only clear siskin-canary hybrid in existence (Plate III, fig. 1). I also own at present a clear buff greenfinch-canary hybrid‡, the only clear greenfinch mule I have seen (Plate III, fig. 4). Also two almost clear twite-canary hybrids§. This list shows that not only the nearly clear but the wholly clear hybrid does occur, although extremely rare.

III. CINNAMON AND OTHER ALLIED WILD SPORTS.

Cinnamon Inheritance. Although many general rules have been found to apply to canary breeding, there are some interesting exceptions, and the mystery of cinnamon blood is one which has long been, and still is, the puzzle of the fancy. It may be well to state first what is already known concerning the breeding of cinnamon canaries, before adding any fresh information.

Cinnamon colour of plumage (Plate I, fig. 1) in young birds can be obtained only by using a cinnamon or cinnamon-bred male bird. If a male having no cinnamon blood be mated to a self-cinnamon female, the young have no cinnamon feathers. If a cinnamon or cinnamon-bred male be mated to a female with no cinnamon blood, all the young which show any cinnamon feathers are females.

In addition to these known facts, I have proved this year that a self-cinnamon male mated to a clear female with pink eyes—the sign of cinnamon descent—may have cinnamon-variegated male progeny.

I have also bred this season a yellow male bird—clear except four small cinnamon-ticked saddle feathers—from two clear parents that were pink-eyed, but had shown no cinnamon feathers in their pedigree for at least two generations.

Crosses of Pink-Eyed ♂s and Dark-Eyed ♀s. My records also show that if a pink-eyed male be mated to a dark-eyed female, all the clear, variegated, and green, dark-eyed progeny are males, and all the clear, variegated-cinnamon, and self-cinnamon, pink-eyed progeny are females: the sexes also occur in equal numbers.

---

* Clear except four small dark ticks, which gained second prize at the Crystal Place in 1907, and other leading prizes.

† First prize, City of Glasgow; first prize, Glasgow and West of Scotland; first, Aberdeen; first, Scottish National at Edinburgh (1908); first prize and championship diploma, Crystal Palace, 1909.

‡ Second Prize, Glasgow and West of Scotland, 1908; second, Aberdeen; second, Scottish National (1908).

§ One of them second prize, Glasgow and West of Scotland, 1908; first, Scottish National (Edinburgh), 1908 (Plate III, fig. 3).

The terms "pink-eye" and "dark-eye" are strictly reserved for those chicks in which these qualities were noted within two days of hatching.

These results agree with those of Doncaster with regard to sex inheritance in the moth *Abraxas grossulariata*, and its variety *lacticolor*, which appear in Report IV to the Evolution Committee of the Royal Society.

The summary of my matings give the following results (see Appendix B, p. 40):

Pink-eyed ♂ × dark-eyed ♀ : 14 matings* = 24 dark-eyed ♂ + 21 pink-eyed ♀ + 4 dark-eyed o + 7 pink-eyed o.

If we add the 4 dark-eyed o to the 24 dark-eyed ♂, and the 7 pinked-eyed o to the 21 pink-eyed ♀, which would be probably a correct procedure, we should have equality of the sexes and 28 dark-eyed males against 28 pink-eyed females.

A fuller discussion of this subject will be found in Section VII (iii) below.

Wild Cinnamon Sports and Cinnamon Hybrids. For some years I have been collecting all the wild cinnamon birds which I could acquire. Any rarity of this sort is usually called a cock in order to enhance its value (a remark which applies also to many museum specimens), but I have now had a fair number in my aviaries, and have always succeeded in getting these cinnamon "cocks" to lay (e.g. Plate II, fig. 2)†. Some of these have also been examined post-mortem and the sex verified. I have also seen and examined many cinnamon wild birds belonging to other fanciers. I have further bred six cinnamon hybrids of different sorts (Plate I, fig. 1), and seen and examined several others. All these cinnamon wild birds and hybrids I have found to be females.

Therefore I feel confident in stating that (*a*) all cinnamon wild birds are females, and (*b*) that male cinnamon hybrids must be very rare, for, so far, I have seen none‡. These statements are based upon :—

1. Twenty cinnamon wild birds and cinnamon hybrids seen at bird-shows, and in bird-rooms.

2. The following living examples at present in my possession:
   (*a*) Four cinnamon and very pale cinnamon (almost white) greenfinches (*L. chloris*) (e.g. Plate II, fig. 2).
   (*b*) One cinnamon yellowhammer (*E. citrinella*).
   (*c*) Three cinnamon canary-greenfinch hybrids (Plate I, fig. 1).

3. The following formerly in my possession:
   (*a*) One self-cinnamon canary-greenfinch hybrid.
   (*b*) One self-cinnamon canary-linnet (*L. cannabina*) hybrid bred by myself.

---

* Sex uncertain = o.

† The cinnamon greenfinch hen usually lays pale cinnamon eggs devoid of markings unless mating has taken place.

‡ John L. Burnett, Govan, informs me that he has bred male cinnamon greenfinch hybrids.

4. The following preserved specimens which died in my possession, the sex being verified post-mortem:

(*a*) Three self-cinnamon greenfinches (*L. chloris*)*.

(*b*) One self-cinnamon goldfinch (*Carduelis elegans*).

(*c*) One self-cinnamon sparrow (*Passer domesticus*).

(*d*) One very pale cinnamon (almost white) sparrow (*P. domesticus*).

(*e*) One self-cinnamon linnet (*L. cannabina*).

5. One very pale cinnamon (almost white) blackbird (*T. merula*) which was shot in October 1908, sent to me by Mr John Dixon, Wigton, and examined post-mortem (Plate II, fig. 3).

6. One cinnamon goldfinch (*Carduelis elegans*) which belonged to Mr John Hector, Aberdeen, was known to be a female during life, and has now been preserved and presented to me.

7. One very pale cinnamon (almost white) starling (*Sturnus vulgaris*) caught by a cat in Aberdeen recently, and now preserved in good condition, and in my possession. This bird has every appearance of being a female.

All of these cinnamon sports and hybrids are of the female sex.

Wild White Sports and White Hybrids. While I have found all self-cinnamon sports in wild birds, and also all that show the faintest shade of cinnamon colour in their plumage to be females, I have also been impressed with the fact that most that show any noticeable amount of pure white plumage are males (e.g. Plate II, fig. 1).

The following have been verified post-mortem, and most of them are in my possession (they are all males):—

1. One white corn bunting (*E. miliaria*)†. This bird has two or three wing quills ticked with dark colour, all the rest of the bird being clear.

2. Two almost clear linnets (*L. cannabina*).

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* Professor Dean kindly prepared sections from corresponding parts of the eyes of one of these cinnamon greenfinch hens, and of a normal wild male greenfinch: the latter had considerably more pigment generally but especially in the posterior hemisphere, and it was much blacker even in the most sparsely distributed areas than in the cinnamon where it was of a rich brown colour.

Professor J. Arthur Thomson and his assistant Dr John Rennie kindly cut sections of the following eyes of canaries:—1. Yellow variegated ♂ bred from cinnamon father (*Fde*). 2. Clear buff crest ♀. 3. Cinnamon-marked (wings) ♀. 4. Clear yellow pink-eyed hen—paternal aunt of the white canary (Plate I, fig. 4).

The amount and colour of the pigment differed in the order given; in 1, it formed a fairly thick black line, with brownish shades in its thinner parts: in 2 the line was thinner, and dark brown in sparsely distributed areas: in 3, there was very much less pigment forming a thin golden brown line, uniformly interrupted with clearer spaces between the pigment cells: in 4, the pigment formed a still paler and thinner golden line, with larger clear inter-spaces between the pigment cells.

† Shot in Durris, Aberdeenshire, in the autumn of 1908, and presented to me by Mr McDonald, Schoolmaster, after being stuffed by the late Mr George Sim, Naturalist.

3. One half-clear redpoll (mealy) (*L. linaria*).
4. One almost clear yellowhammer (*E. citrinella*).
5. One ¾ clear red grouse (*L. scoticus*).
6. One ¾ clear blackbird (*T. merula*).

7. One almost clear chaffinch (*F. coelebs*) shot in Aberdeenshire in Dec. 1908 (Plate II, fig. 1). This beautiful bird, shot ruthlessly by a gamekeeper during the severe snowstorm, when it might easily have been caught alive, is white all over with the following exceptions: (1) the 8th and 9th quills and 3 or 4 coverts of the right wing are of the normal dark colour; (2) the left wing has the 5th quill grizzled, the 7th, 8th and 9th dark normal colour, and 3 or 4 coverts also dark; (3) there are also 3 or 4 ticked scapulars on each side. The upper breast has a faint reddish-brown, and the dorsal region a yellow tinge, which is also seen on the rump (*vid.* coloured figure).

8. Dr Henry of Kemnay, Aberdeenshire, sent me on June 15th, a beautiful pure white male wood pigeon (*Columba palumbus*), which had been shot a day or two previously. It is "clear" (no dark feathers): the eyes are not noticeably abnormal.

I have bred hybrids this season from the following male birds:

1. One ¾ white (clear) linnet (*L. cannabina*).
2. One ⅓ white (clear) linnet (*L. cannabina*).

Also from two other males of the same description in previous seasons.

I do not say that *all* more or less white sports in wild birds are males, for I have at present two linnets with one or two white spots about 4—6 millimetres in size and these are undoubtedly females, and I have seen a pure white female pheasant (*Phasianus colchicus*).

Clear (white) and almost clear female hybrids also occur now and then. I believe, however, that the majority in this case is greatly on the male side, just as we have seen the preponderance in the case of cinnamon hybrids to be on that of the female.

It should be possible to produce male cinnamon hybrids by mating a self-cinnamon cock canary with say a self-cinnamon greenfinch. The nearest approach I have made to this is a male canary-greenfinch hybrid of a peculiar greyish colour (neither the ordinary "dark" mule, nor cinnamon) which I bred last year from a clear yellow cock canary and a self-cinnamon greenfinch—the canary being bred from a clear yellow cock and a buff green hen probably of cinnamon descent.

Origin of the Cinnamon Canary. This question of correlation of sex with cinnamon colour is a most interesting and puzzling one, and may explain the rarity of cinnamon varieties in wild birds. It also increases our interest in the origin of the cinnamon canary.

It would almost appear that we must look for and supply another sort of complementary colour—perhaps greens from cinnamon—before the cinnamon female is capable of producing a male with sufficient cinnamon blood to propagate the variety (*vid.* Greenfinch family in Summary of Conclusions). In the case of the canary it is possible that the cinnamon variety was propagated by the mating of original wild green cocks (Plate I, fig. 2) with the self-cinnamon female sports, the green males from this cross being capable, when mated with cinnamon females, of producing cinnamon males for the propagation of the variety*.

IV. THE CINNAMON CANARY AS THE FOUNDATION OF VARIETY.

The Cinnamon Canary as the Source of all our Varieties. As I believe this interesting canary—the cinnamon—to be the starting point, after the wild green bird, of all the present varieties of canaries, I shall give, at this stage, the grounds for this belief, which has been founded mainly on a study of my cinnamon wild birds, and of the cinnamon and other hybrids I have produced. I have satisfied myself that cinnamon blood, wherever found, indicates the presence of a character, essentially sporting or varying not only in respect to colour and type of plumage, but also to type generally, e.g. size and form of body. I would first draw attention to the following points:

(1) Three rich coloured self-cinnamon greenfinches (e.g. Plate II, fig. 2) acquired last year, and kept outdoors, have this year moulted pale cinnamon.

(2) One of the pale cinnamon greenfinches of last year has moulted paler still this year. It is now creamy white (cf. the blackbird, Plate II, fig. 3), but still shows traces of its original rich cinnamon colour.

We learn from this to recognise several shades and intensities of the cinnamon colour which occurs in wild birds; this fact we must remember later when we discuss the earliest varieties of the canary.

(3) Two cinnamon-variegated canary-greenfinch hybrids (e.g. Plate I, fig. 1) bred by me in 1906 and 1907, each with a small white spot on the nape, and two or three white tail feathers (the rest of the feathers being self-cinnamon), have this year largely increased the white areas of plumage—the heads, tail feathers and coverts of both showing white areas which were previously cinnamon.

(4) A self-cinnamon canary-linnet hybrid bred by me developed several white tail feathers at the second moult.

(5) One of the cinnamon-variegated canary-greenfinch hybrids (Plate I, fig. 1) after taking second prize at Bathgate in perfect plumage, has moulted in its second year into a Dutch Frill hybrid (cf. Plate V, fig. 1), the body feathers, although still cinnamon, showing the most extraordinary twists and turns, just as if it had been pulled backwards through a thick hedge, as Tegetmeier says of the Frizzle Fowl,

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* C. L. W. Noorduijn (Groningen) tells me that at Ponta Delgada, the wild green canary is crossed with "yellow," and that cinnamon-variegated birds are thus obtained.

which must have originated, I believe, in a similar manner, from the cinnamon Cochin. At all events, I can say decidedly in the case of the hybrid, that there was no Dutch Frill blood in the canary parent, which belongs to a strain I have bred for many years.

(6) This same hybrid, along with the frilled feathers, has assumed a semi-upright position, thus indicating the origin of our canaries of shape and position, e.g. Lancashire, Belgian and Scotch Fancy, which are supposed to have come from the Old Dutch variety (cf. Plate V, fig. 1 and fig. 4, p. 23).

(7) I have also noted that cinnamon canaries and hybrids sometimes develop an extra number of tail feathers: one cinnamon-variegated bird of this year has thirteen. I have also two hybrids from cinnamon birds with fourteen each.

(8) I have bred and now possess alive two siskin-canary hybrids, one with a perfectly shaped golden yellow cap (most of the rest of the bird being dark), and the other with a beautiful silver-spangled back (most of the rest of the bird being clear) (Plate I, fig. 3).

The cap and spangled back are both characteristics of the Lizard canary, a specimen of which I never possessed (Plate IV, fig. 3). The strain of the canary parent is entirely free from any Lizard cross. The spangled back appeared at the first moult, just as occurs in the spangling of the Lizard canary. I have also bred several other siskin-canary hybrids with irregular or "broken" Lizard caps. The canary parent in each case was of a strain with cinnamon blood but with no Lizard cross.

In addition to the probability that the Frizzle Fowl has originated from the cinnamon Cochin, it is also likely that the different spangled varieties of poultry have had a similar origin.

We also know that Lizard and London Fancy (Plate IV, figs. 2 and 3) canaries both show the same unstable character of plumage as the cinnamon, both of these varieties being fit to exhibit only during the show season after their first moult, owing to subsequent changes in plumage.

Consequently we may safely infer that both Lizard and London Fancy canaries have been derived from cinnamon canaries.

The "Pink-eye" in Canaries. Before giving my matings of the crested variety and the results of my breeding for colour and quality of plumage, a short reference is here necessary to "pink-eye"—*the* sign of cinnamon blood—in canaries. In the adult canary, pink-eye very seldom shows sufficiently to attract attention while the bird is in the cage—in this respect differing from pink-eyed albinos, e.g. Russian rabbit, and also many wild birds. I have at present, a clear yellow yellowhammer (*E. citrinella*), with pale grey irides, and whose bright red eyes can be seen in any position of the bird in its cage: this bird can see to pick up a mealworm only from a well-illuminated part of the cage bottom (Plate II, fig. 4).

Acuteness of vision is seldom diminished in the pink-eyed canary; in it the pink-eye is best seen by holding the bird with its beak directed to the source of light (a roof-light is best) and to the examiner's face. In this position both brightly illuminated pupils (the fundus reflex) will be seen. The brightness depends mostly on the size of the pupil, and on the amount of top shadow, e.g. a crested bird will show it more clearly than a plainhead.

As an ordinary dark-eyed bird may show a small amount of redness of pupil when held in the position described, and as the degree of brightness depends on the amount of illumination—just as in using the ophthalmoscope in the case of the human eye—the only certain way to tell a pink-eyed canary is to examine the chick immediately after hatching, when the unopened eye region will be seen to be of a distinct pink (fleshy-pink) colour and quite different from that of its dark-eyed nest companions. For this reason also it is preferable to use the term "pink-eye" rather than "red" or "brown-red" eye.

V. THE EVOLUTION OF THE CANARY. EVIDENCE FROM HERVIEUX*.

One is inclined to attach much importance to this author's statements as they bear evidence of being founded on actual experiments, and are not mere repetitions from other writers.

Indeed many of the stories concerning the breeding of those mules, which we, at the present time, regard as impossible, e.g. chaffinch-canary, yellowhammer-canary, have probably arisen from the too free interpretation and misunderstanding of Hervieux's statements.

For he instances the above two hybrids, but only to illustrate his nomenclature. "A male canary being coupled with a female chaffinch, the young which come from them are named Serins mulets de Pinçon" and the others the same:

"Canary mule of the linnet."

"Canary mule of the yellowhammer (Bruant)."

"Canary mule of the goldfinch."

Having stated this he adds:

"Of all those birds of which I am about to speak, those which one pairs most commonly with our canaries, are the goldfinches, male and female, for the others are but seldom used above all at present, so that is an experiment which some new Fanciers (nouveaux Curieux) wish to make, to see what sorts of mules are produced from these different birds."

This clearly proves that Hervieux had no personal knowledge of the yellowhammer-canary and chaffinch-canary hybrids, which subsequent writers (e.g. Buffon) state, on this inadequate evidence, to have been bred.

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* *Traité des Serins de Canarie, par Hervieux* (1713). There are various editions from 1709—1785. The oldest most complete one is that of 1713 which I have used.

This prepares us to receive Hervieux's list of varieties of the canary in 1713 with a considerable amount of assurance that we are dealing with facts, which, being properly interpreted, are of the utmost importance.

But we meet in all translators, and in many subsequent writers (whether professed translators or not) with great errors even in the rendering of the variety-names.

For instance, the writer of the canary article in Rees' *Encyclopaedia* (1819) translates the term "Isabelle" as pink, and gaily proceeds to speak of the Pink Canary, and also the Pink Canary with red eyes! This translation occurs also in Buffon.

Before quoting the varieties, in order to understand the nomenclature, it is desirable to give the following extracts: "Il faut remarquer qu'il y a bien des Serins dont je viens de parler, qui ont outre la queue blanche, des plumes blanches à une aile, et souvent aux deux ailes; mais malgré cette différence particulière, les Curieux ne leur donnent pas un autre nom, que Serin à queue blanche, ou race de Panachez," which may be translated as follows:—

"It is necessary to mention that there are many Canaries of which I am about to speak, which have besides the white tail, the feathers white of one wing, and often of both wings; but in spite of this particular difference, Fanciers (Curieux) do not give them another name than Canary with white tail, or Variegated kind."

It is evident that variegation, as we now know it, was just beginning at that time, and that it arose from the sporting types of which "Gris," "Jaune" and "Blonde" are mentioned. For Hervieux says that we know canaries of these types when they are of the variegated race, as having:—

1. Several white feathers in the tail.
2. Several white claws (ergots).
3. Le duvet.

The transitional stage of the bird is also shown by Hervieux's uncertainty as to the kinds to include under the term "variegated race," for he also adds (p. 272): "I say also that there are canaries which are of the variegated race, which have not, however, any of the three marks, which I have given above, or which have not even one of them; so that it is necessary to leave it to the good faith of those who sell them to you for the variegated race."

These explanations prove that the term "race de Panachez" strictly means with tail white, with a few white body feathers, or with both these variations from self-grey, or self-cinnamon in their different shades.

In our muling experiments we have shown that the first variations to occur are these particular ones, viz. white feathers in tail, and small white spot at the back of the head, or on other parts of the body (Plate I, fig. 1). Our term variegated is denoted towards the end of the list ("which commences with the commonest and finishes with the most rare") by the single word "panaché," e.g. "Serin Panaché commun."

To understand the list, we must also recollect, as I have shown in my experiments with wild cinnamon sports, that self-cinnamon is a varying colour, and frequently changes in the same bird through various shades of pale cinnamon to a creamy white, often with a gloss of yellow on the surface (blond doré) (Plate II, fig. 3).

We must also remember that the colour grey is like cinnamon, one of the pallid variations which occur in nature, e.g. grey greenfinch, a very beautiful example of which, a female, was shown at the Scottish National Show 1909, and is the property of Mr J. W. Bruce, Coldstream. This bird is said to be three years old, and not to have changed its colour—but it is quite likely that other examples might grow paler just as the cinnamon type does—for present day grey and grizzle crests invariably moult lighter each year until ultimately they become clear*.

The mottled or spangled type of variation (Plate IV, fig. 3) (in my opinion, closely allied to cinnamon), also appears in the agate varieties.

Having considered these preliminary points, we are now in a position to interpret the list itself:—

Noms que l'on donne aux Serins, selon leurs différentes couleurs.

Je croy qu'il est apropos de marquer ici les noms que l'on donne communément aux serins, selon leurs différentes couleurs; afin que l'on sçache en quelle classe, ou plûtôt en quel degré de beauté sont les serins que l'on a, ou ceux que l'on souhaite avoir; pour cet effet je me suis proposé de les nommer par ordre, en commençant par les plus communs, et finissant par les plus rares.

1. Serin Gris commun.
   The ordinary grey canary.

2. Serin Gris aux duvets† et aux pattes blanches, qu'on appelle Race de Panachez.
   Slightly variegated frilled canary with white feet.

3. Serin Gris à queue blanche, race de Panachez.
   Slightly variegated frilled canary with white tail.

4. Serin Blond commun.
   The ordinary pale canary.

5. Serin Blond aux yeux rouges.
   The pale canary with pink eyes.

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* By the kindness of Mr Alex. Cochrane, Edinburgh, I now possess a silver-grey linnet (*L. Cannabina*) —also of female sex.

† "Duvets" means the light feathers which adorn the under surface of the body of birds, and may be translated downy or frilled—for it is this part of the bird that first shows the tendency to excess of feather seen in Dutch Frills. Hervieux's explanation of "le duvet" at page 271 may be translated as follows:—"which shows itself, when taking your canary in your hand, you find on it, on blowing it under the body and stomach, a little white down (un petit duvet blanc) and in consequence of a different colour from the natural plumage."

He also adds:—"There are some canaries which have much more of this down than others. This is what one finds with the Fanciers, one they call Serins au petit duvet, that is to say, those which show a little; and the others they call Serins au grand duvet, that is to say, those which have much: this down does not appear usually till near the moult."

6. Serin Blond doré.
   The pale canary glossed with yellow.
7. Serin Blond aux duvets, race de Panachez.
   Slightly variegated pale frilled canary.
8. Serin Blond à queue blanche, race de Panachez.
   Slightly variegated pale canary with white tail.
9. Serin Jaune commun.
   The lemon-yellow canary.
10. Serin Jaune aux duvets, race de Panachez.
    Slightly variegated frilled lemon-yellow canary.
11. Serin Jaune à queue blanche, race de Panachez.
    Slightly variegated lemon-yellow canary with white tail.
12. Serin Agate commun.
    The original Lizard canary.
13. Serin Agate aux yeux rouges.
    The Lizard with pink eyes (showing cinnamon origin).
14. Serin Agate à queue blanche, race de Panachez.
    Slightly variegated Lizard canary with white tail.
15. Serin Agate aux duvets, race de Panachez.
    Slightly variegated frilled Lizard.
16. Serin Isabelle commun.
    The original cinnamon canary.
17. Serin Isabelle aux yeux rouges.
    The cinnamon canary with pink eyes.
18. Serin Isabelle doré.
    The cinnamon canary glossed with yellow.
19. Serin Isabelle aux duvets, race de Panachez.
    Slightly variegated frilled cinnamon.
20. Serin Isabelle à queue blanche, race de Panachez.
    Slightly variegated cinnamon with white tail.
21. Serin Blanc, aux yeux rouges (Plate I, fig. 4).
    The white canary with pink eyes.
22. Serin Panaché commun.
    The original variegated canary.
23. Serin Panaché aux yeux rouges.
    Grey-variegated canary with pink eyes.
24. Serin Panaché de blond.
    Pale cinnamon-variegated canary.
25. Serin Panaché de blond aux yeux rouges.
    Pale cinnamon-variegated canary with pink eyes.

26. Serin Panaché de noir*.
    Green-variegated canary (e.g. figs. 2 and 3).
27. Serin Panaché de noir-jonquille, aux yeux rouges.
    Cinnamon-green variegated canary with pink eyes.
28. Serin Panaché de noir-jonquille et regulier.
    The London Fancy canary (Plate IV, fig. 2).
29. Serin Plein, qui sont à present les plus rares.
    Clear orange-yellow canary, which is, at present, the rarest.

[30†. The crest canary (or rather the crowned) which is one of the most beautiful (Buffon).]

The study of this most instructive list which begins with the commonest and ends with the rarest, combined with a knowledge of the nature and behaviour of sports in wild birds generally, proves distinctly the "sport" origin of all the varieties of the canary.

In classes 1—3 we have the grey canary varying in the direction of frilled and white feathers, and white feet.

In 4—8, the pale type (either of grey or cinnamon) shows the same variations, but in addition a more marked tendency to albinism (pink eye), and towards the differentiation between "yellow" and "buff."

In 9—11, the uniformly lemon-yellow canary shows similar plumage variations.

In 12—15, the original Lizard proclaims its cinnamon descent by having pink eyes, besides the plumage changes like the others.

In *Canary and Cage-Bird Life* for April 16, 1909, Mr L. Butterworth's lecture to the Rochdale Ornithological Club on "The Lizard Canary Fancy Past and Present" is given (Plate IV, fig. 3). In it, this lemon-yellow variation with its tendency to become paler is described in connection with the Lizard canary of forty years ago. At the same time the "duvet" or frilled variety appeared. As these statements from an experienced and observant fancier are important historically, I give them in full, premising that this lemon-yellow colour in mules is well known to be due to cinnamon inheritance.

"When I first started to breed the Lizard canary there was a strain of Lizards which were very plentiful in and around Rochdale known as the lemon Lizard, or lemon jonque, on account of the cap being a pale yellow colour somewhat the colour of a lemon. In its nest feathers it had a back full of straight, narrow rowing, but after its first moult the colour of its cap and the tips of the small feathers were of the same pale yellow colour, the spangle being not nearly so distinct as that of the orange-coloured variety. Breeders, seeing that it stood no chance on the show bench, refused to breed with it, and consequently, in a few years, the strain died out.

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* Noir—Such quills and tail feathers are mostly black or smoky when spread out; but when in position show their yellowish-green edging mainly.

† In the 1793 London edition of Buffon's *Natural History*, Class 30 is included in Hervieux's 1713 list of varieties, where I have been unable to find it.

All the varieties except 29 and 30 are mentioned in the 1709 and 1711 editions of Hervieux.

"About the same time, there was another strain known as the flat or hollow-backed Lizard. This was a class of bird with a back full of large, distinct spangling, or moons as we called them. The moons were distributed all over the back, and not in straight, regular rows as you see them in the Lizards of to-day. This class of bird had very often a *split* or *parting* down the centre of the back, and as it very rarely got into the money at any show, gradually became scarce, until it has almost met with the same fate as the lemon jonque. I should never pair two golds or two silvers together without a special reason*.......

"I remember experimenting in this direction many years ago. I paired a gold cock with a gold hen, and succeeded in breeding some decent young from the pair. Then I inbred with two of the young ones, also both golds. The result was the feathers on the young birds bred from the inbred pair, instead of lying close to the body, grew the wrong way about; they turned up over the back just like those on a *Frizzle* Fowl, which convinced me that you can go too far in that direction" (cf. fig. 1).

In Classes 16—20, the original cinnamon displays similar variations to those in previous classes.

Class 21 is specially interesting to us, as the only white canary ever seen by British fanciers was exhibited at the last Crystal Palace show, and is undoubtedly of cinnamon descent (Plate I, fig. 4).

In 22—26, we have the start of our present day variegated varieties (cf. figs. 2 and 3).

Class 27 is interesting as indicating what I call a cinnamon-green variegation, for these birds, the produce of a cinnamon cock with a dark-eyed hen, are all males and undoubtedly show more pinkness of the eye than other green-variegated birds. They also frequently show a tendency to the dark-green—almost black—London Fancy markings.

In Class 28, we have the start of the London Fancy—now almost extinct—and its occurrence immediately after Class 27 may be of some assistance in re-establishing this beautiful variety (Plate IV, fig. 2).

In Class 29, we have the appearance, of which I have had experience, of a rich orange-yellow bird. The special quality of rich colour which characterizes the Norwich canary has probably its origin here.

Class 30, mentioned by Buffon as being in Hervieux's list, whether there or not, at all events shows that crest was known about 1750.

In our cinnamon muling experiments we have shown that position is correlated with cinnamon sporting and frills.

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* "Gold" is the yellow variety of Lizard, and "silver" the buff.

In this manner our birds of position—Lancashire (fig. 4, p. 23), Yorkshire (Plate I, fig. 4), Dutch Frill (Plate V, fig. 1), Belgian, etc.—have arisen.

We have thus been able to trace the origin of all our present day varieties.

It only remains for some enterprising fancier to follow out the experiments farther and introduce some new varieties, e.g. fantail, trumpeter, black, tumbler, silkie, and many others.

In the English (1718) translation of Hervieux the term "duvet" is taken to mean *rough-footed*, and "panaché" and "race de Panachez" are both translated *copple-crowned*. The one interpretation is as nonsensical as the other, and quite as bad as that of the writer already mentioned who describes "Isabelle" as pink. But the reference to copple-crowns is interesting, as probably indicating the existence of a crested canary in England before 1718. One feels, however, that in the case of this translator one has to deal with a poultry or pigeon, and not a canary fancier.

Note. I may mention that Temminck in his *Histoire Naturelle Générale des Gallinacés* describes the silk fowl under the name of the "Coq à Duvet," and gives it the scientific title of *Gallus Lanatus* (Tegetmeier's *Poultry Book*, 1867). Also in support of my interpretation of "panaché," let me quote the following:—

"Description des Couleurs d'un Canari Panaché, observé avec M. de Montbeillard:"

"The shades and arrangement of the colours of the variegated canaries differ exceedingly; some are black on the head, others not; some are spotted irregularly, and others with great regularity. The differences of colour are commonly perceived only on the upper part of the bird; they consist of two large black spots on each wing, the one before and the other behind, in a large crescent of the same colour placed on the back, pointing its concavity towards the head, and joining by its horns to the two anterior black spots of the wings. Lastly, the tail is surrounded behind by an half-collar of grey, which seems to be a compound colour resulting from the intimate mixture of black and yellow." (Buffon's *Nat. Hist. of Birds*, London, 1793.)

Albin's Song-Birds. At this stage also, let me refer shortly to another old book on song-birds that agrees in every detail with my interpretation of Hervieux's list of varieties of the canary, and also adds some additional information. The title is:

A Natural History of English Song-Birds, and such of the Foreign as are usually brought over, and esteem'd for their Singing, etc. By Mr Eleazer Albin. London 1759 (3rd Edit.).

Albin's knowledge of the song-birds he mentions, and their proper treatment in confinement is so complete and excellent (e.g. his treatment of the goldfinch with regard to hemp seed) that this little book would be an up-to-date guide at the present day. On this account I attach much importance to his list of varieties of the canary given on page 86.

Albin's List of Varieties of Canary in 1759.

(1) "Bright lovely yellow, with jet-black spots." This undoubtedly describes the London Fancy canary, which like the Lizard ("agate" of Hervieux) we believe to be derived from the cinnamon, and which is now almost extinct (Plate IV, fig. 2).

(2) "The Mealy-Bird, so named from the mealy kind of Colour which seems to cover his Feathers." This is the buff bird of the present day.

(3) "Mottled-Birds: their chief colour is white mottled with black or brownish spots." These are our green-variegated and cinnamon-variegated varieties.

(4) "All yellow." Our clear yellow.

(5) "All white" (Plate I, fig. 4).

(6) "Grey." This is the original grey, the "Serin gris" of Hervieux, which is closely allied to the cinnamon canary of to-day.

(7) Other varieties not named.

The Epitome of the Art of Husbandry. London, 1675. By J. B., Gent. At this date in England canaries were green, and variegation had evidently not occurred, for the author—Joseph Blagrove, who is particularly well informed with regard to singing-birds—says (p. 107) "Many Country-People cannot distinguish a Canary from one of our common Green-Birds, etc."

He also writes (p. 106): "The first I shall begin withal is, the Bird called the Canary-Bird, because the original of that Bird came from thence (I hold this to be the best Song-Bird): But now with industry they breed them very plentifully in Germany, and in Italy also; and they have bred *some few here in England,* though as yet not anything to the purpose as they do in other Countries."

VI. COMPARATIVE EVOLUTION OF OTHER DOMESTICATED SPECIES.

The Evolution of the Domestic Fowl. It will probably be found that in other domesticated animals, a similar line of development has been followed, and I have mentioned one or two points of similarity in the domestic fowl. In it, the game varieties seem to form a more or less direct line from *Gallus bankiva,* while those which show greatest diversity in type generally, and in plumage, owe this variability to the original Cochin, which in 1867 not only included a definite cinnamon variety and a white variety in its family, but also had as its prevailing colour Buff* of various shades, e.g. Lemon Cochins, Silver Buff Cochins, etc. (*vid.* Tegetmeier's *Poultry Book,* 1867).

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* "Buff" denotes colour in poultry, and is not used in the technical canary sense.

The Cochin shows the variability which we have seen to occur in cinnamon canaries and hybrids.

(1) According to Tegetmeier (*Poultry Book,* 1867) this variety of fowl has the defect of "twisted primary quill feathers much more frequently than any other."

It is probably because this "defect" was made a disqualifying point in poultry shows, that the Frizzled Cochin has not become an established variety.

(2) The Silk Cochin or Emu Fowl is known.

(3) Grouse and Partridge Cochins represent the spangled varieties.

(4) "There is a tendency in Cochins to produce an extraordinary number of cocks in nearly every brood" (Tegetmeier). We have thus a sexual peculiarity in Cochins just as in cinnamon Canaries (cf. *p. e.* ♂ × *d. e.* ♀ matings).

(5) Cochins are also peculiarly subject to visual defects like albino birds. "The eye should be red...in all cases of blindness pearl-eyed birds" (a further stage of albinism) "have been the sufferers" (Tegetmeier). This pearl eye is said to be "very hereditary" in Cochins (Wright's *Poultry Book,* 1902). Davenport in *Inheritance in Poultry,* quoting McGrew (1904, p. 526), mentions the Buff Cochin as probably the oldest Chinese variety, and cites records of the oldest monastery—Hoangho—to the effect that this fowl was cultivated by the brotherhood 1500 years ago.

From the same author, a very important confirmation of our theory is obtained. Referring to the indigenous Buff Cochin of China, a traveller says that "no two can be found of exactly the same color; some are a chestnut-color, others darker, and some quite light" (McGrew, 1901, p. 527). With regard to the Buff Cochins first imported into England, Wright agrees with Tegetmeier in saying that the colour varied from lightest silver buff and silver cinnamon, through lemons and buffs, to the deepest coloured cinnamons.

Thus we have in the Cochin the same variability that we have seen to occur in our cinnamon sports and hybrids, and we can understand how *Gallus bankiva,* through a cinnamon sport, might be the ancestor of all our present varieties without the aid of a separate ancestor for the Aseel-Malay group. We have, moreover, evidence of the sporting tendency in *G. bankiva,* for Darwin, quoting Mr Blyth, says that the species varies considerably in the wild state, some from near the Himalayas being *paler* coloured than those from other parts of India (Darwin, *The Variation of Animals and Plants under domestication,* 1875, Vol. I. p. 247).

An interesting point also, in connection with this cinnamon-sport origin of all our domestic varieties (which sport occurs, as we have shown, from the female side), is the following statement by Blumenbach, 1831 (given by Tegetmeier): "What we have observed above concerning the aberrations of the formative nisus, namely, that it occurs less frequently in animals of the male sex than in females, is confirmed by the examples of this variety of poultry distinguished by the protuberance on the head: for of this deformity very slight traces indeed are found in the cocks, and those but seldom."

Darwin supports Blumenbach in his statement that this protuberance, with its accompanying crest, was originally confined to the female sex (*loc. cit.* p. 270).

Corroborative Evidence.

(A) Since writing the above, Mr Lewer has sent me a most interesting article on the "Origin of our Breeds of Poultry," by Henry Scherren, F.Z.S., M.B.O.U., which appeared in *Feathered World* for Oct. 11, 1907 (with coloured plate).

This will be found to corroborate my theory. Aldrovandus' classification in 1599 is given:

(1) Common farm poultry—with game characteristics—the female slightly crested.

(2) Paduans—a crested variety with pale coloured (yellow) beak and legs and the wild plumage broken up with white, green, red, and yellow.

(3) A "buskined" or feather-legged race with similar characteristics. Evidently the original Cochin.

(4) A dwarf race—the original bantam.

(5) Turkish fowls—in which Lewis Wright saw a fairly strong resemblance to the Pencilled Hamburghs. Compare my origin of the Lizard canary.

(6) Persians—tailless or rumpless fowls.

Aldrovandus also mentions:—

(7) Frizzled Fowl.

(8) Woolly Fowl—the Silk Fowl of the present day.

(B) *Buff Poultry and Cinnamon Canaries.* The term "buff" in poultry, indicates colour, not quality of feather. The buff colour has been grafted on to the different varieties of poultry, exactly in the same manner as cinnamon in canaries, e.g. cinnamon Norwich, cinnamon Crests, etc.

Davenport, after showing that the buff colour of the Cochin is of high antiquity, and stating that it has been transferred to many other breeds by crossing, e.g. Buff Wyandotte, quotes McGrew (1901, p. 24):

"Two distinct lines were produced under different methods. One was formed from Wyandotte—Buff Cochin cross; the other came through the Rhode Island Red—Wyandotte cross. The Rhode Island Red is, however, as is well-known, a direct descendant of the Buff Cochin. The Buff Plymouth Rocks were derived *directly* or *indirectly* from the Buff Cochin. The history of the Buff Leghorn is the same—the offspring of a yellow Danish Leghorn cock and Buff Cochin pullets, mated with a yellow Leghorn hen. The produce, three-fourths yellow Leghorn and one-fourth Buff Cochin, gave (Wyckoff, 1904, p. 527) the first Buff Leghorns ever shown."

The Buff "Orpingtons"—a highly modern and mongrel breed—have a similar history, being chiefly Buff Cochin and Dorking (Wright, 1902, p. 296).

The behaviour of the buff colour in these crosses is exactly the same as we have shown to occur in the cinnaman colour of canaries—for the Buff Leghorns, for instance, were not obtained directly from the buff mother but from her sons, in the same manner as we have demonstrated cinnamon feathers in canaries, to be inherited from the male side. We may safely assume that the buff varieties of poultry correspond with the cinnamon varieties of the canary, that both are due to an original early cinnamon sport, and that this sport or mutation is, in all probability, the cause of the great diversity of all the varieties of the species.

(C) *Early "Sport" in the Pigeon.* In the case of the pigeon also, evidence is adducible which proves that the first change from the Blue Rock pigeon (*Columba livia*) was one in the direction of albinism.

In *The Dovecot and Aviary* (Rev. E. S. Dixon, 1851) the author gives a passage in full from Varro, who lived from 116—27 B.C. He translates it thus:

"If ever you should establish a Dovery, you would consider the birds your own, although they were wild. For two sorts of pigeons are usually kept in a Dovery: the one belonging to rural districts, and as others call it, a Rock Pigeon, which is kept in towers, and among the beams and rafters (*columinibus*) of a farmhouse, and which is on that account named Columba, since from natural timidity it seeks the highest of roofs; whence it happens that the rustic pigeons especially seek for towers, to which they may at their own pleasure fly from the fields, and return thither. The second kind of pigeons is more quiet; and contented with the food given at home, it accustomes itself to feed within the limits of the gate. This kind is of a *white* colour principally, but the country sort is without white or variegated colours. From these two original stocks a third mixed or mongrel kind is bred for the sake of the produce."

VII. CANARY BREEDING.

(i) *Matings of Crested Variety** (cf. figs. 2 and 3, also Plate IV, figs. 1 and 4). In Appendix I. (p. 33) is given the actual data of my matings with explanation of the symbols used. They may be summarised as follows:

38 crest ♂ × crest-bred plainhead ♀ = 66 crest + 64 plainhead,

or, in symbols, 38 C ♂ × c ♀ = $66C + 64c$.

Again, 44 c ♂ × C ♀ = $60C + 63c$.

Together, 253 offspring, $126C + 127c$.

Further, 15 C ♂ × C ♀ = $34C + 17c$.

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* By the comparatively downless state of its head the newly hatched crest may always be distinguished from the crest-bred plainhead.

From these results it follows that:

(1) Reciprocal matings of crest and crest-bred plainhead ($C \times c$) result in practically an equal number of crested and crest-bred plainhead young*.

(2) Double cresting ($C \times C$) gives twice the number of crests as compared with crest-bred plainheads.

(3) Double crest-bred plainhead breeding ($c \times c$) is so well-known never to produce a single crest that I have not included any matings of that sort in my paper. In this mating crestlessness is recessive (fig. 2).

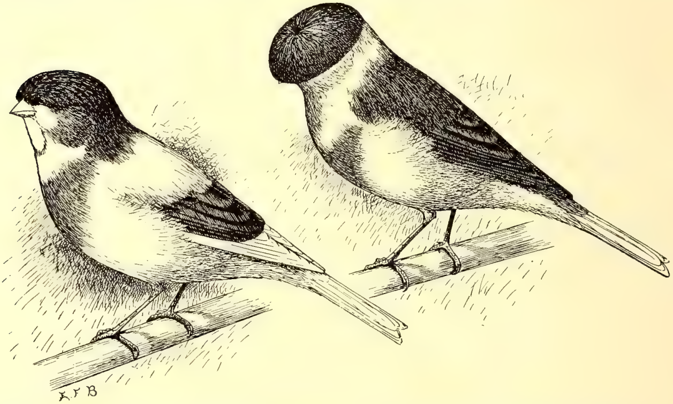

Fig. 2. Variegated crest-bred Norwich Plainhead. First Prize, St Helens, December, 1908.

Fig. 3. Heavily variegated Norwich Crest. First Prize, St Helens, December, 1908.

From an original sketch by E. F. Bailey. By permission of *Canary and Cage Bird Life*.

(4) No crested bird (fig. 3) in my reciprocal matings of crest and crest-bred plainhead has produced crests only, or in sufficient number to be worthy of note: those producing most are:

2. '91 *C. d. y.* ♂ = *C* 7, *c* 1.
5. '98 *C. m. y.* ♀ (*c. gr. b.* × *C. t. y. Lan.* {3. '97}) = *C* 6, *c* 1.
2. '99 *C. gr.* ♂ (*C. m.* × *c. v.* 1. '98) = *C* 6, *c* 1.

(5) Those birds used in double-crest ($C \times C$) matings which produced most crested young were:

5. '97 *C. m.* (*C. m.* × *c. v.*) = *C* 5, *c* 1.
5. '98 *C. t.* = *C* 4.
1. '06 *C. cl.* (*c. m.* × *C. cl. Lan.* 5. '03) = *C* 2, *c* 3.
5. '07 „ = *C* 3.
3. '08 „ = *C* 3.

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* Skulls from this mating were preserved and none showed the condition of *hernia cerebri*.

(6) Thus, reciprocal $C \times c$ matings, resulting in practically 50 $^\circ/_\circ$ crests and 50 $^\circ/_\circ$ crest-bred plainheads, agree with the Mendelian result of a heterozygote × homozygote union; it is difficult to get a corresponding agreement in the $C \times C$ matings, which give twice the number of crests as compared with crest-bred plainheads.

(7) Of 160 crests bred by me, 14 are noted as having bad fronts. The great bulk, however, were high class crests and held principal honours at all Scottish shows. The same is true of the crest-bred plainheads, one of which, "Cock o' the North"—*c. gr.* (*c. v.* × *C. gr.* 8. '93)—defeated "Devastation," the Crystal Palace Champion of two years' standing, under Mr Bexon, one of the best known English judges. Such highly bred birds are less prolific and more delicate than the crests

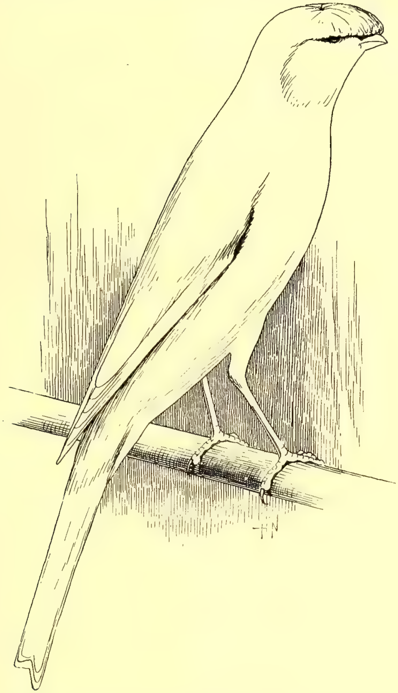

FIG. 4. Typical Lancashire Coppy. To illustrate frontal with absence of back crest, and upright position. From a sketch by H. Norman. By permission of *Cage Birds*.

used by Davenport in his experiments: they are also very subject to lenticular degeneration, many becoming blind from cataract shortly after the first or second moult. This is, no doubt, connected with the excessive development of head feather, which is an essential point in the variety (Plate IV, fig. 1).

(8) I have mentioned that almost $\frac{1}{12}$ of the crests had deficient frontal crest development: it may be well also to state that in the Lancashire variety of crested canary (the "Coppy") we have a crest with a front and no back—this being the characteristic of the Coppy crest (fig. 4). In it also there is no bald occiput, the feather lying close and smooth at the nape like an ordinary crest-bred plainhead.

It is also necessary in breeding the ordinary or Norwich crest (the variety of my experiments) to occasionally introduce Lancashire blood in order to get the best results. Hence we have a complex set of conditions existing, which results in probably $\frac{3}{4}$ of the highest class crests of the day having bald occiputs.

(9) It may be stated definitely that "perfect crest" and "bald occiput" may co-exist (fig. 5).

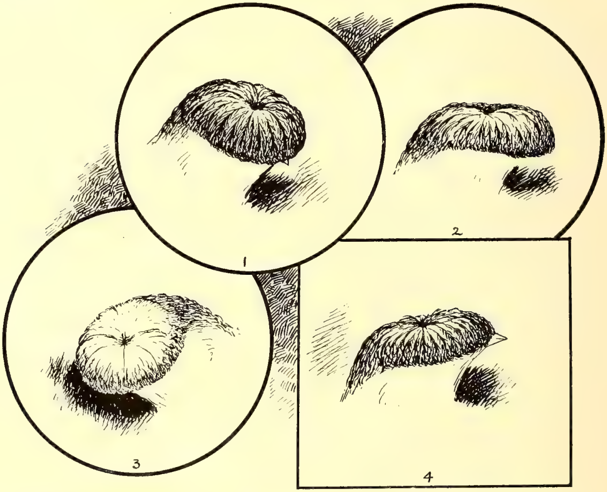

Fig. 5. A Study in "Crests." From sketches taken by H. Norman at Camberwell (L. and P. O. S.) Show, October, 1906. Class 6. Any variety Crested Canary.

No. 1. First Prize. Perfect shape, good density, droop, and radiation.
No. 2. Second Prize. Heavy frontal, weaker back crest, centre too far back.
No. 3. Third Prize. Smaller crest of good shape, droop, and radiation.
No. 4. Fourth Prize. Deficient in size, density, and radiation.

By permission of *Cage Birds*.

It is also a fact that no partly bald plainheads occur in the crested variety—a proportion of $\frac{1}{16}$ is necessary according to Davenport's system of allelomorphs: see his p. 13.

Double-Cresting and Baldness. With regard to double-cresting, of which I have had considerable experience, we have seen that the proportion of crests was doubled. None of these 34 crests and 17 crest-bred plainheads showed any sign of general baldness. I have been, however, careful to select perfectly shaped crests with closely lying backs. I have often heard of, and seen, partially bald-headed crests, which were said to be the result of double-cresting. In these there was always some skin affection of an eczematous or parasitic nature, which would sufficiently account for the condition apart from simple baldness*.

Results of Double-Cresting. In my experience double-cresting may be continued for several generations with the best results as far as shape, droop and radiation (for the present-day crest is not "flat" as Davenport quotes from Blakston in

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* Bateson (*Mendel's Principles of Heredity*, 1909) writes (quoting Evolution Committee Reports and Davenport): "The crest of the Crested Canaries is *always* bred for Shows by mating cresteds with plain-headed birds." (p. 298.)

This is, by no means, universally the case, as double-cresting is a well-known method of pairing to produce winners.

"This neatly-laid appearance (of the Crest) is *only* produced when the bird is heterozygous for the crest factor." (p. 298.)

The following were crests of perfect shape, and prize-winners:

C. cl. ♀ (*C. cl.* × *C. v.* 2 : 2. 06).

C. t. ♀ (*C. cl.* × *C. v.* 2 : 2. '06).

C. m. ♂ and *C. m.* ♀ : 3. '02 had crest-bred father, but both father and mother were bred from double-crests.

In 1894, Mr John Hector, Aberdeen, bred a green crested hen from double-crests, which gained first prizes at Dundee and Aberdeen, and was then sold to Mr Fred. Weinberg, Dundee, who gained many leading prizes with it, and believed it to be the best crest he had ever owned.

Mr Jas. Wallace, Aberdeen, gained 3rd prize, Class 9, Crystal Palace, 1909, with a clear yellow crest which he bred from double-crests.

Mr F. W. Barnett, Fakenham, Norfolk, who is now, and has been for many years, the much envied chief of crest breeders, has favoured me with his experience of double-cresting, which will be found to agree with my own.

Mr Barnett has not exhibited any birds for some years (most winners, however, have been bred by him), but he tells me he won at the Crystal Palace ten or twelve years ago with a crested bird bred from two crests; he has bred several others in a similar way which could have won in good company, had they been shown by him. His practice, when in need of fresh blood, is to mate a Lancashire Coppy (Fig. 4) with a Crest (Fig. 3) which "*hit very well*"; he also agrees with me in stating that indiscriminate double-cresting often throws mop crests, the reverse of bald heads. These mop crests are not the bald heads of the fancier as supposed by Bateson (p. 37).

"The mating of two crested parents is by several authors said to give rise to some bald birds." (p. 37.)

Bateson here, like Darwin, repeats a belief which undoubtedly does obtain in the fancy, but which my experience does not corroborate—for I have never seen baldness which could be attributed to double-cresting, and I have seen much of it that could be easily explained by simpler means, e.g. *lice*, to the attacks of which the *heads* and *necks* of these heavily feathered birds are specially prone, and which are, as a rule, unrecognised by fanciers.

Darwin also states erroneously that there is a feather-footed breed of canary (1875, *The Variation of Animals and Plants under domestication*, Vol. I. p. 311) (cf. p. 17 above and note on p. 13).

1880, *vid.* p. 11) are concerned; but there is certainly, unless care be exercised, a tendency for the bald occiput, not to become greater in area, but to be surrounded by aberrant feathers, which cause a dishevelled appearance at the back of the head—the "horned" crest in fact.

It is possible that this condition might develop into a structure like the shell crest of the Trumpeter pigeon, whose rose crest exactly resembles the crest of a modern crested canary, covering eyes and beak in even, more or less, circular radiation. In this pigeon double-cresting is the rule: there are no crest-bred plainheads, and no baldness occurs (Plate V, fig. 6).

Plumage-colour in the Crested Variety. We have already seen that my results with regard to "mottled × yellow," or more correctly, variegated × clear matings in my non-crested or muling strain did not agree with Davenport's conclusions.

The same is true of plumage-colour in my crested strain, the results of which I now give, with the additional explanation that Davenport's "mottled × green" corresponds with my variegated × green, and his "mottled × mottled" with my variegated × variegated matings.

Matings		Progeny	
	variegated	green	clear
Variegated × variegated, 36 matings =	103	6	12
Variegated × clear, 24 matings =	68	0	17
Variegated × green, 24 matings =	58	13	1

The variegated × variegated proportion is roughly 51 : 3 : 6, which does not correspond, even approximately, to the expected 9 : 3 : 4.

In neither of the other two sets of matings is the expected 50 °/o of variegateds and of clears, of variegateds and of greens obtained.

(ii) *Inheritance of Yellow and Buff Qualities.* The quality of colour being comparatively unimportant in the crested variety, I regret that sufficient attention was not given to noting the relative proportions of yellow and buff progeny. This point was carefully attended to in the notes of my non-crested or muling strain, where colour is of great importance.

There is a wide-spread belief that yellow or jonque feather is inherited much more strongly from the male than from the female parent. The figures from my crested strain are not sufficiently full to be of great importance in definitely deciding this question.

Proportion of Yellow to Buff Progeny in the Crested Variety.

Matings				Progeny		
		Yellows		Buffs		Unknown
18 yellow ♂ × buff ♀	=	22	+	10	+	22
14 buff ♂ × yellow ♀	=	23	+	18	+	16

Thus yellow ♂ × buff ♀ notes 22 yellows in a total of 54, and buff ♂ × yellow ♀ notes 23 yellows in a total of 57.

Such as they are these figures indicate that there is no great difference in the number of yellow progeny obtained, between using a buff cock and a yellow cock. They also show a tendency to partial dominance of yellow, the proportions in those noted being 22 : 10 in the first set, and 23 : 18 in the second. The "unknowns," however, form too large an element in the offspring to allow of any great stress being laid on these results.

"Buff" recessive. It may be stated definitely that no yellow progeny can be obtained from two buff parents. Any reported cases of this sort are probably due to one of the parents being really a poor yellow and not a good or "high" buff.

Some care has to be exercised in distinguishing these birds, but in every case the buff tips of the feathers about the head, neck and saddle—the frost or meal—can be made out in the buff bird, and is absent in the yellow (Plate III, figs. 1 and 2).

Double-buff matings in my Crested Strain.

Number of Matings	Progeny	
65	193 buffs	0 yellows

Reciprocal Yellow and Buff Matings, Muling Strain. The following are the results in detail of reciprocal yellow and buff matings, and double yellow matings in my muling strain since 1891:

Yellow ♂ × Buff ♀. Number of Matings, 26.

	Progeny.	
Yellow	Buff	Unknown (whether yellow or buff)
20 male	19 male	3 male
33 female	15 female	1 female
4 sex unknown	4 sex unknown	6 sex unknown
57	38	10

Proportion of yellows to buffs = 57 : 38, or 3 : 2.
" males to females = 42 : 49.

Buff ♂ × yellow ♀. Number of Matings, 20.

	Progeny.	
Yellow	Buff	Unknown (whether yellow or buff)
20 male	12 male	1 sex unknown
22 female	12 female	
2 sex unknown	4 sex unknown	
44	28	1

Proportion of yellows to buffs = 11 : 7, nearly 3 : 2.
" males to females = 32 : 34.

Putting the reciprocal matings together we have

Yellows = 101 Buffs = 66

Proportion of yellows to buffs = almost exactly 3 : 2.

„ males to females = 74 : 83.

Double-yellow (*yellow* × *yellow*) *Matings. Number of Matings*, 18.

Progeny.

Yellow	Buff	Unknown (whether yellow or buff)
22 male	8 male	4 female
26 female	8 female	
2 sex unknown		
50	16	4

Proportion of yellows to buffs = 3 : 1 nearly.

„ males to females = 30 : 38.

From these tables it will be seen that the belief that more yellow progeny came from yellow male and buff female than from buff male and yellow female matings, is not substantiated. The proportion in both cases is almost exactly 3 : 2.

It is further evident that double-yellowing practically doubles the number of yellows obtained, the proportion then being almost exactly 3 : 1, instead of 3 : 2.

Females are rather more numerous than males in all the matings.

The following birds are those in my muling strain which have produced most yellow progeny :

(1) Variegated yellow ♀ (bred from double-yellows) = 6 yellows—males and females.

(2) Clear yellow ♂ (bred from yellow ♂ × buff ♀) = 3 *cl. y.* ♂ + 3 *cl. y.* ♀.

(3) Cinnamon ticked yellow ♀ = 5 *y.* ♀ + 1 *y.* ♂ + 3 ?

(4) Ticked yellow ♀ (bred from a treble-yellowed ♂ × *m. b.* ♀) = 5 *y.* ♀ + 1 *y.* ♂.

(5) Variegated yellow ♂ (bred from double yellows) = 4 *y.* ♂ + 6 *y.* ♀ + 2 *b.* ♂.

The Effect of Double-yellowing. While double-yellow mating is thus shown to improve the colour of a strain, it must be done cautiously, as feather becomes thin and scanty, and the general health of the stock suffers if this method be used even for three or four generations in succession. It seems to be true that reciprocal yellow and buff matings should be the general rule, if the health of any strain is to be kept good. I have not been able, so far, to prove that any yellow, or any crested bird, was truly homozygous. The yellow quality of feather, like the crest, seems to be of a compound nature; it is associated especially with thinness and scantiness of plumage, and results usually, if double-yellowing be continued to any extent, in a featherless condition of the body, especially on its under aspect.

(iii) *Inheritance of Eye Colour.* I now pass to my matings bearing on pink and dark eyes. The matings are all tabled in Appendix B, and the table below gives a summary of the results reached.

The possible matings are as follows:

I. Pure Pink-eyed ♂ × pure Dark-eyed ♀.

II. Pink-eyed ♂ × Pink-eyed ♀.

III. Pure Dark-eyed ♂ × Pink-eyed ♀.

III. 1. Dark-eyed ♂ × Pink-eyed ♀ from Pink-eyed Father and Dark-eyed Mother.

III. 2. Dark-eyed ♂ from Pink-eyed Father × Pink-eyed ♀.

III. 3. Dark-eyed ♂ from Pink-eyed Mother × Pink-eyed ♀.

III. 4. Dark-eyed ♂ from Pink-eyed Father × Pink-eyed ♀ from Pink-eyed Father and Dark-eyed Mother.

III. 5. Dark-eyed ♂ from Pink-eyed Mother × Pink-eyed ♀ from Pink-eyed Father and Dark-eyed Mother.

IV. Dark-eyed ♂ × Dark-eyed ♀, both pure.

IV. 1. Dark-eyed ♂ × Dark-eyed ♀ from Pink-eyed Mother.

IV. 2. Dark-eyed ♂ from Pink-eyed Mother × Dark-eyed ♀ from Pink-eyed Mother.

IV. 3. Dark-eyed ♂ from Pink-eyed Father × Dark-eyed ♀ (pure).

IV. 4. Dark-eyed ♂ from Pink-eyed Mother × Dark-eyed ♀ (pure).

Now according to the view of Bateson and Punnet every pink-eyed female is alike in gametic character, and every dark-eyed female is also alike. The dark-eyed male may, however, be homozygous or heterozygous in eye colour. Thus there is no distinction between a dark-eyed female of dark-eyed descent, and one which has arisen from a pink-eyed ancestry by one dark-eyed cross. Unfortunately the data are not sufficiently numerous, and the knowledge of sex is so incomplete in the case of the offspring that it is not possible to test effectively some of the fundamental points of this theory which depends on the hypotheses that (*a*) sex is heterozygous in the female and (*b*) that there is a repulsion between femaleness and dark-eyedness.

Thus whether a dark-eyed female comes from a dark-eyed line or not, her gametic constitution is represented by *p. d.* ♂ ♀ and her ova are *p.* ♀ and *d.* ♂, but a dark-eyed male may be either *d. d.* ♂ ♂, the homozygous type, or *p. d.* ♂ ♂, the heterozygous type, the gametes being *d.* ♂, *d.* ♂ and *p.* ♂, *d.* ♂ respectively. It is perfectly easy to deduce from this the expected proportions of offspring both as to sex and eye colour. But the numbers, especially those sexed, in the subclasses III. 1—5, and IV. 1—4, are quite insufficient to draw conclusions as to the fundamental hypotheses. Such conclusions, however, as can be drawn are at no point opposed to the theory above developed.

Let us look at the categories in order:

I. This is *p. p.* ♂♂ × *p. d.* ♂♀ for there is no "pure" dark-eyed female. The result should be 50 °/o pink-eyed females and 50 °/o (*p. d.* ♂♂) dark-eyed heterozygous males. Actually we have ♂ 24 + ○ 4 dark-eyed and ♀ 21 + ○ 7 pink-eyed offspring.

II. This is *p. p.* ♂♂ × *p. p.* ♂♀ and should give 50 °/o of pink-eyed ♂'s and 50 °/o of pink-eyed ♀'s. The result ♂ 9 and ♀ 13 is not opposed to this as there were ○ 13.

III. This is *d. d.* ♂♂ × *p. p.* ♂♀. The result should be all offspring dark-eyed and the sexes in equal numbers. There were 110 dark-eyed and no pink-eyed. Of the canaries alone ♂ 20, ♀ 21, and ○ 9. The results accord with the theory.

III. 1. This is *d. d.* ♂♂ × *p. p.* ♀♂, the same as III. according to the theory. There were 22 offspring all dark-eyed, sexes are not available.

III. 2—5. It would be interesting to consider these separately, but the material is not adequate. According to the theory they are all *p. d.* ♂♂ × *p. p.* ♂♀, and should give rise to 25 °/o pink-eyed females, 25 °/o pink-eyed males, 25 °/o dark-eyed females and 25 °/o dark-eyed heterozygous males. All together there were in fact 10 pink-eyed and 10 dark-eyed offspring, and 3 doubtful. The sexes were not ascertained.

IV. This is on the theory *d. d.* ♂♂ × *d. p.* ♂♀ and should give all dark-eyed, there were 283 offspring all dark-eyed, and equality of sexes. There were ♂ 96 and ♀ 93 and many unsexed.

IV. 1. This is *d. d.* ♂♂ and *d. p.* ♂♀. This again should give 50 °/o heterozygous dark-eyed males and 50 °/o dark-eyed females. There were 16 dark-eyed offspring in all, sex not recorded.

IV. 2 is *d. p.* ♂♂ × *d. p.* ♀♂ and should give 25 °/o pink-eyed females, 25 °/o dark-eyed females and 50 °/o dark-eyed males. There were 15 dark-eyed, and 6 pink-eyed progeny.

IV. 3—4 should also give the same result. We have altogether (IV. 2—4) 43 offspring, 11 pink-eyed and 32 dark-eyed; a result exceedingly close to the expected. There are too few sexed to allow of any definite conclusion. The whole subject deserves to be treated from far greater numbers. Thus far there is nothing in my observations—like the four dark-eyed hens of Durham and Marryat from *d. p.* ♂♀ × *p. p.* ♂♂—at variance with the theory.

VIII. SUMMARY OF CONCLUSIONS.

I. All canary varieties have arisen from a grey or cinnamon sport occurring in the female—the pallid type of variation which occurs at present among many wild birds.

This theory is advanced from a study of:

1. Wild sports generally, in nature and in confinement.

2. Cinnamon, and cinnamon-bred hybrids, which frequently show characteristics of canary varieties arising *de novo.*

3. The earliest canary literature.

4. Collateral evidence of a similar nature in poultry and pigeons.

II. "Dark-eye" and "Pink-eye" are found to behave generally in Mendelian fashion, for from Group IV. (*d. e.* × *d. e.*) it is evident that there is a homozygous type of dark-eyed canary—also from

Group IV. 2, 3, 4, it appears that there is also a heterozygous or impure form occurring in the male as well as the female. The pink-eyed birds being homozygous, if we arrange our groups of matings according to Mendelian principles we find:

Group III. *d. e.* × *p. e.*

Matings 33 Progeny 110 Dark-eyed 110

In this group dark-eye is dominant, and pink-eye recessive.

The following heterozygous matings:

IV. 2.	*d. e. F.* × *d. e. F.*	Matings 5	Progeny 21	*d. e.* 15	*p. e.* 6
„ 3.	*F. d. e.* × *d. e.*	„ 3	„ 12	„ 9	„ 3
„ 4.	*d. e. F.* × *d. e.*	„ 4	„ 10	„ 8	„ 2
			43	32	11

The result closely approximates to 3 : 1.

The following heterozygous × homozygous matings:

III. 2.	*F. d. e.* × *p. e.*	Matings 1	Progeny 3	*d. e.* 1	*p. e.* 2	
„ 3.	*d. e. F.* × *p. e.*	„ 3	„ 12	„ 4	„ 5.	Unknown 3.
„ 4.	*F. d. e.* × *F. p. e.*	„ 1	„ 2	„ 0	„ 2	
„ 5.	*d. e. F.* × *F. p. e.*	„ 2	„ 6	„ 5	„ 1	
			23	10	10	

This result gives the required 50 $^\circ/_\circ$ of each.

One would expect the female of the homozygous type of dark-eyed canary to be homozygous as well as the male, and I have evidence of this in several of my females giving very large percentages of dark-eyed males when mated to pink-eyed males—the proportion of six to one occurring several times. It is probable that a homozygous dark-eyed female would be completely dominant over the cinnamon male, and that male dark-eyed progeny only would result.

I can prove the occurrence of wild heterozygous males by the following most interesting result which I have just obtained, viz. a family of young greenfinches which are leaving their nest to-day (June 8th). The father is a wild caught bird which I selected as being of the colour which I think indicates a heterozygous nature not only in wild birds, but also in canaries, viz. a colour I call cinnamon-green. To any casual observer, however, the bird would pass as a normally coloured greenfinch. The mother is one of my pale cinnamon—almost creamy—white greenfinches already mentioned. The family of five consists of four of a cinnamon and one of a greyish type all much paler than the normally coloured young of greenfinches. It is too early to sex the young, but I hope to have, at last, seen a *male* cinnamon greenfinch*! The cinnamons appear to be males, and the grey a female.

III. Other characters which are seen to behave as recessives are (*a*) buffness, (*b*) crest-bred plainheadedness, and their corresponding qualities (*a*) yellowness, (*b*) crestedness, exhibit more or less imperfect dominance.

There can be no doubt that the majority of crests at present are heterozygous with regard to crest, although this cannot be essential to crest formation as stated by Bateson (*Principles of Heredity,* 1909), for the Trumpeter pigeon, which is homozygous, has a similar arrangement in its rose crest (Plate V, fig. 6). Bateson also, following Davenport, is in error in assuming that the fancier's bald crest is an upstanding crest which shows the bald occiput. Such a crest is the very reverse of bald, and is the well-known "mop" crest of the fancy.

IV. My points of disagreement with C. B. Davenport centre mainly in the material used in his experiments, and in his interpretation of fancy points with regard to crest and plumage colour.

His "green × yellow" matings do not correspond with my "green × clear" partly because his "green" is not green but variegated (*vide* Plate I. fig. 1, where the pied throat shows the bird to be variegated). Similarly nearly all "clears" have dark underflue, or dark spots on bill or feet, which strictly constitutes them variegated. Hence Davenport's "green × yellow" is really variegated × variegated, and naturally from this mating one would expect variegated or "mottled" young.

Moreover, greens often become variegated, variegateds often become ticked, and ticked birds often become clear during their life-time.

Davenport's perfect crest, as has been shown, is far short of what is considered perfect at the present day, and the perfect crest does not depend on the absence of a bald occiput.

The technical qualities "yellow" and "buff" are important in any study of the plumage of the canary, and are not considered.

V. It is essential in studying Mendelian phenomena as occurring in fancy varieties that the most strict definition of the characters under examination be made, and that their nomenclature, and behaviour under varying conditions, be thoroughly understood. A fancier is trained to detect differences which others are quite unable to see, and his success depends on the careful balancing of factors which to the uninitiated are unobservable. It would therefore save much confusion if sharply defined facts only were taken into consideration meantime, and if no assumptions were made such as those referring to "bald occiput," "baldness," "perfect crest," etc.

An expression of my deep indebtedness is due to Professor Karl Pearson, for much assistance and advice in the preparation of this paper.

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* Three of the cinnamons died, and on examination were found to be males.

APPENDIX.

CANARY MATINGS.

A. *Matings of the Crested Variety.*

Meaning of Symbols. Males are mentioned first, females second, e.g. ♂ × ♀. *C.* = crest. *c.* = crest-bred plainhead (from crested stock). *gr.* = self-green. *v.* = variegated to any extent between green and *m* = marked on head, wings and tail, or head and one wing with or without tail marking. *C. d.* = clear body, dark crest. *C. t.* = ticked crest. *c. t.* = ticked crest-bred. Body usually almost clear, but there may be ticking, or grizzling, i.e. faint marking on the head, body, wings, or tail. *cl.* = clear. *Lan.* = Lancashire variety [crested—(coppy)—or plainhead]. *Pl.* = plainhead—not crest-bred. *Cin.* = self-cinnamon. *b.* = buff. *y.* = yellow. Sex of progeny is mentioned when known. The rule in crest breeding is to double-buff, i.e. both parents are generally buff birds in order to increase feather and size. Hence all birds are *b.* (= buff), when *y.* (= yellow) is not mentioned in any mating. If *y.* be mentioned then *b.* is used for those buffs which have been noted, no symbol being given for those unknown.

Example. *C. t. b.* (bad front) (*C. t.* × *c. cl.*: 3. '04) × *Pl. t. y.* = 1 *C. t. y.* ♂ (bad front) + 1 *C. t. b.* ♂ (bad front) + 1 *c. t. y.* ♂ + 1 *c. m. y.* ♀.

The above formula means:

A ticked buff crested cock with bad front to his crest (from mating no. 3. 1904 of ticked crested cock with clear crest-bred hen) mated with ticked yellow plainhead, bred the following young: one ticked yellow crested cock with bad front, one ticked buff crested cock with bad front, one ticked yellow crest-bred cock, one marked yellow crest-bred hen.

Matings and Progeny.

1891. 1. *c. v.* × *C. cl.* (*c. t.* × *C. v.*) = 1 *c. v.* ♂.

2. *C. d. y.* × *c. v. b.* = 3 *C. v.* + 2 *C. t. b.* × 1 *c. v. b.*

1892. 1. *C. m.* (*c.* × *C.*) × *c. v.* (*c* × *C.*) = 2 *C. m.* ♂ + 1 *C. gr.* ♂ + 1 *c. gr.* ♂ + 1 *c. v.* ♂.

2. *C.* × *c. v.* = 1 *c. v.* ♂ + 1 *c. cl.* ♀.

3. *C. m.* (*c.* × *C.*) × *c. v.* = 1 *C. t.* ♂ + 1 *c. v.* ♂.

4. *c. v.* (*c. v.* × *C. cl.*: 1. '91) × *C. v.* = *C. v.* ♀.

5. *c. v.* × *C. v.* = 1 *C. cl.* + 1 *C.*

6. *c. v. y.* × *C. gr. b.* (*c. y.* × *C. v. b.*) = 1 *C. d. y.* ♀ + 1 *C. gr.* ♀ + 2 *c. gr.* ♀ + 1 *c. y. gr.* ♂ + 1 *c. v.* ♀

7. *c. v.* × *C. cl.* (*c. t.* × *C. v.*) = 1 *C. v.* ♀ + 1. *c. v.* ♂.

8. *C. m.* (*c.* × *C.*) × *c. cl. Lan.* = 1 *C. d.* ♀ × 1 *c. cl.* ♀.

9. *C.* × *c. cl. Lan.* = 2 *c. cl.* (one ♀).

10. *C. d. y.* × *c. cl. b. Lan.* = 2 *C. d. y.*

1893. 1. *C. gr.* (*C. m.*×*c. v.*: 1. '92)×*c. cl.* (*C. m.*×*c. cl. Lan.*: 8. '92)=1 *C. m.*♂ +1 *C. m.*♀ +1 *c. v.*♂ +1 *c. v.*♀ +1 *c. m.*♂.
2. *C. m.* (*c.*×*C.*)×*c. cl.* (*C. m.*×*c. cl. Lan.*: 8. '92)=*C. m.*♂.
3. *c. v.*×*C. v.* (*c. v.*×*C. v.*: 4. '92)=1 *C. gr.*♂ +1 *c. v.*♂ +1 *c. cl.*♀.
4. *c. v.* (*c. v.*×*C. cl.*: 1. '91)×*C. cl.* (*c. t.*×*C. v.*)=3 *C. m.*♂ +1 *c. t.*♂.
5. *C. t.* (*C. m.*×*c. v.*: 3. '92)×*c. gr.* (*c. v. y.*×*C. gr. b.*: 6. '92)=1 *C. m.*♂ +1 *C. m.*+2 *c. v.*♀.
6. *c. gr. y.* (*c. v. y.*×*C. gr. b.*: 6. '92)×*C. d. b.* (*C. m.*×*c. cl. Lan.*: 8. '92)=*c. v.*
7. *C. m.* (*c*×*C*)×*C. gr.* (*c. v. y.*×*C. gr. b.*: 6. '92)=1 *C. gr.*♂ +1 *C. v.*♂ +1 *c. v.*♂ +1 *c. v.*♀ +1 *c. v.*
8. *c. v.* (*c. v.*×*C. cl.*: 7. '92)×*C. gr.* (*c. v. y.*×*C. gr. b.*: 6. '92)=*c. gr.*♂.
9. *c. v.*×*C. gr.* (*c. y.*×*C. v.*)=1 *C. gr.*♂ +1 *C. gr.*♀ +1 *C. v.*♀ +2 *c. v.*♂.
10. *C. m.* (*c.*×*C.*)×*C. gr.* (*c. y.*×*C. v. b.*)=2 *C. v.*♂.

1894. 1. *c. gr.* (*c. v.*×*C. gr.*: 9. '93)×*C. v.* (*c. v.*×*C. gr.*: 9. '93)=1 *c. v.*♀ +1 *c. cl.*♀.
2. *c. gr.* (*c. v.*×*C. gr.*: 9. '93)×*C. gr.* (*c. v. y.*×*C. gr. b.*: 6. '92)=1 *C. gr.*♂ +1 *c. gr.*♂.
3. *C. v.* (*C. m.*×*C. gr.*: 7. '93)×*c. cl.* (*C.*×*c. cl. Lan.*: 9. '92)=2 *C. v.*♀ +1 *C. v.*♂.
4. *c. gr. y.* (*c. v. y.*×*C. gr. b.*: 6. '92)×*C. m. b.* (*C. gr.*×*c. cl.*: 1. '93)=*c. v.*♂.

1895. 1. *C. t. y.* (*Lan.*)×*c. cl. b. Lan.*=*C. t.*+*c. m.*♂.
2. *C. v.* (*C. m.*×*C. gr.*: 7. '93)×*c. v.* (*c. gr.*×*C. v.*: 1. '94)=*C. v.*♀.
3. *c. v.* (*c. v.*×*C. cl.*: 1. '91)×*C. m.*=1 *C. m.*×1 *c. v.*♂ +1 *c. v.*+2 *c. m.*
4. *c. gr. y.* (*c. v. y.*×*C. gr. b.*: 6. '92)×*C. b. v.* 1 (*C. v.*×*c. cl.*: 3. '94)=*c. m. y.*

1896. 1. *C. gr.*×*C. v.* 2 (*C. v.*×*c. cl.*: 3. '94)=2 *C. v.*+1 *c. v.*
2. *C. t.*×*C. v.* (*C. v.*×*c. v.*: 2. '95)=2 *c. v.*
3. *c. v.* (*c. v.*×*C. m.*: 3. '95)×*C. t.*=1 *c. cl.*♂.
4. *c. v.* (*c. v.*×*C. cl.*: 7. '92)×*C. gr.*=1 *C. v.*♂ +1 *C. v.*♀ +1 *c. v.*+1 *c. v.*♀ +1 *c. gr.*♂ +1 *c. gr.*
5. *C. t.*×*c. gr.*=1 *c. gr.*+1 *c. t.*

1897. 1. *C. y.* (*Lan.*)×*C. v. b.* (*C. v.*×*c. v.*: 2. '95)=1 *C. m.*+1 *c. v. b.*
2. *C. m.* (*C. m.*×*c. v.*)×*c. v.* (*c. v.*×*C. gr.*: 4. '96)=1 *C. t.*+1 *c. v.*♂ +1 *c. v.*+1 *c. t.*
3. *c. gr. b.* (*c. v.*×*C. gr.*: 4. '96)×*C. t. y.* (*Lan.*)=1 *C. d. y*♀ +1 *C. m. y.*♀ +2 *C. m. b.* (bad fronts)+2 *c. v. y.*+1 *c. v. y.*♀ +2 *c. v. b.*+1. *c. v. y.*♂.
4. *c. m.* (*C. t. y. Lan.*×*c. cl. Lan.*)×*C. v.* (*c. v.*×*C. gr.*: 4. '96)=1 *c. gr.*♀.
5. *C. m.* (*C. m.*×*c. v.*)×*C. v.* (*c. v.*×*C. gr.*: 4. '96)=2 *C. v.*♂ +1 *C. m.*+1 *C. t.*♀.

1898. 1. *C. m.* (*C. m.*×*c. v.*)×*c. v.* (*c. v.*×*C. gr.*: 4. '96)=1 *C. gr.*♂ +1 *C. m.*♀.
2. *c. gr.* (*c. v.*×*C. gr.*: 4. '96)×*C. t.* (*C. m.*×*C. v.*: 5. '97)=1 *C. v.*♂ +1 *C. v.*♀ +1 *c. v.*♀ +1 *c. v.*
3. *c. cl.* (*c. v.*×*C. t.*: 3. '96)×*C. v.* (*c. v.*×*C. gr.*: 4. '96)=1 *C. m.*♂ ×1 *c. v.*♂ +1 *c. v.*♀.
4. *c. v. y.* (*c. gr. b.*×*C. t. y. Lan.*: 3. '97)×*C. b. v.* 2 (*C. v.*× *c. cl.*: 3. '94)=1 *C. m. y.*
5. *C. t. b.*×*C. m. y.* (*c. gr. b.*×*C. t. y. Lan.*: 3. '97)=1 *C. t.*+2 *C. v. b.*♂ +1 *C. v.*
6. *C. t. b.*×*c. v. y.* (*c. gr. b.*×*C. t. y. Lan.*: 3. '97)=1 *C. d.*♀ +1 *C. t. y.*♀ +1 *C. t.*♀ +1 *c. m. y.*♀ +1 *c. cl.*♀ +1 *c. t.*♀

1899. 1. *C. gr. b.* (*C. m.* × *c. v.*: 1. '98) × *c. m. y.* (*C. t. b.* × *c. v. y.*: 6. '98) = 1 *C.* + 2 *C. v. y.* ♀ + 1 *c. v. y.* ♂.

2. *C. gr.* (*C. m.* × *c. v.*: 1. '98) × *c. t.* (*C. t. b.* × *c. v. y.*: 6. '98) = *C. v.* ♂.

3. *C. t. b.* × *c. v. y.* (*c. gr. b.* × *C. t. y. Lan.*: 3. '97) = 1 *C. v. b.* ♀ + 2 *C. t. y.* ♀ + 2 *c. v.* ♂ + 1 *c. v. b.* ♀.

4. *c. v. y.* (*c. gr. b.* × *C. t. y. Lan.*: 3. '97) × *C. t. b.* (*C. m.* × *C. v.*: 5. '97) = *c. v. b.* ♂.

5. *C. m.* (*C. m.* × *c. v.*) × *c. v.* (*c. gr.* × *C. t.*: 2. '98) = *c. v.* ♀.

6. *C. v.* (*C. m.* × *C. v.*: 5. '97) × *c. gr.* (*c. m.* × *C. v.*: 4. '97) = *C. gr.* ♂ (bad front).

7. *c. gr.* (*c. v.* × *C. gr.*: 4. '96) × *C. t.* (*C. t. b.* × *c. v. y.*: 6. '98) = 2 *C. m.* (bad fronts) + 1 *c. m.*

8. *c. cl. b.* (*c. v.* × *C. t.*: 3. '96) × *C. m. y.* (*c. gr. b.* × *C. t. y. Lan.*: 3. '97) = *C. m. y.* ♀ + *C. t. b.* ♀ + *c. cl. b.* ♀.

1900. 1. *C. m.* (*C. m.* × *c. v.*) × *C. t.* (*C. t. b.* × *c. v. y.*: 6. '98) = *C. t.* ♀ + *c. cl.* ♀.

2. *c. gr. b.* (*c. v.* × *C. gr.*: 4. '96) × *C. v. y.* (*C. gr. b.* × *c. m. y.*: 1. '99) = *C. gr. b.* (rough).

3. *C. v. b.* (*C. gr.* × *c. t.*: 2. '99) × *C. t. y.* (*C. t. b.* × *c. v. y.*: 3. '99) = *C. t. y.*

4. *c. v. y.* (*c. gr. b.* × *C. t. y. Lan.*: 3. '97) × *C. v. b.* (*C. t. b.* × *c. v. y.*: 3. '99) = 1 *C. v. b.* ♂ + 1 *C. m.* (bad front) + 2 *C. t.* (one bad front).

5. *c. v.* (*C. t. b.* × *c. v. y.*: 3. '99) × *C. t.* (*c. cl. b.* × *C. m. y.*: 8. '99) = 1 *C. m.* ♂ + 1 *C. v.* ♀ + 1 *c. m.*

1901. 1. *c. cl.* (*c. v.* × *C. t.*: 3. '96) × *C. t.* (*C. m.* × *C. t.*: 1. 1900) = *c.*

2. *C. gr.* (*C. m.* × *c. v.*: 1. '98) × *C. t.* (*C. t. b.* × *c. v. y.*: 6. '98) = 1 *C. m.* ♂ + 1 *C. m.* ♀ + 1 *c. m.* ♂.

3. *C. v.* (*c. v. y.* × *C. v. b.*: 4. 1900) × *c. cl.* (*C. m.* × *C. t.*: 1. 1900) = *C. m.*

4 *c. v. y.* (*c. gr. b.* × *C. t. y. Lan.*: 3. '97) × *C. v. b.* (*C. t. b.* × *c. v. y.*: 3. '99) = 1 *C. v. b.* ♂ + 1 *C. gr. y.* ♂ (bad front) + 1 *C. m. b.* ♀ + 1 *C. v. y* ♀ + 1 *c. v. y* ♂ + 1 *c. v. y.* ♀ + 1 *c. v. y.* ♀ + 1 *c. v. b.* ♂ + 1 *c. m. b.* ♂ + 1 *c. cl. y.* ♀.

1902. 1. *C. v. b.* (*c. v. y.* × *C. v. b.*: 4. 1900) × *c. cl. y.* (*c. v. y.* × *C. v. b.*: 4. '01) = 2 *c. v.*

2. *c. cl.* (*c. v.* × *C. t.*: 3. '96) × *C. v.* (*C. t. b.* × *c. v. y.*: 3. '99) = 1 *C. m.* + 1 *C. t.* ♀ + 1 *c. v.*

3. *c. m.* (*C. gr.* × *C. t.*: 2. '01) × *C. t.* (*C. m.* × *C. t.*: 1. 1900) = 1 *C. m.* ♂ + 1 *C. m.* ♀ (both champions).

1903. 1. *c. v. y.* (*c. gr. b.* × *C. t. y. Lan.*: 3. '97) × *C. v. b.* (*C. t. b.* × *c. v. y.*: 3. '99) = 1 *c. m. y.* + 1 *c. v. y.*

2. *c. v.* × *C. t.* (*c. cl.* × *C. v.*: 2. '02) = 2 *C. m.* (both bad fronts) + *c. v.*

3. *c. m.* (*C. gr.* × *C. t.*: 2. '01) × *C. gr.* = 1 *c. gr.* ♂ + 1 *c. m.*

4. *c. cl.* (*c. v.* × *C. t.*: 3. '96) × *C. m.* = 1 *C. t.* ♂ + 1 *c. m.* ♀.

5. *c. m.* (*C. gr.* × *C. t.*: 2. '01) × *C. cl. Lan.* = 1 *C. cl.* ♂ + 1 *C. t.* + 1 *c. cl.*

1904. 1. *c. gr.* (*c. m.* × *C. gr.*: 3. '03) × *C. t.* = *c. v.* ♂.

2. *C. cl.* (*c. m.* × *C. cl. Lan.*: 5. '03) × *c. m.* = *c. v.* ♂ + *c. cl.* ♀.

3. *C. t.* (*c. cl.* × *C. m.*: 4. '03) × *c. cl.* = 1 *C. t.* ♂ (bad front) × 2 *cl.* ♀.

1905. 1. *C. m.* × *c. cl.* (*C. t.* × *c. cl.*: 3. '04) = 1 *C. v.* ♂ + 1 *C. t.* ♀ + 1 *c. v.* ♂ + 1 *c. m.* ♀.

2. *C. cl. b.* (*c. m.* × *C. cl. Lan.*: 5. '03) × *c. cl. y.* = 3 *C. cl.*

3. *C. t. Lan.* × *c. cl.* (*C. cl.* × *c. m.*: 2. '04) = *C. t* + *c. cl.*

4. *C. m.* × *c. m.* (*c. cl.* × *C. m.*: 4. '03) = 3 *C. v.* ♀ + 1 *C. t.* ♀ + 2 *c. m.* ♂ + 1 *c. m.* ♀ + 1 *c. cl.* ♂ + 3 *c. v.*

5. *C. t. b.* (bad front) (*C. t.* × *c. cl.*: 3. '04) × *Pl. t. y.* = 1 *C. t. y.* ♂ (bad front) + 1 *C. t. b.* ♂ (bad front) + 1 *c. t. y.* ♂ + 1 *c. m. y.* ♀.

1906. 1. *C. cl.* (*c. m.* × *C. cl. Lan.*: 5. '03) × *C. v.* 1 (*C. m.* × *c. m.*: 4. '05) = 1 *C. t.* ♂ + 1 *C. t.* + 2 *c. v.* ♀ + 1 *c. m.* ♀.
2. *C. cl.* (*c. m.* × *C. cl. Lan.*: 5. '03) × *C. v.* 2 (*C. m.* × *c. m.*: 4. '05) = 1 *C. t.* ♂ + 1 *C. t.* ♀ + 1 *C. cl.* ♀ + 1 *C. t.* (bad front) + 1 *C. t.* + 1 *c. m.* + 1 *c. m.* ♀ + 1 *c. cl.* ♂.
3. *c. gr.* (*c. m.* × *C. gr.*: 3. '03) × *C. t.* (*C. m.* × *c. m.*: 4. '05) = *C. v.*
4. *C. t. Lan.* × *c. m.* (*C. m.* × *c. cl.*: 1. '05) = 1 *C. m.* ♂ + 1 *c. v.* ♀ + 1 *c. v.* + 1 *c. v.* + 1 *c. cl.* ♀ + 1 *c. v.* + 1 *c. cl.*
5. *C. m.* × *c. m.* (*c. cl.* × *C. m.*: 4. '03) = 1 *C. v.* ♂ + 3 *C. v.* + 1 *c. cl.* ♀ + 2 *c.*

1907. 1. *C. cl. b.* (*c. m.* × *C. cl. Lan.*: 5. '03) × *self-cin. y.* = 1 *C. v. b.* ♂ (bad front) + 1 *C. v. y.* ♀ + 1 *c. v. b.* ♂.
2. *c. v. y.* (*c. cl. y.* × *self-cin. b.*) × *C. cl. b.* (*C. cl.* × *C. v.* 2: 2. '06) = 1 *C. v. y.* ♂ (bad front) + 1 *C. cl. y.* ♀ + 1 *c. cl. y.* ♀.
3. *c. v. y.* (*c. cl. y.* × *self-cin. b.*) × *C. t. b.* (*C. cl.* × *C. v.* 2: 2. '06) = 1 *C. v. y.* ♂ + 1 *C. v. y.* ♂.
4. *c. cl. y.* (*self-cin. y.* × *c. m. y.*) × *C. v. b.* 2 (*C. m.* × *c. m.*: 4. '05) = *C. m. y.* ♂ + *C. cin. m. b.* ♀ + *c. cl. y.* ♀.
5. *C. cl.* (*c. m.* × *C. cl. Lan.*: 5. '03) × *C. v.* 1 (*C. m.* × *c. m.*: 4. '05) = 1 *C. cl.* + 1 *C. v.* + 1 *C. cl.* + 1 *c. m.* + 1 *c. v.*
6. *C. t. b.* (*C. cl.* × *C. v.* 2: 2. '06) × *c. v. y.* (*c. cl. y.* × *self-cin. b.*) = 1 *C. t.* + 1 *C. v. y.* ♀ + 1 *C. t. y.* ♀ + 1 *c. v. b.* ♀ + 1 *c. v. b.* ♂ + 1 *c. m.* ♂ + 1 *c. m. y.* ♀ + 1 *c. cl.*
7. *C. cl.* (*c. m.* × *C. cl. Lan.*: 5. '03) × *c. cl.* (*C. m.* × *c. m.*: 5. '06) = 1 *C. t.* + 2 *C. cl.* + 1 *c. cl.*
8. *C. t.* (*C. cl.* × *C. v.* 1: 1. '06) × *c. v.* (*C. cl.* × *C. v.* 1: 1. '06) = 1 *C. v.*
9. *C. cl.* (*c. m.* × *C. cl. Lan.*: 5. '03) × *c. v.* (*C. cl.* × *C. v.* 1: 1. '06) = 1 *C. t.* + 1 *C. t.* + 1 *c. v.* ♀ + 1 *c. v.* ♀.
10. *c. cl. y.* (*Pl. cl. y.* × *c. m. b.*) × *C. t. b.* (*C. cl.* × *C. v.* 2: 2. '06) = 1 *C. t.* + 1 *C. t.* + 1 *c. cl. y.* ♀ + 3 *c. v.*

1908. 1. *C. m. y.* (*c. cl. y.* × *C. v. b.* 2: 4. '07) × *C. cl. b.* (*C. cl.* × *C. v.* 2: 2. '06) = 1 *C. m.* + 1 *C. cl.*
2. *c. v. b.* (*C. t. b.* × *c. v. y.*: 6. '07) × *C. v. y.* (*C. cl. b.* × *self-cin. y.*: 1. '07) = 1 *C. m. b.* ♀ + 1 *C. m. y.* ♀.
3. *C. cl.* (*c. m.* × *C. cl. Lan.*: 5. '03) × *C. v.* 1 (*C. m.* × *c. m.*: 4. '05) = 2 *C. t.* ♂ + 1 *C. v.* ♀.

B. *Cinnamon Matings, Dark-eyed Matings and Cross or Hybrid Matings.*

Terms. p. e. = pink-eyed on hatching and pure pink-eyed bred so far as known. *d. e.* = dark-eyed on hatching and pure dark-eyed bred so far as known. *F.* = hybrid nature of inheritance. Other abbreviations as in other tables. *Cin.* = Cinnamon and pink-eyed on hatching. *t.* (ticked) *m.* (marked), and *v.* (variegated) birds are dark-eyed on hatching.

I. *p. e.* × *d. e.* *Pink-eyed Male mated with Dark-eyed Female.*

1898. 1. *cl. y. p. e.* × *t. y.* = 1 *m. y.* ♂ + 1 *cin. v.* ♀ + 1 *cl. y. p. e.* ♀.
2. *cl. y. p. e.* × *cl. b. d. e.* = 2 *cl. y. d. e.* ♂ + 1 *cl. y. p. e.* + 1 *cl. b. p. e.*
3. *cl. y. p. e.* × *cl. b. d. e.* = 3 *cl. y. p. e.* ♀.

1904. 1. *cl. b. p. e.* × Linnet (*L. cannabina*) = 2 *dark* ♂ + 1 *self-cin.* ♀ (linnet mules).
2. *self-cin. b.* × *cl. y. d. e.* = 1 *v. b.* ♂ + 1 *m. y.* ♂ + 1 *v. b. cin.* ♀.

1906. 1. *cl. b. p. e.*×*cl. y. d. e.*=2 *t. y.* ♂ +1 *cl. b. d. e.* ♂.
2. *cl. b. p. e.*×*cl. y. d. e.*=1 *m. b.* ♂ +1 *cl. y. p. e.* ♀ +1 *t. cin. y.* ♀.
3. *cl. b. p. e.*×*m. y.*=1 *t. b. d. e.* ♂ +1 *cl. y. d. e.* ♂ +1 *cl. b. d. e.* ♂ +1 *cl. y. p. e.* ♀.
4. *self-cin. y.*×*v. y.*=1 *gr. y.* ♂ +1 *self-cin.* ♀ +1 *self-cin. y.* ♀ +1 *v. cin. y.* ♀.

1907. 1. *cl. b. p. e.*×*cl. y. d. e.*=1 *d. e.*+1 *cl. y. d. e.* ♂ +3 *cl. b. p. e.* ♀.
2. *self-cin. y.*×*t. b.*=1 *v. y.* ♂ +1 *v. b.* ♂ +1 *v. cin. y.* ♀ +1 *v. cin. b.* ♀.
3. *cl. y. p. e.*×*v. b.*=1 *v. y.* ♂ +1 *v. cin. b.* ♀ +1 *cl. y. p. e.* ♀.
4. *cl. b. p. e.*×*v. y.*=2 *v. y.* ♂ +1 *v. cin. y.* ♀ +1 *d. e.*+4 *p. e.*
5. *self-cin. b.*×*cl. y. d. e.*=3 *v. y.* ♂ +2 *d. e.*+1 *p. e.*

II. *p. e.* × *p. e.* *Pink-eyed Male mated with Pink-eyed Female.*

1898. *p. e.*×*cl. y. p. e.*=1 *cl. y. p. e.* ♂ +2 *cin. t. y.* ♀ +1 *cin. t. y.*

1904. 1. *self-cin. b.*×*cl. y. p. e.*=2 *cin. v. y.* ♀ +1 *cin. v. b.* ♀ +1 *cin. t. b.* ♀.
2. *self-cin. b.*×*cl. b. p. e.*=*cl. b. p. e.* ♂ +*cin. v. b.* ♀.

1905. 1. *cl. b. p. e.*×*cl. y. p. e.*=1 *cl. y. p. e.*+2 *cl. b. p. e.*+3 *cl. p. e.*
2. *self-cin. y.*×*self-cin. b.*=*self-cin. y.* ♂ +*self-cin. y.* ♀ +*self-cin. b.* ♂ +*self-cin b.* ♀ +1 *p. e.*

1908. *self-cin. b.*×*cl. y. p. e.*=2 *cin. v. y.* ♂ +3 *cin. v. b.* ♀ +1 *cin. v. y.* ♀.

1909. 1. *cin. t. b.*×*cl. y. p. e.*=3. *p. e.*
2. *self-cin. b.*×*self-cin. y.*=3 *p. e.*
3. *cin. t. y.*×*cin. t. b.*=2 *cl. p. e.*

III. *d. e.* × *p. e.* *Dark-eyed Male mated with Pink-eyed Female.*

1895. *cl. b. d. e.*×*cin. t.*=*cl. b. d. e.* ♂ +*cl. b. d, e.* ♀ +2 *cl. b. d. e.*+*t. b.*

1899. *d. e.*×*cin. t. y.*=1 *d. e.* (linnet mule).

1901. *t. y.*×*cin. t. y.*=*m. b.* ♂ +*m. y.* ♀ +2 *m.* ♀ +1 *cl. y. d. e.* ♀.

1902. *t. y.*×*cin. t. y.*=*v. y.* ♂ +*t. y.* ♀ +2 *cl. y.* ♀.

1904. 1. *d. e.*×*cl. y. p. e.*=*d. e.* ♂ +*d. e.* (goldfinch mules).
2. *d. e.*×*cin. v. y.*=8 *d. e.* (goldfinch mules).
3. *cl. y. d. e.*×*self-cin. y.*=*m. y.* ♂.
4. *cl. y. d. e.*×*self-cin. y.*=*v. y.* ♂ +*gr.* ♀ +*v.* ♀ +*v. y.* ♀.
5. *cl. y. d. e.*×*self-cin. b.*=*m. b.* ♂ +*v. b.* ♀ +*m.*

1905. 1. *cl. y. d. e.*×*cin. v. y.*=*m. y.* ♂ +*cl. y. d. e.* ♀ +*t. b.* ♀.
2. *d. e.*×*cin. v. y.*=3 *d. e.* (goldfinch mules).
3. *d. e.*×*cin. m. b.*=7 *d. e.* (goldfinch mules).
4. *d. e.*×*cin. v. y.* (*self-cin. b.*×*cl. y. p. e.*)=3 *d. e.* (siskin mules).
5. *d. e.*×*cin. v. y.* (*self-cin. b.*×*cl. y. p. e.*)=2 *d. e.* (goldfinch mules).
6. *cl. y. d. e.*×*self-cin. b.*=*v. y.* ♂ +2 *v. b.* ♂ +*v. y.* ♀.

1906. 1. *cl. y. d. e.* × *cin. m. b.* = 2 *v. y.* ♂ + *v. b.* ♂ + *t. y.* ♂ + *v. y.* ♀.

2. *d. e.* × *cin. m. b.* = 3 *d. e.* (siskin mules).

3. *d. e.* × *cin. m. b.* = 1 *d. e.* (goldfinch mule).

4. *cl. y. d. e.* × *self-cin. y.* = *m. y.* ♂ + *m. y.* + *v. b.* ♂ + 4 *v. y.* ♀.

1907. 1. *C. cl. b. d. e* × *self-cin. y.* = *C. v. b.* ♂ + *C. v. y.* ♀ + *c. v. b.* ♂.

2. *d. e.* × *cl. y. p. e.* = 6 *d. e.* (linnet mules).

3. *d. e.* × *cl. y. p. e.* = 1 *d. e.* (redpoll mule).

4. *t. y.* × *self-cin. b.* = 2 *v. b.* ♂ + 2 *v. y.*

5. *c. cl. d. e.* × *self-cin. y.* = *cl. d. e.*

6. *cl. b. d. e.* × *self-cin. y.* = *v.*

1908. 1. *d. e.* × *cin. m. y.* = 1 *d. e.* (siskin mule).

2. *d. e.* × *cl. y. p. e.* = 1 *d. e.* (siskin mule).

3. *d. e.* × *cl. y. p. e.* = 4 *d. e.* (linnet mules).

4. *d. e.* × *cin. v. y.* = 3 *d. e.* (siskin mules).

5. *d. e.* × *cin. v. b.* = 9 *d. e.* (linnet mules).

1909. 1. *d. e.* × *cl. b. p. e.* = 2 *d. e.* (siskin mules).

2. *d. e.* × *cl. b. p. e.* = 1 *d. e.* (redpoll mule).

3. *d. e.* × *cin. v. b.* = 2 *d. e.* (linnet mules).

III 1. *d. e.* × *p. e. F.* *Dark-eyed Male mated with Pink-eyed Female from Pink-eyed Father.*

1905. *d. e.* × *cin. v. b.* (*self-cin. b.* × *cl. y. d. e*) = 4 *d. e.* (siskin mules).

1907. 1. *d. e.* × *cin. v. y.* (*p. e.* × *d. e.*) = 4 *d. e.* (goldfinch mules).

2. *d. e.* × *cin. v. y.* (*p. e.* × *d. e.*) = 1 *d. e.* (siskin mule).

1908. 1. *d. e.* × *C. cl. y. p. e.* (*p. e.* × *d. e.*) = 3 *d. e.* (linnet mules).

2. *d. e.* × *cin. v. y.* (*p. e.* × *d. e. F.*) = 8 *d. e.* (linnet mules).

3. *d. e.* × *cin. v. b.* (*self-cin. y.* × *d. e.*) = 2 *d. e.* (siskin mules).

III 2. *d. e. F.* × *p. e.* *Dark-eyed Male from Pink-eyed Father mated with Pink-eyed Female.*

1908. *v. y.* (*self-cin. b.* × *d. e.*) × *cin. v. b.* (*self-cin. b.* × *cl. y. p. e.* = 1 *cin. v.* + 1 *cl. p. e.* + 1 *v.*

III 3. *d. e. F.* × *p. e.* *Dark-eyed Male from Pink-eyed Mother mated with Pink-eyed Female.*

1905. 1. *v. y.* (*d. e.* × *p. e.*) × *cl. b. p. e.* = 1 *cl. b. p. e* ♂. + 3 *cl.*

2. *m. y.* (*d. e.* × *p. e.*) × *self-cin. y.* = 3 *d. e.* + 2 *p. e.*

1909. *v. y.* (*d. e. F.* × *p. e.*) × *cin. v. b.* (*self-cin. b.* × *cl. y. p. e.*) = 1 *cin. v.* + 1 *cl. p. e.* + 1 *cl. d. e.*

III 4. *d. e. F.* × *p. e. F.* *Dark-eyed Male from Pink-eyed Father mated with Pink-eyed Female from Pink-eyed Father.*

1908. *v. y.* (*p. e.* × *d. e. F.*) × *cl. b. p. e.* (*p. e.* × *d. e.*) = *cin. t. b.* ♂ + *cl. b. p. e.* ♀.

III 5. *d. e. F.* × *p. e. F.* *Dark-eyed Male from Pink-eyed Mother mated with Pink-eyed Female from Pink-eyed Father.*

1905. *v. y.* (*d. e.* × *self-cin. b.*) × *cin. v. b.* (*self-cin. y.* × *d. e.*) = *cl. p. e.*
1908. *v. b.* (*d. e.* × *p. e.*) × *cin. v. y.* (*self-cin. y.* × *d. e.*) = 5 *d. e.*

IV. *d. e.* × *d. e.* *Dark-eyed Male mated with Dark-eyed Female.*

Vid. Matings of Crested Variety. Appendix A.

IV 1. *d. e.* × *d. e. F.* *Dark-eyed Male mated with Dark-eyed Female from Pink-eyed Mother.*

1905. *d. e.* × *cl. y. d. e.* (*d. e.* × *p. e.*) = 3 *d. e.* (goldfinch mules).
1906. *d. e.* × *cl. y. d. e.* (*d. e. F.* × *p. e.*) = 4 *d. e.* + 1 *d. e.* (goldfinch mules).
1908. *d. e.* × *m. y.* (*d. e.* × *self-cin. y.*) = 6 *d. e.* (goldfinch mules).
1909. *d. e.* × *m. y.* (*d. e.* × *self-cin. y*) = 2 *d. e.* (greenfinch mules).

IV 2. *d. e. F.* × *d. e. F.* *Dark-eyed Male from Pink-eyed Mother mated with Dark-eyed Female from Pink-eyed Mother.*

1905. *v. y.* (*d. e.* × *p. e.*) × *cl. y. d. e.* (*d. e.* × *p. e.*) = 2 *t. y.* ♂ + 1 *cl. y. p. e.* ♀.
1906. *v. y.* (*d. e.* × *p. e.*) × *cl. y. d. e.* (*d. e.* × *p. e.*) = 1 *v. y.* ♂ + 1 *cl. b. d. e.* ♂ + 1 *cl. y. d. e.* ♀ + 1 *cl. y. p. e.* ♀.
1907. 1. *v. y.* (*d. e.* × *self-cin.*) × *cl. b. d. e.* (*d. e.* × *p. e.*) = 1 *v. y.* ♂ + 1 *cl. y. p. e.* ♀ + 1 *cl. y. d. e.* ♀.
2. *v. y.* (*d. e.* × *self-cin.*) × *cl. b. d. e.* (*d. e.* × *p. e.*) = 2 *v. y.* ♂ + 5 *d. e.* + 2 *p. e.*
3. *m. b.* (*d. e.* × *self-cin.*) × *m. y.* (*d. e.* × *self-cin. y*) = 1 *d. e.* + 1 *p. e.*

IV 3. *d. e. F.* × *d. e.* *Dark-eyed Male from Pink-eyed Father mated with Dark-eyed Female.*

1905. *m. y.* (*self-cin. b.* × *cl. y. d. e.*) × *d. e.* (greenfinch ♀) = 4 *d. e.* ♂ + 1 *cin. v.* ♀ + 2 *d. e.* (greenfinch mules).
1908. *C. m. y.* (*p. e.* × *d. e.*) × *C. cl. b. d. e.* = *C. m.* + *C. cl. d. e.*
1909. *C. m. y.* (*p. e.* × *d. e.*) × *C. d. e.* = *C. gr.* + *c. cin. m.* + *c. cl. y. p. e.*

IV 4. *d. e. F.* × *d. e.* *Dark-eyed Male from Pink-eyed Mother mated with Dark-eyed Female.*

1904–07. *v. y.* (*d. e.* × *p. e.*) × *d. e.* (greenfinch ♀) = 4 *d. e.* (greenfinch mules).
1906. *m. y.* (*cl. y. d. e.* × *self-cin. y.*) × *d. e.* (greenfinch ♀) = *cin. v.* ♀ (greenfinch mule).
1908. 1. *m. y.* (*d. e.* × *p. e.*) × *d. e.* (greenfinch ♀) = 3 *d. e.* (greenfinch mules).
2. *m. y.* (*d. e.* × *p. e.*) × *d. e.* (siskin ♀) = 1 *d. e.* + 1 *p. e.* (siskin mules).

SUMMARY OF MATINGS[*].

I. *p. e.* × *d. e.*[†]

Matings 14. Progeny 56. Dark-Eyed ♂ 24, o 4 = 28. Pink-Eyed ♀ 21, o 7[‡] = 28.

II. *p. e.* × *p. e.*

Matings 9. Progeny 35. Pink-Eyed ♂ 9, ♀ 13, o 13 = 35. Dark-Eyed 0.

III. *d. e.* × *p. e.*

Matings 33. Progeny 110. Dark-Eyed 110. Pink-Eyed 0. Of the canaries there were ♂ 20, ♀ 21, o 9.

III 1. *d. e.* × *F. p. e.*

Matings 6. Progeny 22. Dark-Eyed 22. Pink-Eyed 0.

III 2. *F. d. e.* × *p. e.*

Mating 1. Progeny 3. Dark-Eyed 1. Pink-Eyed 2.

III. 3. *d. e. F.* × *p. e.*

Matings 3. Progeny 12. Dark-Eyed 4. Pink-Eyed 5. Doubtful 3.

III. 4. *F. d. e.* × *F. p. e.*

Mating 1. Progeny 2. Pink-Eyed 2.

III. 5. *d. e. F.* × *F. p. e.*

Matings 2. Progeny 6. Dark-Eyed 5. Pink-Eyed 1.

IV. *d. e.* × *d. e.*

Matings 91. Progeny 283. Dark-Eyed 283. Pink-Eyed 0.

IV. 1. *d. e.* × *d. e. F.*

Matings 4. Progeny 16. Dark-Eyed 16.

IV. 2. *d. e. F.* × *d. e. F.*

Matings 5. Progeny 21. Dark-Eyed ♂ 7, ♀ 2, o 6 = 15. Pink-Eyed ♀ 3, o 3 = 6.

IV. 3. *F. d. e.* × *d. e.*

Matings 3. Progeny 12. Dark-Eyed 9. Pink-Eyed 3.

IV. 4. *d. e. F.* × *d. e.*

Matings 4. Progeny 10. Dark-Eyed 8. Pink-Eyed ♀ 1, o 1 = 2.

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[*] ♂ = male, ♀ = female, o = sex unknown.

[†] These matings may also be considered from the standpoint of colour:

2. *y.* × *y.* matings = *y.* ♂ 2 + *y.* ♀ 3
4. *y.* × *b.* matings = *y.* ♂ 4   *b.* ♂ 1
y. ♀ 5   *b.* ♀ 2
Yellows 9   Buffs 3

Sex unascertained in 1 *y. p. e.* and 1 *b. p. e.* both probably ♀

7*b.* × *y.* = *y.* ♂ 10 + *b.* ♂ 5
y. ♀ 4   *b.* ♀ 4
Yellows 14   Buffs 9

Colour unascertained in 4 *d. e.* and 5 *p. e.*

[‡] This mating gave no certain black-eyed ♀s as in the case of Durham and Marryat, ***Report*** *to R. S. Evolution Co.* IV. p. 58, where also it is inaccurately stated that "pink-eyed hen and black-eyed cock gives all young of both sexes black-eyed."

DESCRIPTION OF PLATES.

PLATE I*. *Canaries and Canary Hybrids.*

1. Cinnamon-variegated Buff Canary ♂ × Greenfinch ♀ Hybrid showing Dutch Frill characteristics. A female.

2. The original Wild Canary (*Serinus canaria*).

3. Variegated Buff Siskin ♂ × Canary ♀ Male Hybrid, showing Lizard cap and silver-spangled back. Bred 1908. V.H.C. Crystal Palace, Feb. 1909, in Class 103 (Mixed Hybrids).

4. The White Canary. Bred by W. Kiesel. First prize, Class 66, Crystal Palace, 1909. The first white canary ever seen by living fanciers in this country. By kind permission of the owner, I examined the parents and relations of this bird, and found distinct traces of cinnamon inheritance.

PLATE II*. *Wild Sports.*

1. Almost clear Chaffinch (*Fringilla coelebs*) ♂.

2. Rich Self-cinnamon Greenfinch (*Ligurinus chloris*). One of several living examples in my possession—all females.

3. Very pale Cinnamon (creamy-white) Blackbird (*Turdus merula*), female (examined post-mortem). Other two specimens of a similar nature were shown in Class 123, Crystal Palace, 1909—both females. Also in same class a *pure* white one, with orange bill and red eyes, had third prize—undoubtedly a male specimen.

4. Clear Yellow Yellowhammer (*Emberiza citrinella*)—bright red eyes. At present alive and in my possession. Probably a female.

PLATE III*. *Canary Hybrids.*

1. Clear Yellow Siskin-Canary Hybrid—the only one known. Male. 1st prize, City of Glasgow, 1908; 1st prize, Class 64, Glasgow and West of Scotland, 1908; 1st prize, Class 34, Aberdeen, 1908; 1st prize, Class 87, Scottish National (Edinburgh), 1908–9; 1st prize, Class 95, Crystal Palace, 1909. Also championship diploma and two special prizes for best hybrid, Crystal Palace, 1909.

2. Dark Buff Siskin-Canary Hybrid. Male. I reared twenty-two of these—yellows and buffs—in 1908.

3. Ticked Buff Twite-Canary Hybrid. Male. 2nd prize, Class 65, Glasgow and West of Scotland, 1908; 1st prize, Class 89, Scottish National (Edinburgh), 1908–09; 3rd prize, Class 95, Crystal Palace, 1909.

4. Clear Buff Greenfinch-Canary Hybrid. Male. 2nd prize, Class 64, Glasgow and West of Scotland, 1908; 2nd prize, Class 34, Aberdeen, 1908; 2nd prize, Class 87, Scottish National (Edinburgh), 1908–09; 4th prize, Class 95, Crystal Palace, 1909.

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* From original water-colour drawings by H. Norman.

PLATE IV*. *Canary Varieties.*

1. Buff, wing-marked Dark Crested Canary ♂.
2. Yellow (*Jonque*) London Fancy Canary ♂, in show plumage.
3. Gold Lizard Canary ♂, in show plumage and colour-fed.
4. Clear Yellow Crest-Bred Plainhead Canary ♂.

PLATE V.

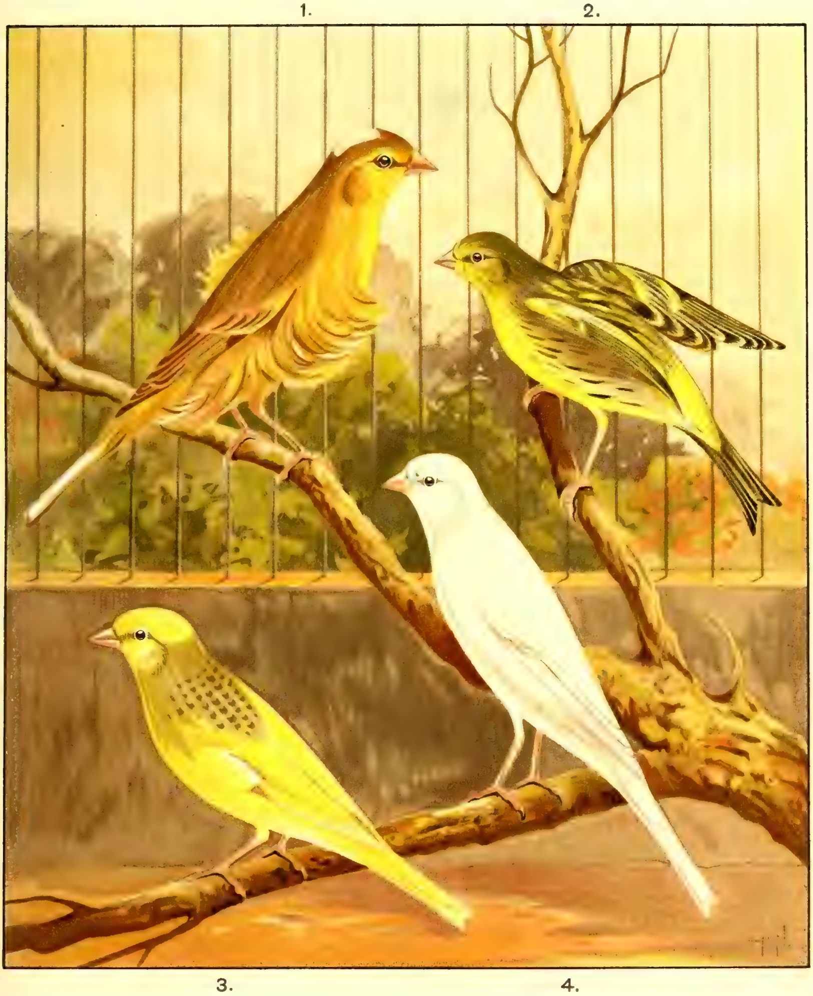
Fig. 1. Variegated Buff Dutch Frill Canary.

Fig. 6. Trumpeter Pigeon.

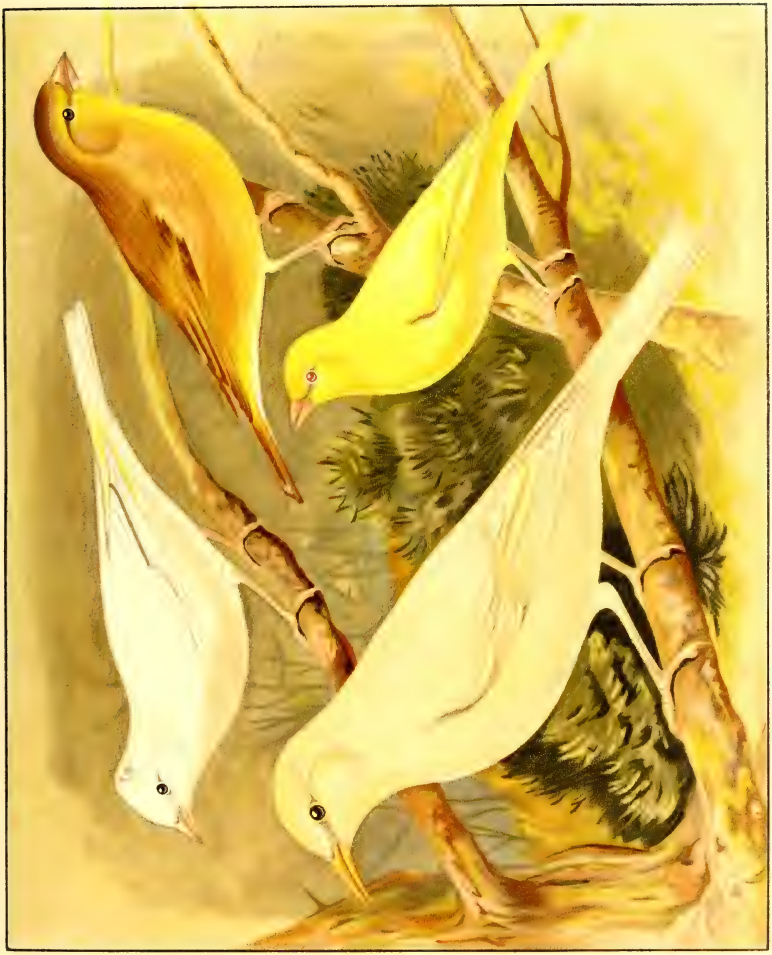
ILLUSTRATIONS IN THE TEXT.

Fig. 2. Variegated Crest-Bred Norwich Plainhead. See p. 22.
Fig. 3. Heavily Variegated Norwich Crest. See p. 22.
Fig. 4. Typical Lancashire Coppy. See p. 23.
Fig. 5. A Study in Crests. See p. 24.

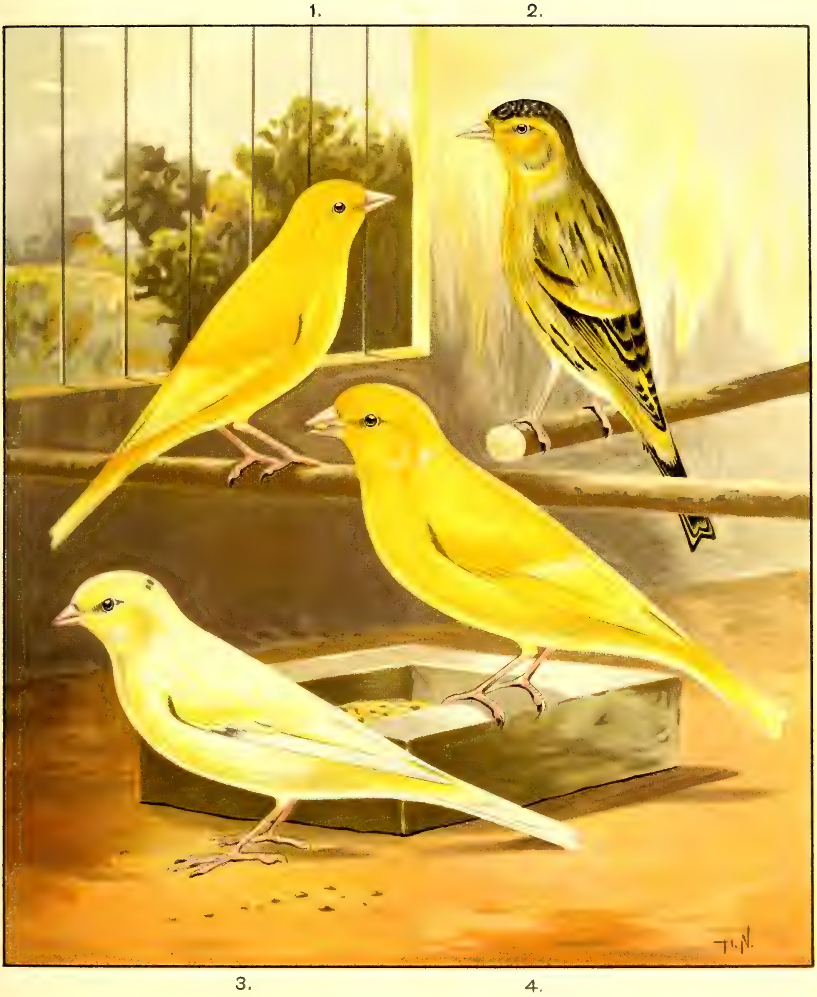
* By permission of *Canary and Cage Bird Life.*



Canary and Canary Hybrids.



Wild Sports.



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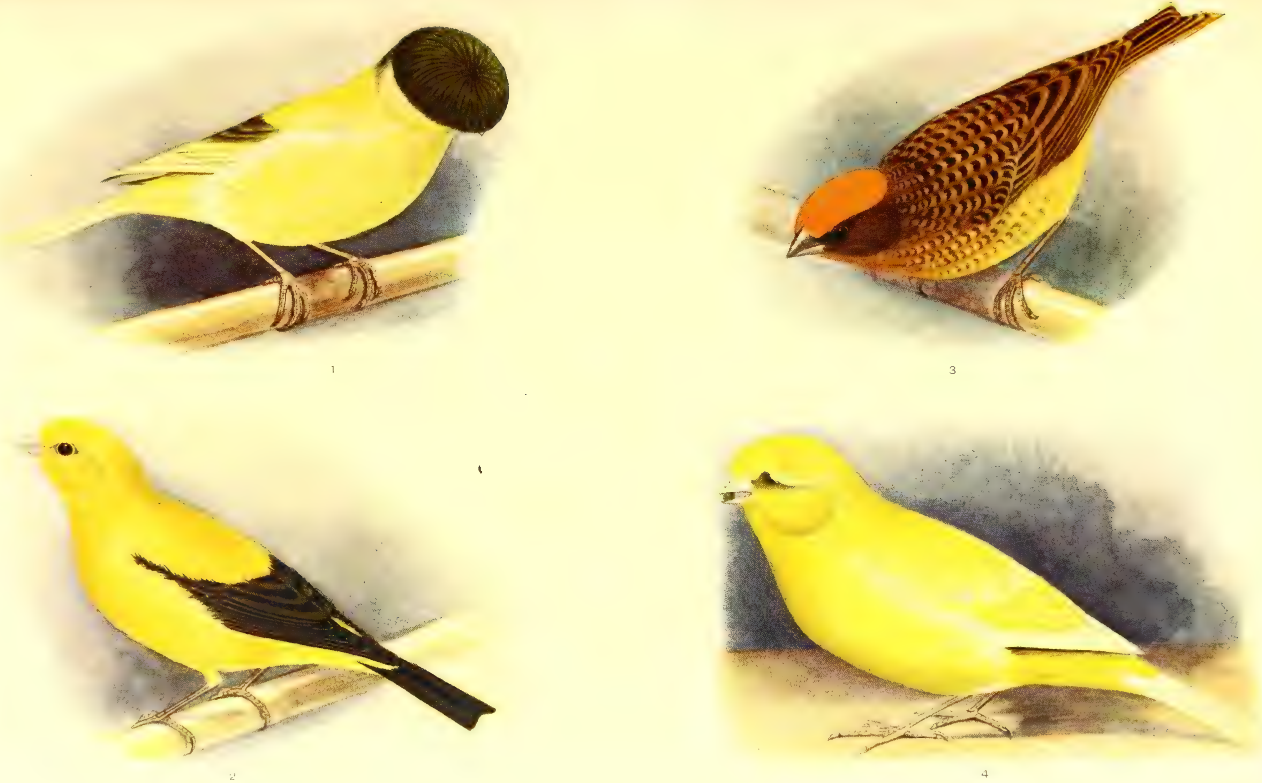

CANARY VARIETIES.

1. CRESTED. 2. LONDON FANCY. 3. LIZARD. 4. CREST-BRED

(Reprinted by kind permission from "CANARY AND CAGE-BIRD LIFE.")

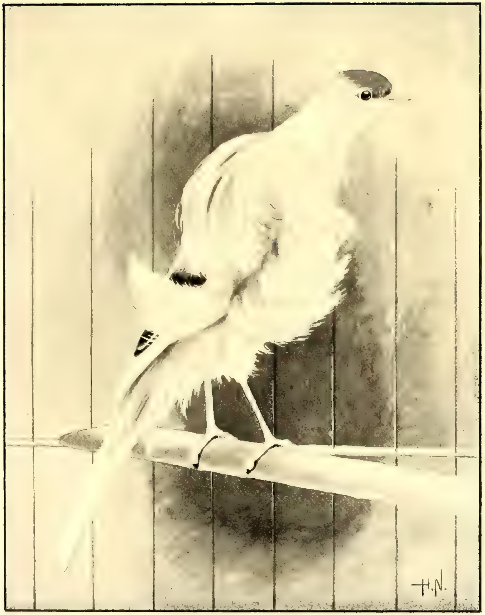

Fig. 1. Variegated buff Dutch Frill Canary. First Prize, Crystal Palace, February 1909. From drawing by H. Norman.

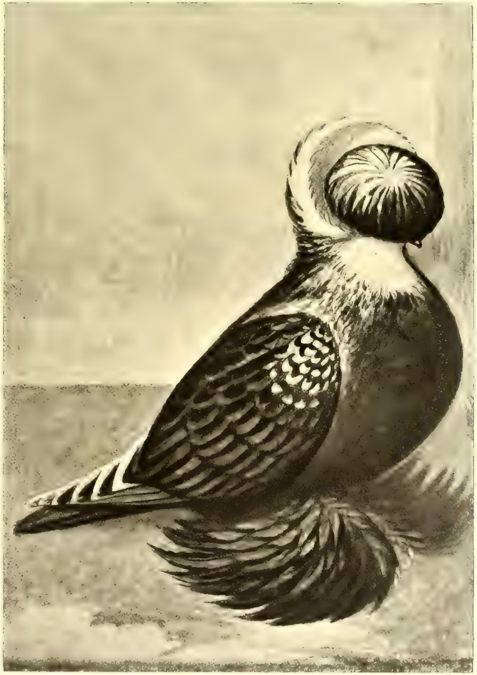

Fig. 6. Trumpeter Pigeon. To illustrate Rose Crest and Shell Crest. By permission of *Feathered World*.

TABLES OF THE Γ-FUNCTION.

THE following tables have been prepared by Mr J. H. Duffell, Associate of the Institute of Actuaries, from the standard tables in Legendre's *Traité des Fonctions Elliptiques*, Paris, 1825—8. Those tables give the values of the log Γ-functions to 12 places of decimals. The present tables provide the values of $\log \Gamma(p)$ from $p = 1{\cdot}000$ to $2{\cdot}000$ proceeding by differences of ·001, and recording *seven* figures. The tabled first differences are the differences of the recorded values, and not the nearest value to 7 figures of the differences of Legendre's tabled values*. It is very usual for biometricians to work with tables of 7 figure logarithms, and the need of a table of $\log \Gamma(p)$ to 7 figures has been long experienced. The original Legendre's table is in a scarce and expensive work and goes to more figures than are required in biometric practice. A number of six-figure $\Gamma(p)$ function tables have been issued, notably the clear table of Mr W. Palin Elderton (*Frequency Curves and Correlation*, C. and E. Layton, 1906). But in view of the forthcoming issue by *Biometrika* of "Tables for Statisticians and Biometricians," it seemed desirable to have stereotyped plates of the log Γ-function to 7 figures, as this number is the working standard adopted in this Journal's numerical tables.

For every value of p greater than 5, and for almost all practical purposes for any value of p greater than 2, we can readily calculate $\log \Gamma(p)$ from the formula†:

$$\log \frac{\Gamma(x+1)}{x^x e^{-x}} = 0{\cdot}399{,}0899 + \tfrac{1}{2}\log x + {\cdot}080{,}929 \sin \frac{25^\circ{\cdot}623}{x},$$

which has the advantage of giving $\Gamma(x+1)/(x^x e^{-x})$, the quantity usually required in biometric work.

For values between 2 and 5 of p, or for values of p less than unity, the formula of reduction $\Gamma(p+1) = p\,\Gamma(p)$ and the present table must still be used.

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* W. Palin Elderton has kindly tested this point theoretically, and finds that the differences of the recorded values are the best to use in such cases. ED.

† Pearson, *Biometrika*, Vol VI. p. 119. In the example line 19 for "$\log x^x e^{-x} = {\cdot}848{,}4081$" read "$\log x^x e^{-x} = 3{\cdot}526{,}3058$."

p	Log Γ (*p*), Negative Characteristic, $\bar{1}$ *0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
1·00	·999,9999	7497	5001	2512	0030	·998,7555	5087	2627	0173	$-\overline{7727}$
1·01	·997,5287	2855	0430	$-\overline{8011}$	$-\overline{5600}$	·996,3196	0798	$-\overline{8408}$	$-\overline{6025}$	$\overline{3648}$
1·02	·995,1279	$-\overline{8916}$	$-\overline{6561}$	$-\overline{4212}$	$-\overline{1870}$	·993,9535	7207	4886	2572	0265
1·03	·992,7964	5671	3384	1104	$-\overline{8831}$	·991,6564	4305	2052	$-\overline{9806}$	$-\overline{7567}$
1·04	·990,5334	3108	0889	$-\overline{8677}$	$-\overline{6471}$	·989,4273	2080	$-\overline{9895}$	$-\overline{7716}$	$-\overline{5544}$
1·05	·988,3379	1220	$-\overline{9068}$	$-\overline{6922}$	$-\overline{4783}$	·987,2651	0525	$-\overline{8406}$	$-\overline{6294}$	$-\overline{4188}$
1·06	·986,2089	$-\overline{9996}$	$-\overline{7910}$	$-\overline{5830}$	$-\overline{3757}$	·985,1690	$-\overline{9630}$	$-\overline{7577}$	$-\overline{5530}$	$-\overline{3489}$
1·07	·984,1455	$-\overline{9428}$	$-\overline{7407}$	$-\overline{5392}$	$-\overline{3384}$	·983,1382	$-\overline{9387}$	$-\overline{7398}$	$-\overline{5415}$	$-\overline{3439}$
1·08	·982,1469	$-\overline{9506}$	$-\overline{7549}$	$-\overline{5599}$	$-\overline{3655}$	·981,1717	$-\overline{9785}$	$-\overline{7860}$	$-\overline{5941}$	$-\overline{4029}$
1·09	·980,2123	0223	$-\overline{8329}$	$-\overline{6442}$	$-\overline{4561}$	·979,2686	0818	$-\overline{8956}$	$-\overline{7100}$	$-\overline{5250}$
1·10	·978,3407	1570	$-\overline{9738}$	$-\overline{7914}$	$-\overline{6095}$	·977,4283	2476	0676	$-\overline{8882}$	$-\overline{7095}$
1·11	·976,5313	3538	1768	0005	$-\overline{8248}$	·975,6497	4753	3014	1281	$-\overline{9555}$
1·12	·974,7834	6120	4411	2709	1013	·973,9323	7638	5960	4288	2622
1·13	·973,0962	$-\overline{9308}$	$-\overline{7659}$	$-\overline{6017}$	$-\overline{4381}$	·972,2751	1126	$-\overline{9508}$	$-\overline{7896}$	$-\overline{6289}$
1·14	·971,4689	3094	1505	$-\overline{9922}$	$-\overline{8345}$	·970,6774	5209	3650	2096	0549
1·15	·969,9007	7471	5941	4417	2898	·969,1386	$-\overline{9879}$	$-\overline{8378}$	$-\overline{6883}$	$-\overline{5393}$
1·16	·968,3910	2432	0960	$-\overline{9493}$	$-\overline{8033}$	·967,6578	5129	3686	2248	0816
1·17	·966,9390	7969	6554	5145	3742	·966,2344	0952	$-\overline{9566}$	$-\overline{8185}$	$-\overline{6810}$
1·18	·965,5440	4076	2718	1366	0019	·964,8677	7341	6011	4687	3368
1·19	·964,2054	0746	$-\overline{9444}$	$-\overline{8147}$	$-\overline{6856}$	·963,5570	4290	3016	1747	0483
1·20	·962,9225	7973	6725	5484	4248	·962,3017	1792	0573	$-\overline{9358}$	$-\overline{8150}$
1·21	·961,6946	5748	4556	3369	2188	·961,1011	$-\overline{9841}$	$-\overline{8675}$	$-\overline{7515}$	$-\overline{6361}$
1·22	·960,5212	4068	2930	1796	0669	·959,9546	8430	7318	6212	5111
1·23	·959,4015	2925	1840	0760	$-\overline{9685}$	·958,8616	7553	6494	5441	4393
1·24	·958,3350	2313	1280	0253	$-\overline{9232}$	·957,8215	7204	6198	5197	4201
1·25	·957,3211	2226	1246	0271	$-\overline{9301}$	·956,8337	7377	6423	5474	4530
1·26	·956,3592	2658	1730	0806	$-\overline{9888}$	·955,8975	8067	7165	6267	5374
1·27	·955,4487	3604	2727	1855	0988	·955,0126	$-\overline{9268}$	$-\overline{8416}$	$-\overline{7570}$	$-\overline{6728}$
1·28	·954,5891	5059	4232	3410	2593	·954,1782	0975	0173	$-\overline{9376}$	$-\overline{8585}$
1·29	·953,7798	7016	6239	5467	4700	·953,3938	3181	2429	1682	0940
1·30	·953,0203	$-\overline{9470}$	$-\overline{8743}$	$-\overline{8021}$	$-\overline{7303}$	·952,6590	5883	5180	4482	3789
1·31	·952,3100	2417	1739	1065	0396	·951,9732	9073	8419	7770	7125
1·32	·951,6485	5850	5220	4595	3975	·951,3359	2748	2142	1541	0944
1·33	·951,0353	$-\overline{9766}$	$-\overline{9184}$	$-\overline{8606}$	$-\overline{8034}$	·950,7466	6903	6344	5791	5242
1·34	·950,4698	4158	3624	3094	2568	·950,2048	1532	1021	0514	0012
1·35	·949,9515	9023	8535	8052	7573	·949,7100	6630	6166	5706	5251
1·36	·949,4800	4355	3913	3477	3044	·949,2617	2194	1776	1362	0953
1·37	·949,0549	0149	$-\overline{9754}$	$-\overline{9363}$	$-\overline{8977}$	·948,8595	8218	7846	7478	7115
1·38	·948,6756	6402	6052	5707	5366	·948,5030	4698	4371	4049	3731
1·39	·948,3417	3108	2803	2503	2208	·948,1916	1630	1348	1070	0797
1·40	·948,0528	0263	0003	$-\overline{9748}$	$-\overline{9497}$	·947,9250	9008	8770	8537	8308
1·41	·947,8084	7864	7648	7437	7230	·947,7027	6829	6636	6446	6261
1·42	·947,6081	5905	5733	5565	5402	·947,5243	5089	4939	4793	4652
1·43	·947,4515	4382	4254	4130	4010	·947,3894	3783	3676	3574	3476
1·44	·947,3382	3292	3207	3125	3049	·947,2976	2908	2844	2784	2728
1·45	·947,2677	2630	2587	2549	2514	·947,2484	2459	2437	2419	2406
1·46	·947,2397	2393	2392	2396	2404	·947,2416	2432	2452	2477	2506
1·47	·947,2539	2576	2617	2662	2712	·947,2766	2824	2886	2952	3022
1·48	·947,3097	3175	3258	3345	3436	·947,3531	3630	3734	3841	3953
1.49	·947,4068	4188	4312	4440	4572	947,4708	4848	4992	5141	5293
1.50	·947,5449	5610	5774	5943	6116	·947,6292	6473	6658	6847	7040

A horizontal bar means that the third figure of the mantissa has changed, a negative sign that it must be *lowered* one unit.

DIFFERENCES :—NEGATIVE down to rule										
0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*p*
2503	2496	2489	2482	2475	2468	2460	2454	2446	2440	*1·00*
2432	2425	2419	2411	2404	2398	2390	2383	2377	2369	*1·01*
2363	2355	2349	2342	2335	2328	2321	2314	2307	2301	*1·02*
2293	2287	2280	2273	2267	2259	2253	2246	2239	2233	*1·03*
2226	2219	2212	2206	2198	2193	2185	2179	2172	2165	*1·04*
2159	2152	2146	2139	2132	2126	2119	2112	2106	2099	*1·05*
2093	2086	2080	2073	2067	2060	2053	2047	2041	2034	*1·06*
2027	2021	2015	2008	2002	1995	1989	1983	1976	1970	*1·07*
1963	1957	1950	1944	1938	1932	1925	1919	1912	1906	*1·08*
1900	1894	1887	1881	1875	1868	1862	1856	1850	1843	*1·09*
1837	1832	1824	1819	1812	1807	1800	1794	1787	1782	*1·10*
1775	1770	1763	1757	1751	1744	1739	1733	1726	1721	*1·11*
1714	1709	1702	1696	1690	1685	1678	1672	1666	1660	*1·12*
1654	1649	1642	1636	1630	1625	1618	1612	1607	1600	*1·13*
1595	1589	1583	1577	1571	1565	1559	1554	1547	1542	*1·14*
1536	1530	1524	1519	1512	1507	1501	1495	1490	1483	*1·15*
1478	1472	1467	1460	1455	1449	1443	1438	1432	1426	*1·16*
1421	1415	1409	1403	1398	1392	1386	1381	1375	1370	*1·17*
1364	1358	1352	1347	1342	1336	1330	1324	1319	1314	*1·18*
1308	1302	1297	1291	1286	1280	1274	1269	1264	1258	*1·19*
1252	1248	1241	1236	1231	1225	1219	1215	1208	1204	*1·20*
1198	1192	1187	1181	1177	1170	1166	1160	1154	1149	*1·21*
1144	1138	1134	1127	1123	1116	1112	1106	1101	1096	*1·22*
1090	1085	1080	1075	1069	1063	1059	1053	1048	1043	*1·23*
1037	1033	1027	1021	1017	1011	1006	1001	996	990	*1·24*
985	980	975	970	964	960	954	949	944	938	*1·25*
934	928	924	918	913	908	902	898	893	887	*1·26*
883	877	872	867	862	858	852	846	842	837	*1·27*
832	827	822	817	811	807	802	797	791	787	*1·28*
782	777	772	767	762	757	752	747	742	737	*1·29*
733	727	722	718	713	707	703	698	693	689	*1·30*
683	678	674	669	664	659	654	649	645	640	*1·31*
635	630	625	620	616	611	606	601	597	591	*1·32*
587	582	578	572	568	563	559	553	549	544	*1·33*
540	534	530	526	520	516	511	507	502	497	*1·34*
492	488	483	479	473	470	464	460	455	451	*1·35*
445	442	436	433	427	423	418	414	409	404	*1·36*
400	395	391	386	382	377	372	368	363	359	*1·37*
354	350	345	341	336	332	327	322	318	314	*1·38*
309	305	300	295	292	286	282	278	273	269	*1·39*
265	260	255	251	247	242	238	233	229	224	*1·40*
220	216	211	207	203	198	193	190	185	180	*1·41*
176	172	168	163	159	154	150	146	141	137	*1·42*
133	128	124	120	116	111	107	102	98	94	*1·43*
90	85	82	76	73	68	64	60	56	51	*1·44*
47	43	−38	−35	−30	−25	−22	−18	−13	− 9	*1·45*
− 4	− 1	+ 4	+ 8	+12	+16	+20	+25	+29	+33	*1·46*
+37	+41	45	50	54	58	62	66	70	75	*1·47*
78	83	87	91	95	99	104	107	112	115	*1·48*
120	124	128	132	136	140	144	149	152	156	*1·49*
161	164	169	173	176	181	185	189	193	197	*1·50*

* Differences change sign at horizontal rule.

p	Log Γ (p), Negative Characteristic, $\bar{1}$									
	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
1·51	·947,7237	7437	7642	7851	8064	·947,8281	8502	8727	8956	9189
1·52	·947,9426	9667	9912	+$\overline{0161}$	+$\overline{0414}$	·948,0671	0932	1196	1465	1738
1·53	·948,2015	2295	2580	2868	3161	·948,3457	3758	4062	4370	4682
1·54	·948,4998	5318	5642	5970	6302	·948,6638	6977	7321	7668	8019
1·55	·948,8374	8733	9096	9463	9834	·949,0208	0587	0969	1355	1745
1·56	·949,2139	2537	2938	3344	3753	·949,4166	4583	5004	5429	5857
1·57	·949,6289	6725	7165	7609	8056	·949,8508	8963	9422	9885	+$\overline{0351}$
1·58	·950,0822	1296	1774	2255	2741	·950,3230	3723	4220	4720	5225
1·59	·950,5733	6245	6760	7280	7803	·950,8330	8860	9395	9933	+$\overline{0475}$
1·60	·951,1020	1569	2122	2679	3240	·951,3804	4372	4943	5519	6098
1·61	·951,6680	7267	7857	8451	9048	·951,9649	+$\overline{0254}$	+$\overline{0862}$	+$\overline{1475}$	+$\overline{2091}$
1·62	·952,2710	3333	3960	4591	5225	·952,5863	6504	7149	7798	8451
1·63	·952,9107	9766	+$\overline{0430}$	+$\overline{1097}$	+$\overline{1767}$	·953,2442	3120	3801	4486	5175
1·64	·953,5867	6563	7263	7966	8673	·953,9383	+$\overline{0097}$	+$\overline{0815}$	+$\overline{1536}$	+$\overline{2260}$
1·65	·954,2989	3721	4456	5195	5938	·954,6684	7434	8187	8944	9704
1·66	·955,0468	1236	2007	2782	3560	·955,4342	5127	5916	6708	7504
1·67	·955,8303	9106	9913	+$\overline{0723}$	+$\overline{1536}$	·956,2353	3174	3998	4825	5656
1·68	·956,6491	7329	8170	9015	9864	·957,0716	1571	2430	3293	4159
1·69	·957,5028	5901	6777	7657	8540	·957,9427	+$\overline{0317}$	+$\overline{1211}$	+$\overline{2108}$	+$\overline{3008}$
1·70	·958,3912	4820	5731	6645	7563	·958,8484	9409	+$\overline{0337}$	+$\overline{1268}$	+$\overline{2203}$
1·71	·959,3141	4083	5028	5977	6929	·959,7884	8843	9805	+$\overline{0771}$	+$\overline{1740}$
1·72	·960,2712	3688	4667	5650	6636	·960,7625	8618	9614	+$\overline{0613}$	+$\overline{1616}$
1·73	·961,2622	3632	4645	5661	6681	·961,7704	8730	9760	+$\overline{0793}$	+$\overline{1830}$
1·74	·962,2869	3912	4959	6009	7062	·962,8118	9178	+$\overline{0241}$	+$\overline{1308}$	+$\overline{2378}$
1·75	·963,3451	4527	5607	6690	7776	·963,8866	9959	+$\overline{1055}$	+$\overline{2155}$	+$\overline{3258}$
1·76	·964,4364	5473	6586	7702	8821	·964,9944	+$\overline{1070}$	+$\overline{2199}$	+$\overline{3331}$	+$\overline{4467}$
1·77	·965,5606	6749	7894	9043	+$\overline{0195}$	·966,1350	2509	3671	4836	6004
1·78	·966,7176	8351	9529	+$\overline{0710}$	+$\overline{1895}$	·967,3082	4274	5468	6665	7866
1·79	·967,9070	+$\overline{0277}$	+$\overline{1488}$	+$\overline{2701}$	+$\overline{3918}$	·968,5138	6361	7588	8818	+$\overline{0051}$
1·80	·969,1287	2526	3768	5014	6263	·969,7515	8770	+$\overline{0029}$	+$\overline{1291}$	+$\overline{2555}$
1·81	·970,3823	5095	6369	7646	8927	·971,0211	1498	2788	4082	5378
1·82	·971,6678	7981	9287	+$\overline{0596}$	+$\overline{1908}$	·972,3224	4542	5864	7189	8517
1·83	·972,9848	+$\overline{1182}$	+$\overline{2520}$	+$\overline{3860}$	+$\overline{5204}$	·973,6551	7900	9254	+$\overline{0610}$	+$\overline{1969}$
1·84	·974,3331	4697	6065	7437	8812	·975,0190	1571	2955	4342	5733
1·85	·975,7126	8522	9922	+$\overline{1325}$	+$\overline{2730}$	·976,4139	5551	6966	8384	9805
1·86	·977,1230	2657	4087	5521	6957	·977,8397	9839	+$\overline{1285}$	+$\overline{2734}$	+$\overline{4186}$
1·87	·978,5640	7098	8559	+$\overline{0023}$	+$\overline{1490}$	·979,2960	4433	5909	7389	8871
1·88	·980,0356	1844	3335	4830	6327	·980,7827	9331	+$\overline{0837}$	+$\overline{2346}$	+$\overline{3859}$
1·89	·981,5374	6893	8414	9939	+$\overline{1466}$	·982,2996	4530	6066	7606	9148
1·90	·983,0693	2242	3793	5348	6905	·983,8465	+$\overline{0028}$	+$\overline{1595}$	+$\overline{3164}$	+$\overline{4736}$
1·91	·984,6311	7890	9471	+$\overline{1055}$	+$\overline{2642}$	·985,4232	5825	7421	9020	+$\overline{0621}$
1·92	·986,2226	3834	5445	7058	8675	·987,0294	1917	3542	5170	6802
1·93	·987,8436	+$\overline{0073}$	+$\overline{1713}$	+$\overline{3356}$	+$\overline{5002}$	·988,6651	8302	9957	+$\overline{1614}$	+$\overline{3275}$
1.94	·989,4938	6605	8274	9946	+$\overline{1621}$	·990,3299	4980	6663	8350	+$\overline{0039}$
1·95	·991,1732	3427	5125	6826	8530	·992,0237	1947	3659	5375	7093
1·96	·992,8815	+$\overline{0539}$	+$\overline{2266}$	+$\overline{3995}$	+$\overline{5728}$	·993,7464	9202	+$\overline{0943}$	+$\overline{2688}$	+$\overline{4435}$
1·97	·994,6185	7937	9693	+$\overline{1451}$	+$\overline{3213}$	·995,4977	6744	8513	+$\overline{0286}$	+$\overline{2062}$
1·98	·996,3840	5621	7405	9192	+$\overline{0982}$	·997,2774	4569	6368	8169	9972
1·99	·998,1779	3588	5401	7216	9034	·999,0854	2678	4504	6333	8165

A horizontal bar means that the third figure of the mantissa has changed, a positive sign that it must be *raised* one unit.

DIFFERENCES :—on this page, POSITIVE										
0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*p*
200	205	209	213	217	221	225	229	233	237	*1·51*
241	245	249	253	257	261	264	269	273	277	*1·52*
280	285	288	293	296	301	304	308	312	316	*1·53*
320	324	328	332	336	339	344	347	351	355	*1·54*
359	363	367	371	374	379	382	386	390	394	*1·55*
398	401	406	409	413	417	421	425	428	432	*1·56*
436	440	444	447	452	455	459	463	466	471	*1·57*
474	478	481	486	489	493	497	500	505	508	*1·58*
512	515	520	523	527	530	535	538	542	545	*1·59*
549	553	557	561	564	568	571	576	579	582	*1·60*
587	590	594	597	601	605	608	613	616	619	*1·61*
623	627	631	634	638	641	645	649	653	656	*1·62*
659	664	667	670	675	678	681	685	689	692	*1·63*
696	700	703	707	710	714	718	721	724	729	*1·64*
732	735	739	743	746	750	753	757	760	764	*1·65*
768	771	775	778	782	785	789	792	796	799	*1·66*
803	807	810	813	817	821	824	827	831	835	*1·67*
838	841	845	849	852	855	859	863	866	869	*1·68*
873	876	880	883	887	890	894	897	900	904	*1·69*
908	911	914	918	921	925	928	931	935	938	*1·70*
942	945	949	952	955	959	962	966	969	972	*1·71*
976	979	983	986	989	993	996	999	1003	1006	*1·72*
1010	1013	1016	1020	1023	1026	1030	1033	1037	1039	*1·73*
1043	1047	1050	1053	1056	1060	1063	1067	1070	1073	*1·74*
1076	1080	1083	1086	1090	1093	1096	1100	1103	1106	*1·75*
1109	1113	1116	1119	1123	1126	1129	1132	1136	1139	*1·76*
1143	1145	1149	1152	1155	1159	1162	1165	1168	1172	*1·77*
1175	1178	1181	1185	1187	1192	1194	1197	1201	1204	*1·78*
1207	1211	1213	1217	1220	1223	1227	1230	1233	1236	*1·79*
1239	1242	1246	1249	1252	1255	1259	1262	1264	1268	*1·80*
1272	1274	1277	1281	1284	1287	1290	1294	1296	1300	*1·81*
1303	1306	1309	1312	1316	1318	1322	1325	1328	1331	*1·82*
1334	1338	1340	1344	1347	1349	1354	1356	1359	1362	*1·83*
1366	1368	1372	1375	1378	1381	1384	1387	1391	1393	*1·84*
1396	1400	1403	1405	1409	1412	1415	1418	1421	1425	*1·85*
1427	1430	1434	1436	1440	1442	1446	1449	1452	1454	*1·86*
1458	1461	1464	1467	1470	1473	1476	1480	1482	1485	*1·87*
1488	1491	1495	1497	1500	1504	1506	1509	1513	1515	*1·88*
1519	1521	1525	1527	1530	1534	1536	1540	1542	1545	*1·89*
1549	1551	1555	1557	1560	1563	1567	1569	1572	1575	*1·90*
1579	1581	1584	1587	1590	1593	1596	1599	1601	1605	*1·91*
1608	1611	1613	1617	1619	1623	1625	1628	1632	1634	*1·92*
1637	1640	1643	1646	1649	1651	1655	1657	1661	1663	*1·93*
1667	1669	1672	1675	1678	1681	1683	1687	1689	1693	*1·94*
1695	1698	1701	1704	1707	1710	1712	1716	1718	1722	*1·95*
1724	1727	1729	1733	1736	1738	1741	1745	1747	1750	*1·96*
1752	1756	1758	1762	1764	1767	1769	1773	1776	1778	*1·97*
1781	1784	1787	1790	1792	1795	1799	1801	1803	1807	*1·98*
1809	1813	1815	1818	1820	1824	1826	1829	1832	1835	*1·99*

A SECOND COOPERATIVE STUDY OF *VESPA VULGARIS*. COMPARISON OF QUEENS OF A SINGLE NEST AND QUEENS OF A GENERAL POPULATION.

BY E. Y. THOMSON, JULIA BELL, M.A., AND KARL PEARSON, F.R.S.

(1) *Scope and Material.*

IN a first biometric study of *Vespa vulgaris* made two years ago* its authors compared the size, variability and correlation of wing parts of queens, drones and workers from a single nest, which contained 129 queens, 150 drones and several hundred workers. In the present study the same measurements have been repeated on 188 queens, presumably belonging to many nests and collected in a different district. The nest of the first paper was taken in the autumn of 1905 in the neighbourhood of Godalming. The queens for this second investigation were obtained in the spring of 1908 in the neighbourhood of Gerard's Cross. As soon as the first queen wasp appeared, a reward of 1*d.* was offered for each queen wasp brought, and considerably over 200 were thus rapidly collected. Of this number certain individuals had to be rejected because their wings were damaged; a few others were rejected because their colour raised suspicion that they had been thrown in from an old nest. The specimens came in small numbers, sometimes one at a time, often alive, and there is no reason to doubt that they represent a genuine sample of the queen wasps of the autumn of 1907, which survived the winter and were starting in the spring of 1908 to establish their nests.

Now this difference of season is a very important factor to be kept constantly in view. For perfect comparison of the mean characters of a nest and of the general population, we ought to have collected our queens from a couple of hundred Surrey nests, one or two from each, in the autumn of the same year as the single nest was taken. But the difficulty of procuring the number of nests required stood largely in the way, and we determined, at any rate in a second study, to investigate the general population queen in the spring, when collecting was easier. The change of district was due to the fact that we found at Gerard's Cross in Dr Alice Lee a ready helper, who organised in association with the local schoolmaster a comprehensive search party. Mr O. H. Latter of Charterhouse most kindly went through the material to exclude members of other races.

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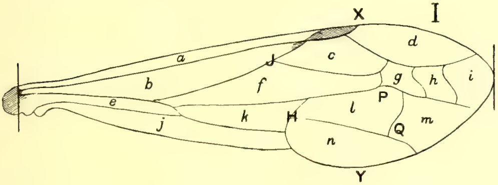
* A Cooperative Study of Queens, Drones and Workers in *Vespa vulgaris*. By Alexandra Wright, Alice Lee and Karl Pearson, *Biometrika*, Vol. v. p. 407.

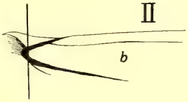
The microscopic measurements made by Miss E. Y. Thomson followed very closely the lines indicated in the earlier paper, an ocular micrometer and a Leitz 1*b* objective being used. The magnification in her case was one ocular unit = 1·46 mm. As this differs slightly from the value used by Miss Wright*, the measurements of both series when compared together in this paper have been reduced to absolute values.

(2) *Characters measured.*

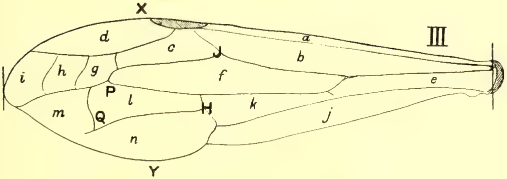
These are precisely the same as those of the first study, and the cuts of that figure reproduced below indicate the lengths measured.

RIGHT WING. *Vespa Vulgaris.*





LEFT WING. *Vespa Vulgaris.*



(A) Total length of wing reckoned from the tip of wing outside of cell *i* to the inside of the vein at the end of cell *b*. Cf. Figs. I. and II.

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* The setting of the microscope differed slightly.

(B) Greatest length of cell *b* taken inside thickened end of veins.

(C) Length of cell *f* from point to further inside point on the *fg* vein.

(D) Greatest breadth of wing from X to wherever found, i.e. XY.

(E) Distance apart of J and H, measured inside veins of cell *f*.

(F) Greatest length of cell *d*, measured inside cell.

(G) Distance apart of points P and Q measured inside cell *l*.

The same indices were also determined by Miss Thomson, namely:

$$H = 100\ D/A, \qquad K = 100\ B/A,$$
$$I = 100\ E/D, \qquad L = 100\ C/A,$$
$$J = 100\ G/D, \qquad M = 100\ F/A.$$

The correlations selected for comparison were those of the previous memoir, i.e.

A and D,	G and F,	I and K,	I and L.
A and C,	G and D,	M and J,	
A and E,	B and E,	L and J,	

A few additional correlations were ascertained, as it was thought desirable to put on record the degree of resemblance between right and left wings in at least one species of wasp.

The analytical determination of the biometric constants was carried out by Miss J. Bell, and as there were 188 individuals it seemed sufficient in this case to form grouped correlation tables and calculate the correlations in the usual product moment manner, adjusting with Sheppard's corrections.

(3) *On the relative Sizes of the mean General Population and of the mean Nest Queens.*

Table I gives the general results.

Now, judged by the probable errors, we should conclude that the mean population queen was in all absolute sizes of the wing less than the mean queen of our special nest[*]. But a word ought to be said as to the possibility of a persistent difference between the two measurers. We fully recognised the difficulty of changing the measurer in the middle of the work, but circumstances did not permit of other action. Accordingly, to diminish personal equation, Miss Thomson practised for some time on Miss Wright's slides, and after a time adjusted her tube length until she got ocular readings in absolute agreement for all practical purposes with Miss Wright's[†]. But on determination of the magnifying power,

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[*] E and G are the only characters about which any doubt could arise, and as the means of the other characters are in all cases significantly less, these doubts may be discarded.

[†] The degree of accordance may be illustrated by the following measurements on Queen 100 of Miss Wright's series:

Miss E. Y. Thomson:	9·20	5·34	4·35	2·68	·99	2·22	·97	for A to G respectively.
Miss A. Wright:	9·20	5·35	4·36	2·68	·99	2·23	·97	,,

1 scale unit was now found to be 1·46 mm., as against Miss Wright's 1·48 mm. The solution of this difference may lie in (*a*) a constant relative personal equation of the two measurers, (*b*) a change in Miss Wright's material during the two years which have elapsed since the measurements were made. In the case of both series the slides were dried for about a month before measurements were made; it is possible, however, that an interval of two years would produce a still further effect.

TABLE I.

Mean Values of Characters in Nest and Population Queens.

	Character	Population Queen		Nest Queen		Ratio *P. Q.*/*N. Q.*	
		Right Wing	Left Wing	Right Wing	Left Wing	Right Wing	Left Wing
Absolute Sizes	*A*	13·223 ± ·015	13·219 ± ·016	13·623 ± ·008	13·617 ± ·009	·9706	·9708
	B	7·770 ± ·010	7·818 ± ·010	8·042 ± ·006	8·058 ± ·007	·9662	·9702
	C	6·185 ± ·008	6·177 ± ·008	6·388 ± ·004	6·390 ± ·004	·9682	·9667
	D	3·933 ± ·006	3·950 ± ·007	3·989 ± ·003	3·983 ± ·003	·9860	·9917
	E	1·446 ± ·003	1·452 ± ·003	1·457 ± ·002	1·467 ± ·001	·9925	·9898
	F	3·031 ± ·005	3·045 ± ·005	3·175 ± ·003	3·191 ± ·003	·9546	·9542
	G	1·414 ± ·002	1·405 ± ·003	1·420 ± ·002	1·415 ± ·002	·9958	·9929
					Mean	·9763	·9766
Indices	*H*	·2975 ± ·0004	·2992 ± ·0005	·2928 ± ·0002	·2925 ± ·0002	1·0161	1·0229
	I	·3672 ± ·0005	·3675 ± ·0005	·3651 ± ·0004	·3687 ± ·0003	1·0057	·9967
	J	·3605 ± ·0005	·3565 ± ·0004	·3560 ± ·0004	·3552 ± ·0004	1·0126	1·0037
	K	·5878 ± ·0003	·5917 ± ·0003	·5904 ± ·0003	·5918 ± ·0003	·9956	·9998
	L	·4680 ± ·0003	·4680 ± ·0003	·4690 ± ·0002	·4694 ± ·0002	·9979	·9970
	M	·2293 ± ·0002	·2303 ± ·0002	·2331 ± ·0002	·2344 ± ·0002	·9837	·9825
					Mean	1·0019	1·0004

The difficulty about a personal equation lies in the fact that it would not extend in either case to the determination of the magnifying powers, for these were found in agreement when determined in the two cases by third observers. If the difference be due to the complete dryness of the material after two years' storage, this difference is of no importance for our present purposes, as both series were measured at practically the same interval after the preparation of the slides.

Even if we could attribute the divergence to personal equation, i.e. if Miss Thomson would have found two years ago, as she does now, that on an average she must reduce Miss Wright's measurements by the factor ·986, i.e. by about 1·4 p.c., this will not account for the average ratio of ·976, i.e. a reduction of 2·4 p.c., found between the population of queens and the queens of a single nest. That we are dealing with a real differentiation is furthermore indicated by the fact that the ratios of the absolute values differ so widely from one character to a second, the range being from ·954 to ·996. There is, we think, no doubt that the absolute dimensions of the wings differ in the case of any single nest by as much as 1 to 4 p.c. from those of the general population. We have however still to bear in mind the possibility that this differentiation is not that of a nest against a general population, but may be that of a Surrey against a Buckinghamshire local race. Finally a third possibility is to be remembered, our spring queens may be severely selected as against the young autumn queens*. If this were so, the conclusion must be that the smaller queens have somewhat the better chance of surviving. The reader will observe that our results for left and right wings were closely in accordance, the maximum difference in the ratio of the means of the two series being in D, where it amounts to ·006; whereas between different characters of the same wing it can be as large as ·041. This is sufficient evidence that the differences are not due to the same source.

If we turn to the population queen, the right wing is larger for the characters A, C and G, and the left wing for the characters B, D, E and F. In the nest queen the right wing is larger for the characters A, D and G, and the left in B, C, E and F. The characters which have changed places are C and D. Actually E and F were the only characters in the nest queen and B and G in the population queen which showed significant deviations in right and left wings, owing to the size of the probable errors. But the general accordance between the two series seems to indicate that a real differentiation between right and left wing might be demonstrable with larger material.

The last column of Table I shows that when we deal with the relative proportions of the wings, there is a far closer resemblance between population queen and nest queen than for absolute dimensions. The average ratio of the indices for the right wing for the two series differs from unity by only about 2 in 1000, and in the left wing by still less. Considering the probable errors of the differences of the indices of the two series, most of these differences are undoubtedly significant, but, I think, we may assert that the degree of resemblance in pattern is greater than it is in size.

The general conclusion to be drawn from examination of our 26 characters is that there is slight but quite definite differentiation between our single nest and our population of queens; but that until further measurements are made we cannot on the basis of this table alone attribute the results without hesitation to: (*a*) local race differences, (*b*) individuality of the nest arising from heredity or nurture, or (*c*) the selective action of the winter in differentiating autumn from spring queens†.

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* Mr Latter suggests as a possibility that the "nest-queens" had fluid in their wings, while the "population queens" had theirs thoroughly dried by hibernation. It is not easy to determine how far this differentiation would be removed by pickling and again drying with the slide. But further observations will be made in this direction.

† We might even suppose that without a selective death-rate the wing might to some extent be worn in six months; the fact that the internal cells like *f* and *b* are also smaller, appears to negative, however, such a hypothesis.

(4) *On the relative Variability of the General Population and of the Nest Queens. Absolute Values.*

Table II gives the absolute variabilities of the 26 characters determined, and we at once observe most marked differences between the two series.

TABLE II.

Standard Deviations of Characters for Nest and Population Queens.

	Character	POPULATION QUEEN		NEST QUEEN		RATIO *P. Q./N. Q.*	
		Right Wing	Left Wing	Right Wing	Left Wing	Right Wing	Left Wing
Absolute Sizes	*A*	·3124 ± ·0109	·3165 ± ·0110	·1428 ± ·0060	·1464 ± ·0061	2·1877	2·1619
	B	·2099 ± ·0073	·2026 ± ·0070	·1089 ± ·0046	·1174 ± ·0049	1·9275	1·7257
	C	·1644 ± ·0057	·1678 ± ·0058	·0666 ± ·0028	·0704 ± ·0030	2·4685	2·3835
	D	·1343 ± ·0047	·1438 ± ·0050	·0525 ± ·0022	·0528 ± ·0022	2·5581	2·7235
	E	·0527 ± ·0018	·0552 ± ·0019	·0329 ± ·0014	·0246 ± ·0010	1·6018	2·2439
	F	·0991 ± ·0034	·1044 ± ·0036	·0462 ± ·0019	·0468 ± ·0020	2·1450	2·2308
	G	·0495 ± ·0017	·0524 ± ·0018	·0376 ± ·0016	·0385 ± ·0016	1·3165	1·3610
					Mean	2·029	2·119
Indices	*H*	·00914 ± ·00032	·00925 ± ·00032	·00329 ± ·00014	·00314 ± ·00013	2·7781	2·9459
	I	·01022 ± ·00036	·01058 ± ·00027	·00628 ± ·00026	·00526 ± ·00022	1·6274	2·0114
	J	·01022 ± ·00036	·00881 ± ·00031	·00717 ± ·00030	·00733 ± ·00031	1·4254	1·2019
	K	·00614 ± ·00021	·00616 ± ·00021	·00422 ± ·00018	·00504 ± ·00021	1·4550	1·2222
	L	·00531 ± ·00018	·00540 ± ·00019	·00281 ± ·00012	·00285 ± ·00012	1·8897	1·8947
	M	·00487 ± ·00017	·00483 ± ·00017	·00360 ± ·00015	·00345 ± ·00014	1·3528	1·4000
					Mean	1·755	1·779

Confining our attention to the absolute variabilities, we see that the variability of the queens in a single nest is not quite half the variability of the queens of the general population. Dr Warren has recently compared the absolute variability for the head index of Natal Termites for the caste of small soldiers* in the two cases of the general population and the population of a single nest. He found for the general population small soldiers a standard deviation of 17·08, and for the average nest one of 7·37, giving a ratio of 2·32. This is of the same order as our results, 2·02 to 2·12. But Dr Warren noticed that between November and March the nest variability could alter considerably (9·00 to 6·56). Further, he was dealing with a head index, and we have compared it with absolute measurements on the wing. If we consider our index measurements on the wing they give (lower part of Table II) 1·76 to 1·78, as against his 2·32. When we consider, however, that he has only taken a single character against our averages of 14 absolute and 12 index characters, and that we get ratios above and below his value, it is difficult to lay much stress on the differences between us. In round figures we may say that the population of a nest is about half as variable as the general population.

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* *Biometrika*, Vol. VI. pp. 336 and 339.

Now it might well be supposed that the queens who survive the winter would be a somewhat highly selected group. Such selection would usually tend to reduce variability, or we might expect that the variability of the general population of queens in the autumn would be, if anything, still larger relatively to the nest population. Is then the large variation of the general population due to a great variability of the nest means resulting from local environmental influence, as Dr Warren suggests in the case of termites? Unfortunately we have in our case only one nest worked out, and we have no data on local variability or the effect of season. But first we ought to consider whether this difference is really incompatible with the effect of heredity on the single nest. Let us suppose the ancestry inbred for n generations, and the ancestral coefficients to diminish in a geometrical series:

$$r_n = \alpha\beta^{n-1},$$

thus obeying the ancestral law of heredity. Suppose the coefficient of assortative mating to be ϵ, for each generation, the mating being that of a brother and sister. Then the correlation of the nth and sth midparent will be obtained by considering the correlation of the means of two individuals only. Let these be H_n and H_s, then, following the lines and notation of a memoir on the ancestral law*, we have, if N be total population of pairs,

$$H_n = \tfrac{1}{2}(h_n + mh'_n), \qquad H_s = \tfrac{1}{2}(h_s + mh'_s),$$

$$S(H_nH_s) = N\rho_{ns}\Sigma_n\Sigma_s = \tfrac{1}{4}\{S(h_nh_s) + mS(h_nh'_s) + mS(h_sh'_n) + m^2S(h'_nh'_s)\},$$

or
$$\rho_{ns}\Sigma_n\Sigma_s = \tfrac{1}{4}(\sigma_n\sigma_s r_{ns} + m\sigma_n\sigma'_s r_{ns'} + m\sigma_s\sigma'_n r_{n's} + m^2\sigma'_n\sigma'_s r_{n's'}).$$

Now, as in that memoir, take $m = \sigma_n/\sigma'_s = \sigma_s/\sigma'_n$, supposing no secular change. Further, note that we have

$$\Sigma_n^2 = \tfrac{1}{4}(\sigma_n^2 + m^2\sigma'^2_n + 2m\sigma_n\sigma'_n r_{nn'})$$

and $r_{nn'} = \epsilon$; thus

$$\Sigma_n^2 = \tfrac{1}{4}\,2(1+\epsilon)\,\sigma_n^2 = \frac{1+\epsilon}{2}\sigma_n^2 = \Sigma_s^2;$$

whence
$$\rho_{ns} = \frac{2}{1+\epsilon}\,\frac{r_{ns} + r_{ns'} + r_{n's} + r_{n's'}}{4}.$$

If now we suppose the ancestral influence of drone and queen to be equal, or not being equal we take r_{n-s} to represent the mean of the four values, we have for the correlation between an nth and sth midparent, which may be represented by ρ_{n-s}, for it depends only on the difference,

$$\rho_{n-s} = \frac{2}{1+\epsilon}r_{n-s} = \frac{2\alpha}{1+\epsilon}\beta^{n-s-1}$$

$$= 1/\gamma\,.\,\beta^{n-s-1},$$

if we take
$$\gamma = (1+\epsilon)/2\alpha.$$

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* *R. S. Proc.* Vol. 62, p. 388.

We have next to consider the correlation between an nth midparent and the actual offspring. Let us represent it by ρ_{0n}. Then

$$N\Sigma_n\sigma_0\rho_{0n} = \tfrac{1}{2}S(h_0h_n) + m\,\tfrac{1}{2}\Sigma(h_0h'_n),$$

or,

$$\Sigma_n\sigma_0\rho_{0n} = \tfrac{1}{2}(\sigma_0\sigma_n r_{0n} + m\sigma_0\sigma_{n'}r_{0n'}),$$

$$\begin{aligned}\rho_{0n} &= \frac{\sigma_n}{\Sigma_n}\tfrac{1}{2}(r_{0n}+r_{0n'})\\ &= \sqrt{\frac{2}{1+\epsilon}}\,r_n = \sqrt{\frac{2}{1+\epsilon}}\,\alpha\beta^{n-1} = \sqrt{\frac{\alpha}{\gamma}}\,\beta^{n-1},\end{aligned}$$

if r_n be again the mean correlation value of the ancestors of the two sexes in the nth degree and the offspring.

We have now to evaluate the determinant Δ and the minor Δ_{11}, obtained by cutting out its first row and column. Taking n generations of inbreeding:

$$\Delta = \begin{vmatrix} 1, & \rho_{01}, & \rho_{02}, & \rho_{03}, & \cdots & \rho_{0n}\\ \rho_{01}, & 1, & \rho_{12}, & \rho_{13}, & \cdots & \rho_{1n}\\ \rho_{02}, & \rho_{12}, & 1, & \rho_{23}, & \cdots & \rho_{2n}\\ \hdotsfor{6}\\ \rho_{0n}, & \rho_{1n}, & \rho_{n2}, & \rho_{n3}, & \cdots & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1, & \sqrt{\alpha/\gamma}, & \sqrt{\alpha/\gamma}\,\beta, & \sqrt{\alpha/\gamma}\,\beta^2, & \ldots & \sqrt{\alpha/\gamma}\,\beta^{n-1}\\ \sqrt{\alpha/\gamma}, & 1, & \dfrac{1}{\gamma}, & \dfrac{1}{\gamma}\beta, & \ldots & \dfrac{1}{\gamma}\beta^{n-2}\\ \sqrt{\alpha/\gamma}\,\beta, & \dfrac{1}{\gamma}, & 1, & \dfrac{1}{\gamma}, & \ldots & \dfrac{1}{\gamma}\beta^{n-3}\\ \hdotsfor{6}\\ \hdotsfor{6}\\ \sqrt{\alpha/\gamma}\,\beta^{n-1}, & \dfrac{1}{\gamma}\beta^{n-2}, & \dfrac{1}{\gamma}\beta^{n-3}, & \dfrac{1}{\gamma}\beta^{n-4}, & \ldots & 1 \end{vmatrix}$$

$$= \alpha\gamma \begin{vmatrix} \dfrac{1}{\alpha\gamma}, & \dfrac{1}{\gamma}, & \dfrac{\beta}{\gamma}, & \dfrac{\beta^2}{\gamma}, & \ldots & \dfrac{\beta^{n-1}}{\gamma}\\ \dfrac{1}{\gamma}, & 1, & \dfrac{1}{\gamma}, & \dfrac{\beta}{\gamma}, & \ldots & \dfrac{\beta^{n-2}}{\gamma}\\ \dfrac{\beta}{\gamma}, & \dfrac{1}{\gamma}, & 1, & \dfrac{1}{\gamma}, & \ldots & \dfrac{\beta^{n-3}}{\gamma}\\ \hdotsfor{6}\\ \dfrac{\beta^{n-1}}{\gamma}, & \dfrac{\beta^{n-2}}{\gamma}, & \dfrac{\beta^{n-3}}{\gamma}, & \dfrac{\beta^{n-4}}{\gamma}, & \ldots & 1 \end{vmatrix}$$

$$= \frac{\alpha\gamma}{\gamma^{n+1}} \begin{vmatrix} \dfrac{1}{\alpha}, & 1, & \beta, & \beta^2, & \ldots & \beta^{n-1}\\ 1, & \gamma, & 1, & \beta, & \ldots & \beta^{n-2}\\ \beta, & 1, & \gamma, & 1, & \ldots & \beta^{n-3}\\ \hdotsfor{6}\\ \beta^{n-1}, & \beta^{n-2}, & \beta^{n-3}, & \beta^{n-4}, & \ldots & \gamma \end{vmatrix}.$$

Subtract β times the second row from the first and β times the second column from the first.

$$\Delta = \frac{\alpha}{\gamma^n} \begin{vmatrix} \frac{1}{\alpha} + \gamma\beta^2 - 2\beta, & 1 - \gamma\beta, & 0, & 0, & \cdots & 0 \\ 1 - \gamma\beta, & \gamma, & 1, & \beta, & \cdots & \beta^{n-2} \\ 0, & 1, & \gamma, & 1, & \cdots & \beta^{n-3} \\ 0, & \beta, & 1, & \gamma, & \cdots & \beta^{n-4} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0, & \beta^{n-2}, & \beta^{n-3}, & \beta^{n-4}, & \cdots & \gamma \end{vmatrix}$$

$$= \frac{\alpha}{\gamma^n} \left\{ \left(\frac{1}{\alpha} + \gamma\beta^2 - 2\beta \right) D_{n-2} - (1 - \gamma\beta)^2 D_{n-3} \right\},$$

where

$$D_{n-2} = \begin{vmatrix} \gamma, & 1, & \beta, & \cdots & \beta^{n-2} \\ 1, & \gamma, & 1, & \cdots & \beta^{n-3} \\ \beta, & 1, & \gamma, & \cdots & \beta^{n-4} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \beta^{n-2}, & \beta^{n-3}, & \beta^{n-4}, & \cdots & \gamma \end{vmatrix}.$$

We now turn to the first minor Δ_{11}, for which we have:

$$\Delta_{11} = \begin{vmatrix} 1, & \frac{1}{\gamma}, & \frac{1}{\gamma}\beta, & \cdots & \frac{1}{\gamma}\beta^{n-2} \\ \frac{1}{\gamma}, & 1, & \frac{1}{\gamma}, & \cdots & \frac{1}{\gamma}\beta^{n-3} \\ \frac{1}{\gamma}\beta, & \frac{1}{\gamma}, & 1, & \cdots & \frac{1}{\gamma}\beta^{n-4} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{1}{\gamma}\beta^{n-2}, & \frac{1}{\gamma}\beta^{n-3}, & \frac{1}{\gamma}\beta^{n-4}, & \cdots & 1 \end{vmatrix}$$

$$= \frac{1}{\gamma^n} \begin{vmatrix} \gamma, & 1, & \beta, & \cdots & \beta^{n-2} \\ 1, & \gamma, & 1, & \cdots & \beta^{n-3} \\ \beta, & 1, & \gamma, & \cdots & \beta^{n-4} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \beta^{n-2}, & \beta^{n-3}, & \beta^{n-4}, & \cdots & \gamma \end{vmatrix}$$

$$= \frac{1}{\gamma^n} D_{n-2}.$$

Now if s_n be the standard-deviation of a single nest:

$$s_n = \sigma_n \sqrt{\Delta/\Delta_{11}} = \sigma_n \sqrt{\alpha \left\{ \left(\frac{1}{\alpha} + \gamma\beta^2 - 2\beta \right) - (1 - \gamma\beta)^2 \frac{D_{n-3}}{D_{n-2}} \right\}}.$$

We have accordingly to find D_{n-3}/D_{n-2} when n is large. Returning to D_{n-2}, subtract β times the second row from the first, and β times the second column from the first. There results:

$$D_{n-2} = \left| \begin{array}{cccccc} \gamma(1+\beta^2) - 2\beta, & 1-\gamma\beta, & 0, & 0, & \ldots & 0 \\ 1-\gamma\beta, & \gamma, & 1, & \beta, & \ldots & \beta^{n-3} \\ 0, & 1, & \gamma, & 1, & \ldots & \beta^{n-4} \\ 0, & \beta, & 1, & \gamma, & \ldots & \beta^{n-5} \\ \multicolumn{6}{c}{\dotfill} \\ 0, & \beta^{n-3}, & \beta^{n-4}, & \beta^{n-5}, & \ldots & \gamma \end{array} \right|$$

$$= \{\gamma(1+\beta^2) - 2\beta\} D_{n-3} - (1-\gamma\beta)^2 D_{n-4}.$$

Assume $D_{n-2} = C\xi^{n-2}$, and we have

$$\xi^2 - \{\gamma(1+\beta^2) - 2\beta\}\xi + (1-\gamma\beta)^2 = 0.$$

Hence if ξ_1 and ξ_2 be the roots

$$D_{n-2} = C_1\xi_1^{n-2} + C_2\xi_2^{n-2},$$

and

$$\frac{D_{n-3}}{D_{n-2}} = \frac{C_1\xi_1^{n-3} + C_2\xi_2^{n-3}}{C_1\xi_1^{n-2} + C_2\xi_2^{n-2}}.$$

Now if ξ_1 be $> \xi_2$, this when n is large rapidly becomes equal to $\frac{1}{\xi_1}$.

Thus we conclude that for a great number of generations of inbreeding of brother and sister, as will usually be the case in wasps,

$$\frac{\sigma_n}{s_n} = \frac{1}{\sqrt{\alpha\left\{\left(\frac{1}{\alpha} + \gamma\beta^2 - 2\beta\right) - \frac{(1-\gamma\beta)^2}{\xi_1}\right\}}},$$

where ξ_1 is the greater root of the quadratic

$$\xi^2 - \{\gamma(1+\beta^2) - 2\beta\}\xi + (1-\gamma\beta)^2 = 0.$$

The result may be written

$$\sigma_n/s_n = 1/\sqrt{1 + \alpha(\xi_1 - \gamma)}.$$

We have worked out this result numerically for a few cases, assuming ϵ the coefficient of assortative mating to be $= {\cdot}5$ the resemblance of brother and sister in most mammals. (This step, of course, from mammals to insects yet needs verification. Cf. Warren for Aphis and Daphnia, *Biometrika*, Vol. I. p. 147.)

Case (i). The average values of biometric work are $\alpha = \frac{1}{2}$, $\beta = \frac{2}{3}$. Hence $\gamma = 1{\cdot}5$, $1 - \gamma\beta$ will be nearly zero and $\xi_1 = {\cdot}8000$. We find

$$\sigma_n/s_n = 1{\cdot}23.$$

Case (ii). Let the correlations be equal to the *gametic* correlations on the Mendelian hypothesis. Then $\alpha = \frac{1}{2}$, $\beta = \frac{1}{2}$. Hence $\gamma = 1{\cdot}5$ and we have

$$\xi^2 - {\cdot}875\,\xi + {\cdot}0625 = 0,$$

which leads to $\xi_1 = {\cdot}7966$, and

$$\sigma_n/s_n = 1{\cdot}24.$$

Case (iii). Let the correlations be equal to the *somatic* correlations on the Mendelian hypothesis. Then $\alpha = \frac{1}{3}$, $\beta = \frac{1}{2}$, $\gamma = 2{\cdot}25$ and

$$\xi^2 = 1{\cdot}8125\,\xi + {\cdot}0156 = 0,$$

giving $\xi_1 = 1{\cdot}8041$, and leading to

$$\sigma_n/s_n = 1{\cdot}08.$$

Case (iv). We assumed the fraternal correlation was as high as ·7, following Warren's results for Aphis and Daphnia. The ratio for $\alpha = \frac{1}{2}$, $\beta = \frac{1}{2}$ was

$$\sigma_n/s_n = 1{\cdot}19,$$

and no sensible change resulted.

The values of the biometric and Mendelian gametic correlations fit the facts better than those of the Mendelian somatic correlations. But neither give a sufficiently reduced variability when we pass from general population to a special nest. We have at most 1·24 as against 1·8 to 2·1. Two only, and these the indices J and K for the left wing, out of the 26 characters, give values 1·20 and 1·22 comparable with the above theoretical values. We are therefore forced to conclude that (*a*) there is some defect in the above algebraic analysis; the problem is not an easy one, and there may be a slip, or (*b*) the wasps, as Dr Warren supposes in the case of the termites, are markedly influenced by the local environment of the nest, and thus a considerable part of the observed large variability ratio has an environmental and not a hereditary source, or (*c*) the result may be due to winter selection, if we suppose that selection of a twofold character, i.e. that from some unexplained cause the rather larger and rather smaller queens have a somewhat better chance of surviving than the mediocre queens.

Undoubtedly the more prudent course at present is not, however, to manufacture hypotheses, but to wait till further material has been measured and till more light can be thrown on this most interesting point. Unless we were most unfortunate in our selection of the original nest*, Table I shows that remarkably little effect has been produced by either selection or growth on the *mean* values of the indices during the winter months. Yet the variability of these indices is much larger than we can possibly attribute in the present state of our knowledge to the hereditary factor, when compared with their values for an individual nest. We should, *a priori*, conceive that local environment would give the general population a wide range of variability by making a large range of local means. The crux is that our one set of local index means is extraordinarily close to the general population index means. Personally, we admit that we are inclined to attribute the result to the continual inbreeding of the wasp, but we also confess that without assuming values for the hereditary coefficients incompatible with our present knowledge, we cannot reach such a variability ratio as we find indicated for both wasps and termites.

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* I.e. just chancing to hit a nest with all its indices closely in agreement with the general population means.

(5) *On the Relative Variability of the General Population and of the Nest Queens. Relative Values.*

In Table III the coefficients of variation of population and nest queens for the 26 characters are placed side by side. We see at once that the variability ratios are as marked when we treat relative as when we treat absolute variation—the general results owing to the small changes in the average values of the characters as we pass from nest to general population being closely alike. Although the coefficients of variation for the general population are for absolute sizes double as large as for the single nest, we see that the conclusion of the first wasp paper, *Biometrika*, Vol. v. p. 415, is confirmed. The wing of the queen of *Vespa vulgaris* possesses extraordinarily small variability. If we deal with indices, even smaller values are found than Dr Warren found for his termites, and our population values for absolute sizes are comparable with his nest values. As far as our present experience reaches, the wing of the queen wasp appears to be the least variable organ that has as yet been studied. We may note that the population queen is about as variable as the worker of an individual nest (*Biometrika*, Vol. v. p. 417).

TABLE III.

Coefficients of Variation of Characters for Nest and Population Queens.

	Character	Population Queen		Nest Queen		Ratio *P. Q.*/*N. Q.*	
		Right Wing	Left Wing	Right Wing	Left Wing	Right Wing	Left Wing
Absolute Sizes	*A*	2·36 ± ·08	2·39 ± ·08	1·05 ± ·04	1·09 ± ·05	2·25	2·19
	B	2·70 ± ·09	2·59 ± ·09	1·36 ± ·07	1·46 ± ·06	1·99	1·77
	C	2·66 ± ·09	2·72 ± ·09	1·05 ± ·04	1·10 ± ·05	2·53	2·47
	D	3·41 ± ·12	3·64 ± ·13	1·31 ± ·06	1·32 ± ·06	2·60	2·76
	E	3·65 ± ·13	3·80 ± ·13	2·13 ± ·09	1·68 ± ·07	1·71	2·26
	F	3·27 ± ·11	3·43 ± ·12	1·45 ± ·06	1·46 ± ·06	2·26	2·35
	G	3·50 ± ·12	3·73 ± ·12	2·65 ± ·11	2·70 ± ·11	1·32	1·38
	Mean	3·08	3·19	1·57	1·54	2·09	2·17
Indices	*H*	3·07 ± ·11	3·09 ± ·11	1·12 ± ·05	1·07 ± ·05	2·74	2·89
	I	2·78 ± ·10	2·88 ± ·10	1·72 ± ·07	1·43 ± ·06	1·62	2·01
	J	2·83 ± ·10	2·47 ± ·09	2·01 ± ·08	2·04 ± ·09	1·41	1·21
	K	1·04 ± ·04	1·04 ± ·04	0·72 ± ·03	0·85 ± ·04	1·44	1·22
	L	1·13 ± ·04	1·15 ± ·04	0·60 ± ·03	0·61 ± ·02	1·88	1·88
	M	2·12 ± ·07	2·10 ± ·07	1·54 ± ·06	1·47 ± ·06	1·38	1·42
	Mean	2·17	2·12	1·28	1·25	1·75	1·77

Looking at the results now obtained, it is not possible to assert any marked difference in variability between the two wings, thus the result of the earlier paper (p. 417) is confirmed.

(6) *Coefficients of Correlation of Characters in the General Population and Nest Queens.*

The results are given in Table IV. Examining first the index correlations of the population queens we see that as in the case of the nest queens they are all slight and of small significance. Yet they are now all *positive*, and do not take the somewhat erratic values found in the case of the nest queens. It may be doubted whether, considering their probable errors, any weight whatever can be laid on these correlations. The small variation of these indices and their low correlations demonstrate, however, that they might be of considerable service for classificatory purposes. If we turn to the correlations of the absolute sizes, we see at once a marked difference. With the exception of one pair to be referred to especially below, in which the nest correlation is higher for both wings, and one pair, where in the right wing they are sensibly equal, the correlation is higher, and very markedly higher in the general population, than in the nest queens. Now this is precisely what the theory of correlation would lead us to expect; if we select a small element of a total population we thereby reduce the correlation. Now let us omit the correlations of length and breadth of wing, where clearly some special cause has been at work. We find for the mean correlation of population queens for the remaining six pairs on both sides ·64. The

TABLE IV.

Coefficients of Correlation for Characters of Nest and Population Queens.

	Characters correlated	POPULATION QUEEN		NEST QUEEN		RATIO *P. Q.*/*N. Q.*	
		Right Wing	Left Wing	Right Wing	Left Wing	Right Wing	Left Wing
Absolute Sizes	*A* and *D*	·478 ± ·038	·579 ± ·033	·558 ± ·040	·619 ± ·037	·86	·94
	A and *C*	·917 ± ·008	·894 ± ·010	·838 ± ·018	·857 ± ·016	1·09	1·04
	A and *E*	·531 ± ·035	·647 ± ·029	·238 ± ·056	·259 ± ·053	2·23	2·50
	G and *F*	·413 ± ·041	·396 ± ·042	·164 ± ·058	·051 ± ·059	2·52	7·76
	G and *D*	·672 ± ·027	·781 ± ·019	·677 ± ·032	·694 ± ·031	·99	1·13
	B and *E*	·506 ± ·037	·617 ± ·031	·233 ± ·056	·280 ± ·055	2·17	2·20
	B and *F*	·677 ± ·027	·671 ± ·027	·180 ± ·057	·139 ± ·058	3·76	4·83
	Mean	·599	·655	·413	·414	1·95	2·92
Indices	*I* and *K*	·123 ± ·048	·054 ± ·049	– ·021 ± ·059	·037 ± ·059	—	—
	M and *J*	·050 ± ·049	·137 ± ·048	– ·138 ± ·058	– ·185 ± ·057	—	—
	L and *J*	·097 ± ·049	·132 ± ·048	– ·367 ± ·051	– ·186 ± ·057	—	—
	I and *L*	·137 ± ·048	·203 ± ·047	·039 ± ·059	·194 ± ·057	—	—
	Mean	·102	·131	– ·122	– ·035	—	—

general reduction of variability for the absolute characters as shown by Table II is about $\frac{1}{2}$*. Now the effect of double selection on the variation of two characters is known to be given by the formula

$$R_{12} = r_{12} \frac{\mu_1 \mu_2}{\sqrt{1 - r_{12}^2 (1 - \mu_1^2)} \sqrt{1 - r_{12}^2 (1 - \mu_2^2)}} .$$

See *Biometrika*, Vol. VI. pp. 111—2.

Putting $r_{12} = \cdot 64$ and $\mu_1 = \mu_2 = \frac{1}{2}$, we find

$$R_{12} = r_{12}/2{\cdot}77.$$

Turning to the last column of Table IV, headed Ratio *P. Q./N. Q.*, and omitting the pair *A* and *D*, we find the mean value of the ratio of the correlation coefficients of the remaining 12 pairs to be 2·68. This is sufficiently near to 2·77 for us to conclude that the decrease in correlation, when we pass from general population to a nest population, is quite compatible with the result which naturally flows when we look upon the nest as a selection of reduced variability made from the general population. It will indeed be noted that the three least correlation ratios involve *A* and *D*, the characters which correlated together give the exceptional case of a *reduced* variation when we pass from the general to a nest population. Can any explanation be offered of this anomalous case? To begin with, the possibility of a stringent winter selection of these characters so that the correlation would be reduced below that of a nest is hardly tenable. Table II shows us that the variability ratios for the *A* and *D* characters are above the average. They ought therefore to show reduced correlation for the nest as compared with the general population. I would draw attention to the fact, however, that *A*, the total length of wing, and *D*, the breadth, are the two characters which are most likely to be affected by wear and tear of the wing. All damaged wings on which the measurement of *A* or *D* was difficult or impossible were, of course, omitted. But it will be clear that the measurement of internal cells is less likely to be influenced by any action of the nature of weathering than these external measurements. The possibility is therefore before us—we do not lay much stress on it—that in the reduced correlation of the length and breadth of the wing, we have the one definite effect of the hibernation. At the same time, the fact that we do not find a reduced correlation in the case of right and left wings for either the characters of length or breadth (see Table V) is somewhat against this hypothesis. It would indicate that the action, whatever its character, is the same for the lengths of both wings or for the breadths of both wings, and can be nothing of the nature of local wear, affecting one wing here and another there.

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* Actually (*Biometrika*, Vol. VI. 112) μ should be found from

$$\Sigma_1^2 = \sigma_1^2 \mu_1^2 \{1 - r_{12}^2 (1 - \mu_2^2)\} / \{1 - r_{12}^2 (1 - \mu_1^2)(1 - \mu_2^2)\}$$

by putting Σ_1/σ_1 = variability ratio for the characters taken weighted with the number of times they are taken. But the r_{12}^2 terms have little effect, e.g. if $\Sigma_1/\sigma_1 = \cdot 5$, $\mu_1 = \mu_2 = \cdot 53$, and with this value $R_{12}/r_{12} = 2{\cdot}52$ which is of the same order. In fact the observed mean value lies between the values of R_{12}/r_{12} for $\mu = \cdot 50$ and $\mu = \cdot 53$.

(7) *Comparison of Correlation between Characters in Right and Left Wings for the two Series.*

TABLE V.

Correlations of Right and Left Wings in Nest and Population Queens.

Character	Population Queen	Nest Queen	Ratio *P. Q./N. Q.*
A	·954 ± ·004	·893 ± ·012	1·07
B	·911 ± ·008	·831 ± ·018	1·10
C	·964 ± ·004	·858 ± ·016	1·12
D	·862 ± ·013	·600 ± ·038	1·44
E	·694 ± ·026	·428 ± ·049	1·62
F	·932 ± ·006	·605 ± ·038	1·54
G	·856 ± ·013	·787 ± ·023	1·09
Mean	·882	·714	1·28

If we apply our selection formulae to these cases we must, since r_{12} is large, $= {\cdot}882$, deduce the true value of μ. We have

$$\Sigma_1^2/\sigma_1^2 = \tfrac{1}{4} \text{ about} = \frac{\mu^2\{1 - r_{12}^2(1-\mu^2)\}}{1 - r_{12}^2(1-\mu^2)^2},$$

or, calling $1 - \mu^2 = v$,

$$1 - r_{12}^2 v^2 = 4(1-v) - 4r_{12}^2 v(1-v),$$

$$5r_{12}^2 v^2 - 4v(1 + r_{12}^2) + 3 = 0,$$

$$v^2 - 1{\cdot}8284v + {\cdot}7713 = 0,$$

of which the required root is $v = {\cdot}6603$, giving $\mu = {\cdot}5828$. Substituting this in the correlation formula

$$R_{12} = r_{12}\frac{\mu^2}{1 - r_{12}^2(1-\mu^2)},$$

we deduce

$$R_{12} = r_{12}/1{\cdot}43.$$

The average correlation ratio 1·43 thus provided is somewhat greater than the observed average value, 1·28. It is, however, greater than four and less than three of the individual values, and it is quite possible that if the process had been applied to each individual character, a closer mean result would have been obtained than by using the mean correlation, ·882. The general agreement is sufficient to show the reduction of correlation found as we pass from general population to nest population is just of the sort of order which the reduction in variability would lead us to anticipate. It is clear, therefore, that the correlation reductions are really what we might expect on the assumption that the nest is a selection from the general population. The outstanding point is the extreme reduction in variability which marks this selection.

It is worth noting that the lowest correlations are those that concern homologous *breadth* measurements on the wings. The highest correlations are those which concern the total length of the wings and the lengths of cells *b*, *f*, and *d* (see Fig. I, p. 49). If we confine our attention to these homologous lengths of the wasp's wings we see that the range ·911 to ·964 is quite comparable with the values obtained by Whiteley and Pearson* for the first joint of the human fingers, ·904 to ·934; with those obtained by Whiteley and Lewenz† for homologous bones of the two hands, ·793 to ·990; and by Dr Warren‡ for homologous long bones in man, ·86 to ·97. The right and left wings of the wasp appear as closely related as the right and left members in man.

(8) *Conclusions.*

The results reached in this paper really centre on the determination of what meaning we are to give to the high variability ratio between the population of a single nest and the general population. The relatively close agreement of the means of this individual nest with those of the general population for the indices, does not seem to indicate that we are to look for an explanation of the difficulty in differentiated environmental effect on the different nests. *A priori* reasonable values of the heredity coefficients do not appear to allow of a reduction of variability to 50 °/ₒ of the general population value. Once the source of this reduction in variability is accounted for, the correlation changes between nest and general population queens are seen to be of the order which should theoretically be associated with it. The only comparable material is that of Dr Warren for Natal termites, which gives a variability reduction of approximately the same value; and this reduction is considered by him to be due in that case to differential environmental effect. It will be of interest to obtain, if possible, a series of autumn queens from the general population, and compare the variability of these with that of the spring queens dealt with in this paper.

Note. It occurred to me after completing the above memoir that selection of *midparents* might give a greater variability than the selection of actual parents in each generation. Testing, however, for the case of immediate parents, this does not appear to be the case. The variability of the offspring:

$$s = \sigma\sqrt{\frac{(1 - r_1^2 - r_2^2 - \epsilon^2 + 2r_1 r_2 \epsilon)}{1 - \epsilon^2}} = \sigma\sqrt{1 - \frac{2r_1^2(1 - \epsilon)}{1 - \epsilon^2}},$$

if the parental correlations be equal. Thus:

$$s = \sigma\sqrt{1 - \left(r_1\sqrt{\frac{2}{1 + \epsilon}}\right)^2} = \sigma\sqrt{1 - \rho_{01}^2},$$

by p. 54, if ρ_{01} be the correlation of the offspring with the first midparent. Thus the midparent simplification of the analysis does not appear to account for the theoretically large variability in the single nest as compared with observation.

K. P.

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* *R. S. Proc.* Vol. 65, pp. 349—350.

† *Biometrika*, Vol. I. p. 358.

‡ *Phil. Trans.* Vol. 189, B, p. 178.

THE OPSONIC INDEX—A MEDICO-STATISTICAL ENQUIRY*.

By W. F. HARVEY, M.A., M.B., D.P.H., Captain I.M.S. and ANDERSON M^cKENDRICK, M.B., Ch.B., Captain I.M.S†.

Much misunderstanding seems to exist as to the relationship which should hold between the statistician, anxious to scrutinise the validity of inferences drawn from observations, and the observer of the facts. The latter looks with suspicion on the former and is inclined to doubt whether the intrusion of his fellow scientist into his domain is for any more worthy purpose than simply to show that he is totally wrong in his conclusions. The mathematician is apt to be impatient at the want of knowledge of his technical terms displayed by the observer and at the often openly expressed contempt for some of his most cherished elementary principles. But the statistical mathematician must get his data from the doctor —for it is of medical science in particular that we speak here—and the doctor at present is dependent on or would be wise to obtain the assistance of the mathematician for the refining of his conclusions. The doctor may also obtain much help from the criticism of the mathematician regarding the numerical soundness of his control observations. The statistician takes very little account of the non-quantitative data which bulk so largely in support of the claims of medical science. He takes merely certain of the characters which lend themselves easily to analysis, such as case-mortality or its complement case-recovery, attack-rate and the like and on this basis alone examines the validity of the conclusions arrived at. The degrees of correlation between such characters and particular methods of treatment are, through the controversies to which they have given rise, becoming familiar to medical men. But it must be pointed out that these correlations measure only the numerical aspect of single pairs of phenomena, admittedly important though these be. In some cases they do not measure the actual facts under discussion but rather some one quantitative relation of these facts. The improvement in the condition of the patient expressed in terms of feeling-of-well-being, the check in the spread of disease processes, the extension in the duration of life produced by particular methods of treatment have scarcely at all been taken into account because they are not available in a form suitable for statistical treatment. These data may be grouped under the commonly used term, experience. But experience is largely a personal matter and can only carry limited weight in argument or at most appeal to the comparatively few. Professor K. Pearson has shown how the degree of association of non-quantitative characters can be worked out. It is incumbent upon us therefore as medical men to provide material for the calculation of coefficients of contingency, as they are called, as well as for coefficients of correlation in future considerations of efficacy of treatment and the indications afforded by special tests.

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† In explanation of the joint authorship of this paper I should say that an earlier unpublished paper on the same subject by Captain McKendrick has been embodied in this paper. Being responsible for the wording of the paper I have used the pronoun "I" in the text. W. F. H.

Another argument which enters implicitly or explicitly into medical dissertation is the argument from analogy. Logicians warn us against the pitfalls of such a method, and yet it is, when discreetly used, a most valuable adjunct to evidence. Thus the prophylactic value of vaccination against smallpox having been established it seems reasonable to suppose that a similar method would be capable of giving some result in other diseases such as rabies, anthrax, cholera, and typhoid fever. If the evidence for the therapeutic value of inoculation in cases of furunculosis, erysipelas and acne is accepted we may reason further to the trial of specific vaccines in the treatment of other affections. If improvement in the condition of a patient and the development of a feeling of well-being usually synchronise with a rise of the opsonic power of his serum we may—until it is disproved—infer that there is a causal connection between them. Finally if over a wide range of diseases, tuberculosis, staphylococcal, streptococcal, pneumococcal, and gonococcal diseases, glanders, dysentery, typhoid and Malta fevers and others there would seem to be accumulating evidence in favour of the arrival of a new era of treatment we may reasonably anticipate that these hopes will ultimately prove, if properly recorded, to be based on statistically significant foundations. But we must discard the fetish of experience, and set to work at the expense of some labour to record the essence of that experience so that all the world may judge of its real value. The work requires co-operation, and the realisation of this fact may speed the advance of exact measurement in the domain of medicine.

In the following treatment of the subject-matter of this paper I have endeavoured to indicate some of the aspects from which this recently much criticised quantitative measurement may be viewed rather than to deal with it exhaustively.

The phagocytic count affords a peculiarly suitable case for figure analysis. We have here a typical frequency distribution, that is to say, the numerical degree to which a given character is possessed. This is most conveniently shown by stating the actual number of individuals, in a sufficiently large and unselected sample, that possess the different gradations of the character in question. The individuals are the leucocytes and the character the amount of phagocytosis. We set this forth by stating the number of leucocytes in a count say of 100 which possess the varying numbers of bacteria.

Number of leucocytes	12	16	22	17	11	6	5	3	3	2	1	1	1	Totals 100
Number of contained bacteria	0	1	2	3	4	5	6	7	8	9	10	11	12	311

The number of leucocytes which contain no bacteria is 12; those containing one bacterium are sixteen, and so on. If now we plot in the usual way number of leucocytes against number of contained bacteria we get, when we join up the tops of all the ordinates, what is called a frequency polygon.

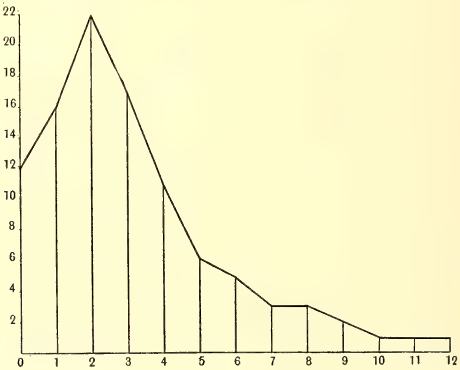

Fig. 1. Frequency Polygon.

The way in which the numbers gradually increase and then gradually decrease suggests that we might be able to replace the polygon with a mathematical curve expressing what is the degree of phagocytosis under the conditions of experiment. But if we counted a second, a third and further hundreds from our slide, we should find that the frequency distribution was never twice the same. We might then be in a position to realise to what extent our method was only a rough approximation to the true state of affairs. We might take very naturally the usual form of average—the Arithmetic Mean—and say we shall deal only with averages of distributions and so get over the difficulty of this variation in our data. But we find that in so doing we have exactly the same difficulty to contend with; the averages, as determined from successive samples taken from the same material, vary just like the distributions. And yet we cannot help feeling that a definite law of some kind governs the degree of phagocytosis. If we appreciate the point that a deficiency in the number of leucocytes containing any given number of bacteria is balanced by an equal excess distributed amongst the numbers of leucocytes containing different numbers of bacteria and assume that all ordinates representing these magnitudes are interdependent owing to their being governed by some law, we can see how the help of the mathematician becomes indispensable: he will determine, if supplied with sufficiently large numbers to avoid gross error, the smooth curve which will most closely fit the totality of observations. We may then be willing to concede that such a curve will give us a near approach to the condition of things as they ought to be, but which are not so owing to experimental error and to smallness of data.

It is necessary in the first instance to afford the mathematician sufficient data by which to construct his curve but once constructed it may become of the utmost utility as a test of the degree in which we are approximating to the ideal by the method which considerations of convenience lead us to adopt. The same sort of standard of comparison might be set up by the bacteriologist himself if he took the pains to collect very large data. The larger the number of leucocytes counted for example, the fewer would be the irregularities, and finally, when the numbers were very large indeed, we may imagine that our frequency polygon would be identical with the mathematician's curve. But the labour of such a task would be very great, and if it had to be done in many cases would take up too much of the observer's already too short life. It would be folly then to refuse the shorthand expressions of the mathematician in the shape of curve or formula, or his statements as to degree of error, simply because he has deduced these from the data supplied to him on the supposition of the operation of a law governing the varying magnitudes of the observations. In many cases it is quite unnecessary to undertake the labour of fitting a curve, since a single measure like the Arithmetic Mean serves to give all the information which is really necessary. But we must at least know what are the limits of error of our average, and this is almost totally neglected by medical observers.

To return to the example given, we should naturally, in seeking to give expression briefly to what is exhibited here in a somewhat lengthy form, take the Arithmetic Mean of the whole distribution. This is 3·11. In words the average number of bacteria taken up per leucocyte is 3·11. Obviously no leucocytes take up this number of bacteria, but long familiarity with such a measure has made all men acquainted with what is meant by the expression. No leucocyte contains this number nor ever will contain this number; nor can even the majority of the leucocytes be said to contain approximately this number. The Arithmetic Mean then in this case does not correspond to the character possessed by the majority. This is due in reality to the want of symmetry of the distribution. This asymmetry or skewness is typical of all phagocytic distributions which have been as yet investigated from this standpoint, and is itself characteristic of the particular law which governs these distributions. It has an important bearing on the claim of the Arithmetic Mean to be called the best descriptive average. Another measure of the phagocytic frequency distribution has been proposed, namely the point corresponding to the maximum ordinate or mode. This is in many ways much to be preferred to the average as a descriptive constant for a skew distribution. For one thing it has a more obvious physical meaning, as it represents the state of affairs which most frequently occurs, i.e. is most probable. Moreover the mode is not affected to nearly the same extent as the average by extreme numbers, which may in some cases be due to extraneous causes. The mode is, however, much more difficult to evaluate, and the ultimate test of its utility in ordinary practice will be the ease of its determination. Nevertheless it may be questioned whether, if we are going to search for the single numerical quantity which will best characterise the phagocytic count, either the mean or the mode is the best for the purpose. If the variability of the distribution, the extent to which it ranges and the frequency with which deviations from the mean or mode occur are the essential points of difference between a normal serum effect and an immune serum effect, then it is certain that neither the mean nor the mode will be the best single measure available. We may look into the point later on but it is one which would have to be settled by the statistician.

The stage then which we have reached in this consideration is this, that the Arithmetic Mean of the phagocytic frequency distribution is the most natural if not the most accurate single expression for the totality of our count. The statement, for instance, that the average number of bacteria per leucocyte is 3·11 is no more true than that the Scottish express travels between London and Edinburgh at 50 miles an hour: in fact it is less true, if only for the reason that the express does at times in the course of every journey travel at this speed. At the same time the average does express something which we can perfectly well understand. It has the great convenience of being readily and rapidly evaluated, and also obviates in practice the necessity for deciding, when two or more leucocytes are in juxtaposition, whether a bacterium situated on a boundary is to be placed in the one or the other.

But it may be objected that the opsonic index is not simply the average of the phagocytic count. Such an average varies, as we know, with thickness of bacterial emulsion, age of serum, length of incubation and so on. It was with the idea of getting rid of these variations that Wright and Douglas devised the opsonic index as a useful gauge of degree of phagocytic activity. The index is defined as the ratio of the averages of the counts for test and normal sera, and is a number intended to show how much more, or, as the case may be, less active a test serum is than a standard normal serum. Normal serum activity—supposed to be more or less constant—supplies the unit, and test serum activity is given in terms of this unit. But apart from the possibility of variations in normal sera the unit is variable owing to the impossibility of standardising the bacterial emulsions used in the test. This difficulty is got over by making a fresh evaluation of the unit for each separate group of determinations. The index is, however, at bottom what it is defined to be, the ratio of two averages, and it is with the value of the particular average used, as a measure of the phagocytic distribution, that we are chiefly concerned. Whether, with varying concentrations of emulsion, longer or shorter incubation, the ratio of the averages of normal and test serum distributions remains the same is quite a separate question and involves the discussion of the comparability of indices estimated under different conditions and at different times. But if we were merely to limit ourselves to an opinion as to whether a given serum is subopsonic, normal or hyperopsonic, we must still know whether it is sufficient for this purpose to compare averages based upon a limited count and what that count ought to be.

We distinguish therefore two questions—in reality they merge into one another:—(1) the question as to whether the average of the phagocytic count as ordinarily carried out affords a characteristic which is practically sufficient to differentiate blood sera; (2) whether, if the average of a test serum is in every case expressed by a separate determination in terms of the average of a normal serum taken under the same conditions, we shall actually get a number which is not affected by peculiarities of the observer or other sources of variation. Question (1) in the first place requires an answer either in the affirmative or the negative, and in the second place necessitates a statement of the approximate limits within which that differentiation is justifiable. Now no one who has worked much with opsonic indices should have much hesitation in saying that they *will* serve to differentiate a highly specific serum from a normal one. The results which we obtain in the case of the glanders bacillus, staphylococci and streptococci, the Micrococcus melitensis, and the organisms of typhoid and Malta fevers are all so striking as to leave no shadow of doubt upon this simple point. It becomes more difficult, however, to dogmatise on the minimum limits within which we may accept difference of index as equivalent to significant differentiation. This is a point on which mathematician and observer might well work hand in hand to reach a conclusion. It will be surmised by workers on the subject that a different answer might have to be given to this question in the case of different bacteria. Such differences—as far as the specific bacterium alone is concerned—may well be connected with the capability of the bacillus to act as an antigen or its capability of being opsonised. The susceptibility of one of the reacting substances (the opsonisable substance) to the action of the other (the opsonin) is, I suspect, one of the elements which enter into the differentiation of phagocytosing leucocytes as regards the number of bacteria ingested by them. If the organism whether through age, quality of culture medium, virulence or some such cause is hard to opsonise, then weakly opsonic sera will not give that differentiation from a normal serum which they would do were the antigen somewhat more susceptible to the influence of the antibody. Dr Greenwood in an admirable paper on this subject has given us some idea of the limits of reliability for differentiation of the index in the case of the tubercle bacillus for counts of 25 leucocytes. This reliability lay outside the range 0·85 to 1·3. It still remains to examine in similar fashion what the reliability is for counts of 100 instead of 25. The case chosen for illustration, however, is one which bears most hardly upon this question of the reliability of the opsonic index. The tubercle powder used for making the emulsions invariably consists of old, dead and dried bacilli. It is scarcely to be expected that the reactibility (if I may use the term) of the organism under these circumstances would stand very high. The degree of variability therefore which one could hope to get with such a reagent would not be very extended, and this is borne out by experience. The records of the Inoculation Department of St Mary's Hospital, London, now afford a very considerable body of data which would I think well repay the analysis of some kindly mathematician. The records concerning the tuberculo-opsonic index are particularly large, and although by taking this index as the basis of our argument we state the case for reliability in the most depreciatory manner, still its supreme importance requires that it should be considered before all others. The rule at St Mary's Hospital is to take from different persons in the laboratory several normal sera against which to compare the test sera for the day. We have therefore here material for testing variability of normal and test serum indices. Suppose we have, corresponding to any given batch of test sera, three normal sera 1, 2, and 3, then the average of any one of these normal sera distributions may be supposed equally valid as supplying the normal unit for the estimation of the index. Taking the three normal sera by themselves we might calculate a set of "normal" serum indices by dividing each normal average by the other, that is to say 1 by 2, 1 by 3, 2 by 1, 3 by 1, 2 by 3 and 3 by 2. In the same way we might obtain "test" serum indices by dividing the average for each "test" serum by the averages of the several normal sera in turn. By doing this systematically in every case and collating results we should obtain two frequency distributions, and be in a position to state what is the degree of variability amongst normal and test sera, at least as tested by means of an emulsion of old and dried tubercle bacillus. Further if we assume that degree of variability is the basis of the differentiation of an opsonic serum from a normal serum then the degree of variability of the corresponding averages or ratios of averages may give us some idea at any rate of the limits within which their differentiation becomes justifiable. The objection to the procedure is: that we introduce into the "normal" index frequency distribution both indices, and their reciprocals. It would probably be better then to use some other method which would not have this effect. It is essential too, in selecting any one normal average for use in the determination either of normal or of test indices, that it should be altogether a random selection. The plan adopted to obviate the first difficulty and to comply with the requirements of random selection was to take only one normal average as divisor. Amongst the two or three normal averages from which I had to select in any given batch of observations, if I took the first recorded in the one case, then I took the second in the next and the third in the next, or if there was no third went back to the first. This method was followed for both "normal" index and "test" index distributions. In order to compare the variabilities in question, one forms the frequency distributions and then the frequency curves corresponding to normal and to test indices. The individual indices were based on counts of 100 leucocytes. The distributions were as follows:—

Tuberculo-Opsonic Indices. Normal Sera 30. 11. 08 *to* 6. 3. 09.

Index	Frequency	Index	Frequency	Limits of Grouping
·70	1	*1·00*	74	·70=·695 to ·715 and so on.
·72	0	*1·02*	56	
·74	0	*1·04*	61	
·76	1	*1·06*	39	
·78	2	*1 08*	37	
·80	4	*1·10*	24	
·82	2	*1·12*	18	
·84	12	*1·14*	11	
·86	12	*1·16*	7	
·88	16	*1·18*	5	
·90	20	*1·20*	3	
·92	48	*1·22*	1	
·94	43	*1·24*	1	
·96	49			
·98	63			
		Total	610	

Tuberculo-Opsonic Indices. Test Sera 30. 11. 08 *to* 6. 3. 09.

Index	Frequency	Index	Frequency	Index	Frequency	Limits
·28	1	*·76*	12	*1·24*	15	·28=·275 to ·295
·30	0	*·78*	12	*1·26*	15	
·32	0	*·80*	18	*1·28*	11	
·34	0	*·82*	29	*1·30*	9	
·36	0	*·84*	28	*1·32*	5	
·38	1	*·86*	47	*1·34*	3	
·40	0	*·88*	36	*1·36*	6	
·42	1	*·90*	47	*1·38*	6	
·44	1	*·92*	49	*1·40*	4	
·46	0	*·94*	47	*1·42*	4	
·48	0	*·96*	52	*1·44*	1	
·50	0	*·98*	51	*1·46*	0	
·52	1	*1·00*	57	*1·48*	1	
·54	0	*1·02*	50	*1·50*	3	
·56	0	*1·04*	43	*1·52*	1	
·58	2	*1·06*	40	*1·54*	1	
·60	3	*1·08*	52	*1·56*	2	
·62	7	*1·10*	38	*1·58*	3	
·64	6	*1·12*	36	*1·60*	1	
·66	7	*1·14*	20	*1·62*	2	
·68	8	*1·16*	24	*1·64*	0	
·70	5	*1·18*	16	*1·66*	0	
·72	9	*1·20*	20	*1·68*	1	
·74	15	*1·22*	15			
				Total	1000	

These are the distributions as they were worked out. By using a coarser grouping we obtain a more regular and more workable series as given in the following tables:—

Normal Serum.

Index	Frequency	Index	Frequency	Limits
·70	1	*1·06*	137	·70 = ·695 to ·755
·76	3	*1·12*	53	
·82	18	*1·18*	15	
·88	48	*1·24*	2	
·94	140			
1·00	193			
		Total	610	

Test Serum.

Index	Frequency	Index	Frequency	Limits
·28	1	*1·00*	158	·28 = ·275 to ·335
·34	0	*1·06*	135	
·40	2	*1·12*	94	
·46	1	*1·18*	60	
·52	1	*1·24*	45	
·58	5	*1·30*	25	
·64	20	*1·36*	15	
·70	22	*1·42*	9	
·76	39	*1·48*	4	
·82	75	*1·54*	4	
·88	130	*1·60*	6	
·94	148	*1·66*	1	
		Total	1000	

We may notice now some of the points of difference between the two distributions. In the normal case the range of indices is between ·70 and 1·24, whereas for test sera the range is much greater, being between ·28 and 1·66. But whilst we may be prepared to admit that such indices as ·70 and 1·24 are quite improbable for normal sera, we still want to know the limits within which we are to diagnose a serum normal or abnormal. This point can to some extent be answered from the figures before us without further treatment. Thus, confining our attention to the data for normal sera alone, we note that we may expect roughly 96 to 97 °/ₒ of normal sera to afford indices ranging within the limits ·89 and 1·19. So also we observe, on the same basis, the following expectations regarding the normality of more limited ranges:

Limits	
·83 to ·89	2·95 %
·89 to ·95	7·87 %
·95 to 1·01	22·95 %
1·01 to 1·07	31·64 %
1·07 to 1·13	22·46 %
1·13 to 1·19	8·69 %
Total	96·56 %

Now how do these percentages compare with those found amongst test sera? If our "test" sera indices had all referred to the first examinations of patients, we should then have been in a position to say how frequently any given index occurred with the serum obtained from an individual suspected of being tubercular. With such data we should also have been able to state definitely what were the probabilities for and against a given index being normal or abnormal. This point has been treated of by Noon and Fleming*. The figures given above are unfortunately not those of patients examined only for the first time. They are simply the indices of patients examined from day to day, irrespective of whether they had presented themselves for the first time, were under inoculation treatment, or had received some such special treatment as massage, exercise, Bier's bandage and so on, in order to bring out more distinctly the normality or abnormality of their condition. Indeed a large proportion must have been under treatment at the time of the examinations here recorded. We should therefore expect that a great number of those who were originally subnormal were being maintained at normal by the inoculations, whereas, owing to the well recognised difficulty of maintaining a hypernormal index in tuberculosis, those amongst the originally normal who passed over into the category of the hypernormal were not so numerous. Still the figures as they stand give us a considerable amount of information. They show the much greater extent to which test sera vary as compared with normal sera, and how in spite of the selection implied by the inclusion of an excess of normal individuals there still remains a much greater deviation of indices from the mean than with normal sera. A comparison of the two distributions will also give us some idea of the extreme limits within which a serum can be called normal. It will show us too the relative frequency with which given indices occur in normal and abnormal cases—always remembering that in this comparison the proportions, for the reason given, are likely to be overlaid with odds in favour of normality. With this qualification we set down the figures.

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* *Lancet*, April 25, 1908.

Limits	Normal Sera	Test Sera
·83 to ·89	2·95 °/ₒ	7·5 °/ₒ
·89 to ·95	7·87 °/ₒ	13·0 °/ₒ
·95 to 1·01	22·95 °/ₒ	14·8 °/ₒ
1·01 to 1·07	31·64 °/ₒ	15·8 °/ₒ
1·07 to 1·13	22·46 °/ₒ	13·5 °/ₒ
1·13 to 1·19	8·69 °/ₒ	9·4 °/ₒ
Totals	96·56 °/ₒ	74·0 °/ₒ

But the best comparison between the distributions is only made after the roughnesses associated with each individual observation are allowed for, that is to say, after we have determined the smooth curve which shall represent the distributions with a minimum of error.

Let I denote the normal serum distribution and II the test serum distribution; then the following are the equations and constants of the curves corresponding to them. Both curves come under the head of Pearson's Type IV, namely

$$y = y_0 \left(1 + \frac{x^2}{a^2}\right)^{-m} e^{-\nu \tan^{-1}\frac{x}{a}}.$$

I. $$y = 167{\cdot}71\left(1 + \frac{x^2}{(5{\cdot}4950)^2}\right)^{-10{\cdot}5796} e^{2{\cdot}5281 \tan^{-1}\frac{x}{5{\cdot}4950}}.$$

Standard Deviation = 1·30, Mean = 1·0039,
Mode = 1·0080.

II. $$y = 131{\cdot}71\left(1 + \frac{x^2}{(7{\cdot}9997)^2}\right)^{-5{\cdot}3714} e^{1{\cdot}6666 \tan^{-1}\frac{x}{7{\cdot}9997}}.$$

Standard Deviation = 2·93, Mean = 1·0048,
Mode = 0·9878.

Curve I is represented by 610 observations while Curve II is based on 1000; therefore in order to compare them we must proportionately magnify the first to give an equal area with the second. Better still we may reduce Curve I so that the highest point upon it shall just touch the point corresponding to it in Curve II. The reduction is shown in Fig. 2. Then the area of Curve I as shown now in this reduced form will represent the degree of occurrence of as many indices out of the test sera indices as can possibly be considered normal, i.e. this area within the larger area represents the maximum computation of normal serum indices. The method of reduction adopted gives a somewhat undue preponderance to the normals over and above that already given. We must keep this point in mind in drawing any deductions from the curves. It is taken for granted here that the curve of normal sera indices represents a true random sample.

We may now examine our curves more particularly. Take the index 0·92 as shown on the base line belonging to the two curves of Fig. 2 and we observe that the ordinate corresponding to this point would be nearly bisected by the

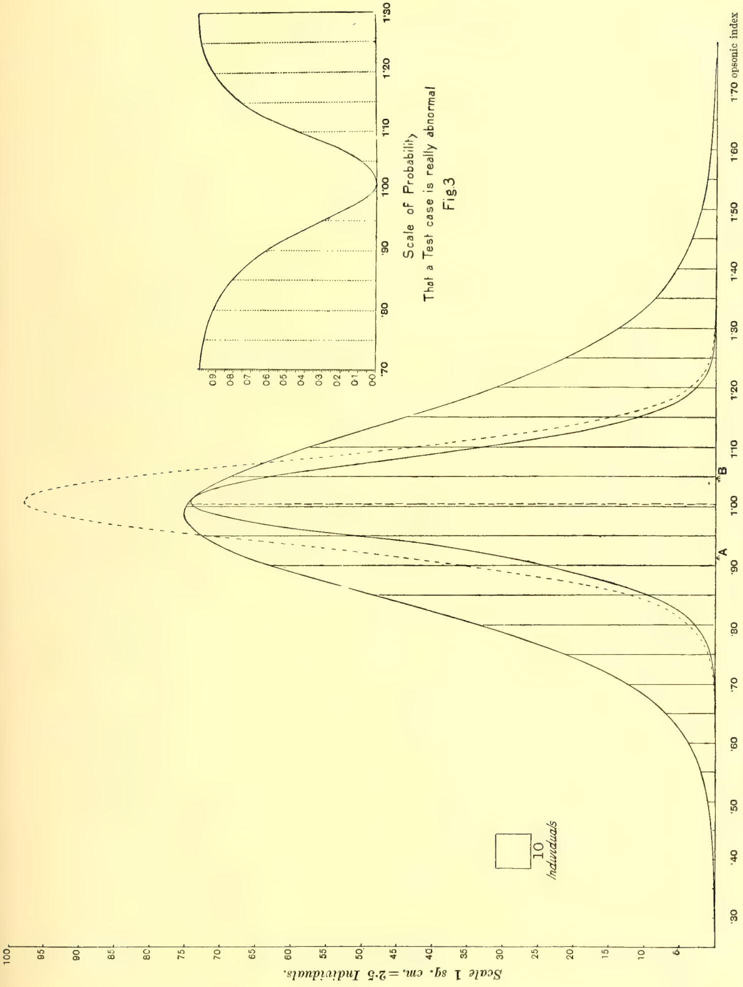

FIG. 2. Comparison of frequency curves of opsonic indices derived from normal and test sera respectively.

A and *B* are the coordinate origins of the equations to the curves.

inner of the two curves. This signifies that for this index the odds in favour of abnormality are not more than those in favour of normality, i.e. the odds are even. In the case of an index unity the probabilities that we are dealing with an abnormal index are seen to be very small, while for indices of ·7 or 1·3 they are very considerable. Fig. 3 shows to scale the actual probabilities for all indices ranging between ·7 and 1·3. In the neighbourhood of the index 1 we see that probability is practically zero. As we proceed outwards in either direction we find the chances of abnormality are becoming greater and greater. The values are as follows:—

Chances in Favour of Abnormality.

Index	Probability	Index	Probability
·70	·990	*1·05*	·081
·75	·965	*1·10*	·428
·80	·914	*1·15*	·750
·85	·801	*1·20*	·907
·90	·614	*1·25*	·976
·95	·303	*1·30*	·991
1·00	·018		

Within the limits ·8 and 1·2 the probability curve slopes rapidly to zero. If then we postulate that for a definite statement on a single examination a probability of at least ·9 is required—and this signifies that our statement would at the worst be wrong once in ten times—we fix our limits of normality at approximately ·8 to 1·2. Outside these we can be reasonably certain that we are dealing with abnormal sera.

These being the facts and the analysis of the facts, it seems reasonable to conclude that even in the admittedly difficult case of the tubercle bacillus we have a means of differentiating within certain limits a hyperopsonic or subopsonic serum from a normal one.

Before proceeding to consider our second point as to the comparability of indices, we may look at the question of simple differentiation from another point of view. We shall leave aside for a moment the index altogether, and fall back upon the frequency distributions from which the averages which enter into the index are calculated. Take the following tuberculo-opsonic distributions, the names attached to which sufficiently explain them.

Number of Bacteria ingested	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	Mean	Index
Number of leucocytes:											
1. Normal Serum	10	29	28	18	9	2	2	1	1	2·12 ± ·08	—
2. Test Serum I	12	16	24	30	8	6	1	1	2	2·45 ± ·11	1·16
3. Test Serum II	14	18	26	18	9	6	6	2	1	2·48 ± ·13	1·17

Standard Deviation (S. D.) Normal = 1·53.
" " I = 1·65.
" " II = 1·85.

We may ask ourselves what are the probabilities that the distributions corresponding to Test Serum I and Test Serum II are significantly different from that of the Normal Serum here used and regarded as a type? Let us understand thoroughly what is meant by the use of the expression "significantly different." Had we made a large number of counts of 100 for this normal serum, we should have found that no two of them would be alike. If then we counted at random a large number of such 100's, what are the probabilities that in the long run we should get a distribution like that given in the case of Serum I or Serum II? Put in another form the problem is:—How many successive 100's should we on the average expect to have to count from the slide representing normal serum before we should get a distribution like either of those given by the test sera. Now for distributions like the above, if they contain a sufficiently large number of observations, and still better if they have had an ideal frequency curve fitted to them, the probabilities for and against the chance here referred to can be calculated. In the case we are considering the numbers are much too small for an accurate determination, but they will serve to give us a rough idea of the order of the significance or absence of significance of the differences exhibited by the distributions. Test Serum I works out as significantly different ($P = {\cdot}002$) from the normal serum here utilised in 998 cases out of 1000, and Test Serum II ($P = {\cdot}005$) in 995 cases out of 1000; in other words for this case it would be safe to the extent named to say that we are dealing with sera producing a different degree of phagocytosis from a normal serum. Let me repeat that owing to the small number of observations the figures are only rough approximations to the order of the chances for and against. Further be it noticed that I limit my comparison to this particular normal serum assumed to be a type, as it would not be safe to state the matter any more strongly without a reference to the degree of variability of normal sera. Now as all work on the subject goes to show that opsonic activity is a specific character, our finding may be said to amount to this: that a diagnosis that the serum of these patients was associated with the presence of tubercular disease or the subjection to treatment with a tubercular vaccine would be correct in some 99 cases out of 100.

I have adverted to the point that the selection of the tuberculo-opsonic index as a test case for the reliability of the inferences which are based upon its estimation, was scarcely a fair one, if the point were generalised to the extent of saying that the adverse findings for this particular index applied to the indices obtained with other bacteria. Take the case of the bacillus of glanders, of which—through the kindness of Dr Fleming—I can afford an instance which was only one of similar results continuing in the particular case for week after week. Indeed, the diagnosis of glanders was based upon the results of opsonic index determinations many months before the actual isolation of the organism set all doubts at rest—if there could be any possible doubt in such a case. At the time of taking the serum, which gave the following phagocytic result, the patient was receiving a glanders vaccine.

Number of Bacteria	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*
No. of Leucocytes :																			
1. Normal Serum	13	13	12	10	15	13	10	3	1	3	1	—	—	—	—	—	—	—	—
2. Test Serum	3	5	8	5	9	10	16	10	10	4	5	3	1	5	3	1	1	1	1

Mean 1=3·36 ± ·16. S. D. 1=2·36.
Mean 2=6·66 ± ·27. S. D. 2=3·96. Index =2·00.

We note here the much larger standard deviation which we get in the case of the glanders bacillus as compared with the tubercle bacillus, both as regards normal and test sera. In this case, although the bacillus is used killed, it is not old nor dried. The divergence in this instance between test and normal sera, as judged by standard deviations, is much greater than in the instances chosen for the tubercle bacillus. As regards the order of the probabilities in favour of significant difference between the distributions, we are unable here to consider the case of the test serum distribution arising as a random sample from the normal serum distribution, because the normal serum range for the small numbers here recorded does not extend so far as that of the test serum. We can, however, reverse the problem and ask what are the probabilities against the test serum giving a distribution like that of the normal serum. When we do so, we find that the probability against such being the case is so enormous that the tables provided for the purposes of its evaluation (*Biometrika*, Vol. I. p. 159) do not go far enough to enable us to obtain it. All we can say is that the chances against our normal serum representing merely one random sample of the test serum, are something greater than 999,999 to 1.

If these probabilities give us an indication of the degree of correctness and incorrectness of our inferences regarding opsonic activity, and if we can regard them as representing those for correctness of diagnosis, we should be in a position to compare them with our impressions of the degree of correctness of the usual clinical diagnosis in which some element of doubt exists. Let us take a concrete example. It would, I think, be satisfactory to know that our expressed opinion as to whether a chronically swollen joint represented a gonorrheal or a tubercular infection would be wrong only once in 200, or once in 100, or once in 50, nay even once in 10 times. Now these are just the sort of cases in which an estimation of an opsonic index or, as the case may be, a phagocytic distribution gives us the greatest help.

There are also confirmatory tests which may be, and in fact usually are, applied. One of these is the method of multiple testing. When an important diagnosis has to be made, it is seldom given on merely a single index. The index is determined on several occasions, and if, as usually happens, the indices show characteristic divergences from the normal, the evidence is considered as greatly strengthened for a positive statement. Or again, special methods are applied in order to accentuate the differences from normal or the fluctuations from an original value, due to the presence of infection. The affected part is massaged, or a Bier's bandage put on above a joint for half an hour, so as to flush it with the retarded blood. If after this procedure the opsonic index shows the fluctuations which might be expected from the entrance of either antibodies or antigens into the blood stream, the diagnosis becomes clear. The following examples will illustrate this point:

(1)

April 8, 1908	INDEX	
	To Gonococcus	To Tubercle Bacillus
Before Bier	1·40	1·06
½ hour after	1·24	·89
5 hours after ...	1·36	·90
20 hours after ...	1·84	1·00

(2) *Effect of Massage. B. tuberculosis.*

Before	·87
½ hour after	·95
3 hours after	·97
21 hours after	1·19.

(3) *Effect of Exercise. B. tuberculosis*, 25. 5. 08.

Before	·87
Just after	·97
7 hours after	1·12
19 hours after	1·15.

One of the defects of these illustrations is that we are ignorant of the effects produced on the variability of normal serum indices by, say, the application of a Bier's bandage to a normal joint, massage or exercise. The point is one, however, which could be easily settled.

We proceed now to consider how far opsonic indices are comparable with one another. This is a much more difficult question to answer offhand. It would seem certain that if they are, it can only be under some strict limitations of technique. We are dealing in this determination with a number of varying factors. Some of the chief possibilities of variation and problems are:—

(1) *The Leucocyte.* Number in unit volume; degree of admixture of red blood corpuscles; variation in vitality; degree of settling.

(2) *The Bacterium.* Number in unit volume, i.e. the concentration of the emulsion; virulence; whether laboratory culture or autogenous; degree of agglutination by the sera used; age of culture; dead or living; whether diplobacteria or larger aggregations are to be regarded as single ingestions or more.

(3) *The Serum.* Whether, with the quantities of serum emulsion and leucocytes used, we have, so to speak, the opsonin and antigen present in exact combining

proportions; whether the serum agglutinates the red blood corpuscles; age of serum; length of time of action; variability of normal sera, either of one and the same individual at different times, or of different individuals at the same time; heated or unheated.

The factors here enumerated by no means exhaust the list which might be made. Nor are they in any sense classified; many are not mutually exclusive classes. Moreover, as it is the comparability of indices that we are now considering, we postulate throughout equality of conditions for individual observers. Thus, when we say that age of serum may be a factor affecting the opsonic index, it is not intended by this to raise the question as to whether an old serum might be compared with a fresh one—that, we know, it cannot be. But if we have test serum and normal serum taken at precisely the same time, will the index remain the same if estimated at once, and at varying intervals after clotting of the bloods? Or take the case of length of time of action of the serum in the opsonic mixture. Is it all the same whether we give the mixture 15, 20 or 30 minutes in the opsoniser? Will the ratio of the averages at the different periods remain the same? Might we not get certain of the leucocytes more rapidly surfeited under the action of one serum than the other and so put out of action? In such a case additional time for ingestion would not be of any use to these leucocytes. A great number of such questions arise in the mind of every observer, and it is certain that they cannot be answered without set experiment. A most valuable paper which deals with this side of the subject is that by Dr A. Fleming in *The Practitioner* for May, 1908. But there are many problems still to be solved before the answer can be given as to the limits within which indices are comparable. We may, however, illustrate a few of the points, which have been raised by actual examples.

Example 1. What is the bearing of concentration of emulsion upon degree of phagocytosis? The organism here used was the *Bacillus coli* and the emulsion a very thick one. The emulsion was diluted down in varying degrees, and tested at each dilution against the same serum and the same leucocytes with the following result (Table, p. 81).

The result is a curious one. We see that the frequency distribution for a concentrated emulsion exhibits a smaller variation than that given by the same emulsion when considerably diluted—the standard deviations being as 1 to 2 for a tenfold dilution. Nor was there any agglutination of the bacteria obvious enough to account for this result. The probability is that we were dealing here with a toxic effect—the so-called *aggressin* effect. Again, when we reach the very high dilutions, we notice that there is no significant difference between the distributions given by the 80- and the 100-fold dilutions. Perhaps at this point we have reached a degree of separation of leucocytes and bacteria such that the opportunity for phagocytosis is so small that no difference can be brought out between different dilutions in the time allowed for action. It would be interesting to have a graded series of frequency distributions showing the transition from the inhibitory action of toxins to the maximum development of phagocytosis. Again, it would be interesting to observe the gradual shrinking of the range of phagocytosis with thinning of the emulsion. We may learn from this example that there is probably an optimum concentration for the production of maximum phagocytosis. Doubtless the same phenomenon would be shown by immune sera and by some other organisms than the one here tested. The question immediately arises as to whether this transition would take place at the same point for both an immune and a normal serum. If it did not, we should certainly not get indices which were the same for different degrees of concentration of emulsion, although the normal and immune sera were the same throughout.

Number of Bacteria Ingested	DISTRIBUTION OF LEUCOCYTES WITH						
	Original Emulsion	5 fold dilution	10 fold dilution	20 fold dilution	40 fold dilution	80 fold dilution	100 fold dilution
0	317	40	43	112	192	632	620
1	345	92	101	180	278	282	282
2	172	112	137	184	221	65	79
3	82	142	139	151	139	16	16
4	47	121	130	128	91	4	2
5	20	99	115	96	49	1	1
6	11	100	111	63	21	—	—
7	4	86	74	44	4	—	—
8	1	64	55	22	4	—	—
9	0	45	32	10	1	—	—
10	1	41	24	4	—	—	—
11	—	17	12	3	—	—	—
12	—	11	9	3	—	—	—
13	—	9	7	—	—	—	—
14	—	5	6	—	—	—	—
15	—	5	2	—	—	—	—
16	—	2	1	—	—	—	—
17	—	2	1	—	—	—	—
18	—	3	0	—	—	—	—
19	—	2	1	—	—	—	—
20	—	2	—	—	—	—	—
Means	1·34	5·03	4·54	3·05	1·94	0·48	0·50
Standard Deviations	1·41	3·38	2·98	2·29	1·62	0·74	0·74

Example 2. The effect of extracellular agglutination of the bacteria of the emulsion by the test serum. The organism used was the *Bacillus Friedlander* and the emulsion was again a concentrated one. The index given by the test serum as determined in the ordinary routine work of the laboratory with a thin emulsion was 2·17. What a contrast to this is presented by the index obtained with a very thick emulsion—viz. 0·75! The determination, it is true, was made on the following day, but age of serum will not account for this amount of lowering of the index.

Number of Bacteria Ingested	Distribution of Leucocytes with Normal Serum	Distribution of Leucocytes with Immune Serum
0	20	41
1	26	52
2	36	57
3	45	42
4	42	30
5	39	24
6	29	20
7	22	8
8	13	3
9	10	7
10	9	2
11	2	6
12	2	3
13	1	1
14	2	1
15	0	0
16	1	1
17	0	1
18	1	0
19	—	0
20	—	0
21	—	0
22	—	1
Means	4·43 ± ·07	3·33 ± ·12
Standard Deviations	2·98	3·18

Opsonic Index 0·75.

In all probability the further dilution of the sera would bring out the real differences between them. This was done at the time but the slides have not yet been counted. The example serves to show how a strongly opsonic serum may yet under certain conditions give an index actually below normal. The reason for this paradox was that strong agglutination of the bacteria took place with the test serum, whereas no visible agglutination took place with the normal serum. The resulting effect was, we may suppose, that for the test serum the agglutination so reduced the concentration of the emulsion as to diminish the phagocytosis below that which took place for a normal serum and a concentrated emulsion. Bacteriolytic action may have contributed to the diminution in the case of the immune serum. With both immune and normal sera a considerable amount of intracellular digestion had taken place. Now although agglutination is often rapid with concentrated emulsions of bacteria, it is much slower for thin emulsions. Opsonic action, on the other hand, is still very rapid even with thin emulsions. With dilution of the emulsion the agglutinating action would be slower and the opsonic action would proceed without this disturbing factor. The same argument would in all likelihood apply to a dilution of a strongly agglutinating serum, even when the emulsion remained concentrated. It is frequently stated that a highly agglutinating serum may contain no opsonin, but before any such statement is accepted we require to know whether it holds good (*a*) on successive dilution of the serum, (*b*) on successive dilution of the emulsion. Our example serves to show that agglutination may play a large part in vitiating the comparability of indices. But agglutination may be slight and test sera may not be so active as the one here used. Therefore it again becomes necessary to determine, for agglutinating sera of all grades, what limitations agglutination imposes on the comparability of indices. It must also be evident that this factor must be a varying one for different laboratories.

Example 3. One of the remarkable features of an opsonic frequency distribution is the variability or degree of dispersion exhibited. Sir Almroth Wright has suggested that this may be due in part at any rate to unequal opportunity for phagocytosis. Leucocytes and red blood corpuscles are heavy as compared with bacteria, and tend rapidly to settle down. Although the capillary tubes are placed horizontally in the opsoniser, there will nevertheless be a certain amount of settling and deposition in layers. Those leucocytes in the lowest layers will have much less opportunity to ingest bacteria than the topmost ones. Consequently we may expect to get every degree of ingestion from those containing no bacteria up to those containing as many as they can take up in the time, and with the particular concentration of emulsion. Now if we could keep the tubes in constant rotary movement, we might expect on the above hypothesis that the degree of variation would be greatly reduced, and this is what does happen: but instead of finding that all the leucocytes, as on the basis of equal opportunity they should, had taken up more or less the same number of bacteria, we find that the diminution in variability has been brought about by an increase in the number of those leucocytes which have taken up few or no bacteria at the expense of those which took up many. Of course, we could scarcely expect to find in any form of experiment that all leucocytes had taken up nearly the same number of bacteria, for leucocytes are themselves living organisms showing, in any one sample, all the stages of development, maturity and decay, correlated with which we should be prepared to find a variable phagocytic activity. Still the diminution of variability of this experiment was probably to be traced to mechanical interference with phagocytosis due to the turning movements. The tubes were not turned continuously, but given a half-twist every 60 seconds. The organism used was the *Streptococcus foecalis* and the emulsion a thick one. The test serum was highly opsonic, giving in the course of routine examination an index of 2·14 on the day previous to this experiment. See Table on p. 84.

Example 4. To test whether it is immaterial if organisms adhering in couplets or triplets should be counted as 2's or 3's or as single organisms in the estimation of an index. The organism taken was the *Streptococcus foecalis*, which, together with the immune serum, was kindly supplied to me by Dr Matthews. This organism makes a fairly good emulsion and the clumps remaining are not greater

Number of Bacteria Ingested	Frequency Distribution of Leucocytes with			
	Normal Serum	Test Serum	Normal Serum with ½ turn every 1′	Test Serum with ½ turn every 1′
0	32	17	72	48
1	36	18	76	49
2	49	33	86	62
3	45	29	64	50
4	71	27	49	48
5	31	34	43	42
6	38	25	32	36
7	34	28	29	35
8	42	22	14	26
9	25	37	12	33
10	22	27	10	16
11	14	20	4	14
12	20	28	3	8
13	7	24	2	8
14	13	23	2	7
15	5	14	0	6
16	6	16	1	5
17	2	9	0	1
18	5	7	1	1
19	1	9	—	3
20	1	9	—	0
21	0	10	—	0
22	1	4	—	0
23	—	3	—	1
24	—	6	—	1
25	—	4	—	—
26	—	5	—	—
27	—	3	—	—
28	—	1	—	—
29	—	0	—	—
30	—	3	—	—
31	—	1	—	—
32	—	0	—	—
33	—	0	—	—
34	—	1	—	—
35	—	1	—	—
36	—	0	—	—
37	—	0	—	—
38	—	0	—	—
39	—	0	—	—
40	—	0	—	—
41	—	1	—	—
42	—	1	—	—
Means	5·96 ± ·13	9·93 ± ·21	3·47 ± ·09	5·22 ± ·13
Standard Deviations	4·28	6·94	3·00	4·21
Opsonic Index	—	1·67	—	1·50

than couplets or triplets. A count was made of 500 leucocytes in both ways for both normal and immune sera. The results were as follows:—

I. The case where every organism ingested was separately counted whether in a clump or isolated.

Number of Bacteria Ingested	FREQUENCY DISTRIBUTION LEUCOCYTES	
	Normal Serum	Test Serum
0	32	17
1	36	18
2	49	33
3	45	29
4	71	27
5	31	34
6	38	25
7	34	28
8	42	22
9	25	37
10	22	27
11	14	20
12	20	28
13	7	24
14	13	23
15	5	14
16	6	16
17	2	9
18	5	7
19	1	9
20	1	9
21	0	10
22	1	4
23	—	3
24	—	6
25	—	4
26	—	5
27	—	3
28	—	1
29	—	0
30	—	3
31	—	1
32	—	0
33	—	0
34	—	1
35	—	1
36	—	0
37	—	0
38	—	0
39	—	0
40	—	0
41	—	1
42	—	1
Means	5·96 ± ·13	9·93 ± ·21
Standard Deviations	4·28	6·94
Opsonic Index	—	1·67

II. The case where couplets and triplets are counted as single ingestions.

Number of Bacteria Ingested	Frequency Distribution Leucocytes	
	Normal Serum	Test Serum
0	32	17
1	57	39
2	87	44
3	70	41
4	52	55
5	57	48
6	53	37
7	33	47
8	21	32
9	16	33
10	7	25
11	5	16
12	3	15
13	5	18
14	1	8
15	0	8
16	1	6
17	—	7
18	—	4
19	—	2
20	—	2
21	—	2
22	—	0
23	—	2
24	—	0
25	—	0
26	—	0
27	—	2
Means	4·10 ± ·09	6·65 ± ·14
Standard Deviations	2·86	4·74
Opsonic Index	—	1·62

So far as the actual index goes there would not appear to be any great difference produced on counting by either method. Probably a better way of determining the point would be to fit frequency curves to each of the distributions and then test for goodness of fit. The one in which the fit was worse—if there were a marked difference—would represent the worse method of counting. I have carried out this test in one case of a tuberculo-opsonic distribution which, though not conclusive even for the organism used, is at least suggestive. The frequency distributions and the corresponding curves are here given:—

Number of Bacteria Ingested	FREQUENCY DISTRIBUTION OF LEUCOCYTES	
	(1) Couplets and triplets counted as 2's and 3's	(2) Couplets and triplets counted as single ingestion
0	194	194
1	244	271
2	228	249
3	150	146
4	97	91
5	60	57
6	38	22
7	20	2
8	3	2
9	3	3
10	3	3
Means	2·20	2·00
Standard Deviations	1·86	1·67

Both distributions come under the head of Pearson's Type I and are given by the equations

$$(1)\quad y = 268{\cdot}7601\left(1+\frac{x}{1{\cdot}086}\right)^{{\cdot}484}\left(1-\frac{x}{14{\cdot}567}\right)^{6{\cdot}491}.$$

$$(2)\quad y = 297{\cdot}574\left(1+\frac{x}{1{\cdot}825}\right)^{1{\cdot}838}\left(1-\frac{x}{193{\cdot}113}\right)^{194{\cdot}484}.$$

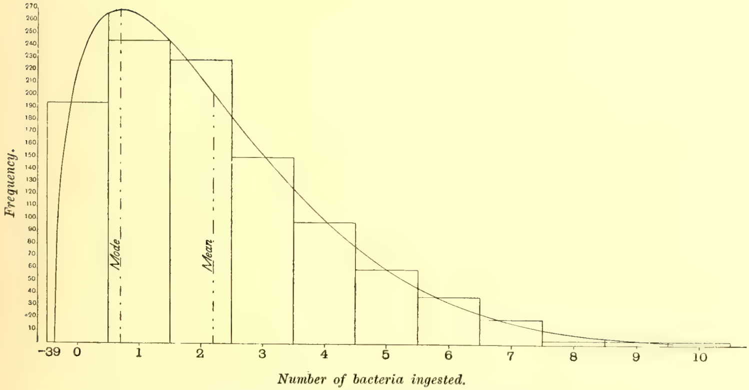

FIG. 4. Couplets and triplets counted as 2's and 3's.

When these curves are tested for goodness of fit to the observations, it is found that this is much better for the case in which couplets and triplets are counted as 2's and 3's than for the case where they are counted as single ingestions, $P = \cdot 51$* as against $P = \cdot 05$. It is quite conceivable that a still better fit would be obtained if we were to regard a couplet for purposes of estimation, say as 1·5 or some other intermediate value between 1 and 2. If such were the case we might conclude that, provided the criterion is sound, this would be the proper value to assign to a couplet inclusion.

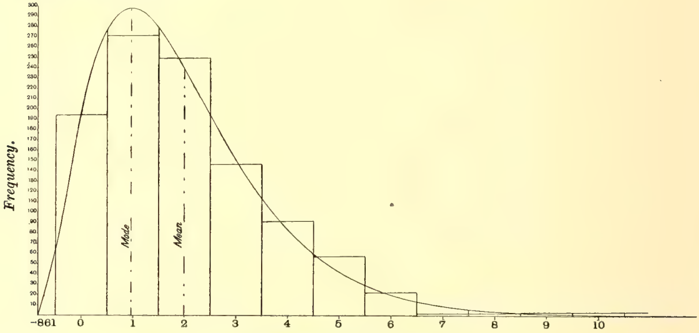

FIG. 5. Couplets and triplets counted as single ingestions.

Other points than those illustrated or mentioned will occur to every worker in these fields. Further, those that are illustrated are single trials, which require repetition both with regard to the particular organism used and to other organisms. They are given here for the purpose of focussing attention on the points which must be considered in discussing the question of the reliability or comparability of the opsonic index as determined by different observers. When, however, such points as the possibility of there being *any* specific opsonic variation demonstrable in tuberculous sera and still more in the case of other sera are disputed, it seems to me that the proof that they are is so overwhelming as to be incontrovertible. All honour then to those who have elaborated so important a method as that here considered; for they have added to the list of quantitative estimations applicable to medical diagnosis and therapeutics one of incalculable value. Its chief drawback is the difficulty of its determination.

It remains now only to consider some of the variations which have been proposed upon the original and very natural measure proposed by Wright and Douglas. Much more criticism seems to have been levelled at the method of measuring the effect of vaccino-therapy than upon the efficacy of vaccino-therapy itself, which seems somewhat of a reversal of their relative importance. One would imagine that those who invented the opsonic index and initiated vaccino-therapy would prefer to have their names more closely associated with the latter than with the former.

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* $P = 1$ represents perfect fit.

Let us take up the discussion, however, on the minor point as to whether an improvement could be made on the method of estimation of the opsonic index. The average is, as we have already stated, but a rough measure of an opsonic frequency distribution. The suggestion has emanated from America that in place of the ratio of averages we should take the ratio of the percentages of leucocytes which have failed to phagocytose at all. The objection which will occur to every one is that this may very likely include a certain number of dead leucocytes. Again, it is a cardinal point in determining the best measure for expressing the features of a frequency distribution that it should be based on the whole number of observations. This the "zero index"—as we may call it—is not. At the same time it must be conceded that the "zero index" is more rapidly determinable than the "average index," and so would allow of the errors associated with its determination being lessened by affording more time to increase the number of observations. It would be specially of advantage where the emulsions used were at all concentrated and where consequently the strain and difficulty of counting the numerous bacteria ingested become very great. It might be of use also in cases where it is not easy to obtain uniform emulsions, and where the leucocytes may contain chains or clumps of bacteria. On the other hand, with concentrated emulsions the number of leucocytes containing no bacteria becomes greatly diminished, and may in fact simply come to represent those which are actually dead. In this case, instead of taking these leucocytes as our gauge of phagocytic power, we might take those containing only one bacterium instead. I have made a small number of observations on the degree of correspondence of the two indices in the case of *B. tuberculosis*. The slides from which the two indices were determined were those which had served for the routine estimations of the laboratory. All the indices were calculated from countings done by myself alone. The distributions of the two were as follows:—

Average Indices	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*	*1·0*	*1·1*	*1·2*	*1·3*	*1·4*	*1·5*	*1·6*	Mean	S. D.
Frequency	0	1	5	4	6	17	17	12	17	13	1	3	2	2	·96 ± ·02	·24

Zero Indices	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*	*1·0*	*1·1*	*1·2*	*1·3*	*1·4*	*1·5*	*1·6*	Mean	S. D.
Frequency	6	5	6	5	10	10	16	10	7	8	5	6	3	3	·91 ± ·02	·35

We note at once a marked difference between the distributions. The "average index" shows a greater concentration about the mean than the "zero index." In fact the zero index shows a greater variability than the other, a result which we should have expected from the fact that the zero index is based on only one observation whereas the other is based on the total number of observations. The result is that the probable error of the former will be greater than that of the latter. The coefficients of variation are

Zero Index = 38·6,
Average Index = 25·4.

The actual detailed correspondence between the two is given in the following table from which we obtain a correlation coefficient of $r = 0{\cdot}76$.

"Average" Index.

"Zero" Index.	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*	*1·0*	*1·1*	*1·2*	*1·3*	*1·4*	*1·5*	*1·6*	Totals
·3	—	1	3	1	—	1	—	—	—	—	—	—	—	—	6
·4	—	—	1	1	—	1	2	—	—	—	—	—	—	—	5
·5	—	—	—	1	4	—	1	—	—	—	—	—	—	—	6
·6	—	—	—	1	—	3	—	—	1	—	—	—	—	—	5
·7	—	—	1	—	—	2	3	—	2	2	—	—	—	—	10
·8	—	—	—	—	1	4	2	2	—	1	—	—	—	—	10
·9	—	—	—	—	1	4	6	2	3	—	—	—	—	—	16
1·0	—	—	—	—	—	2	1	3	3	—	1	—	—	—	10
1·1	—	—	—	—	—	—	—	4	1	2	—	—	—	—	7
1·2	—	—	—	—	—	—	2	1	3	2	—	—	—	—	8
1·3	—	—	—	—	—	—	—	—	1	4	—	—	—	—	5
1·4	—	—	—	—	—	—	—	—	3	1	—	1	—	1	6
1·5	—	—	—	—	—	—	—	—	—	1	—	1	—	1	3
1·6	—	—	—	—	—	—	—	—	—	—	—	1	2	—	3
Totals	0	1	5	4	6	17	17	12	17	13	1	3	2	2	100

If we designate the "average index" by A and its mean value by $\bar{A}$ and the "zero index" by Z and its mean value by $\bar{Z}$, we have as an equation connecting the two indices regarded as variables

$$A = \bar{A} + {\cdot}53\,(Z - \bar{Z}) \pm {\cdot}01,$$

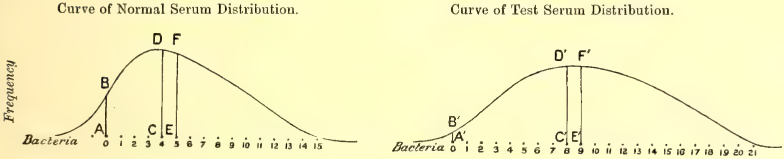
i.e.

$$= {\cdot}96 + {\cdot}53\,(Z - {\cdot}91) \pm {\cdot}01,$$

and the degree of relationship between the one index and the corresponding mean value of the other is shown in the following table:—

Zero Index	AVERAGE INDEX	
	Calculated	Observed
·3	·64	·55
·4	·70	·74
·5	·75	·72
·6	·80	·82
·7	·86	·94
·8	·91	·89
·9	·96	·91
1·0	1·01	1·01
1·1	1·07	1·07
1·2	1·12	1·06
1·3	1·17	1·18
1·4	1·23	1·25
1·5	1·28	1·40
1·6	1·33	1·47

As a matter of fact the zero index probably has its greatest exactness for very thin emulsions, where the number of leucocytes containing no bacteria approximates to the modal ordinate in our frequency curve. This leads me to refer again to a third measure—the mode—which has been proposed as being more descriptive of the phagocytic frequency distribution, and was given by Dr Greenwood (Discussion, Pathological Section, Royal Society of Medicine). There is no doubt that it would have considerable advantages over the arithmetic mean, but it is a difficult value to calculate directly. Further, it would be necessary in every count to set out the frequency distribution, and this is not necessary in the estimation of an arithmetic mean. We may show these different indices in graphic form.


Curve of Normal Serum Distribution.

Curve of Test Serum Distribution.

$$\text{Average Index} = \frac{A'E'}{AE}.$$

$$\text{Zero Index} = \frac{AB}{A'B'}.$$

$$\text{Modal Index} = \frac{A'C'}{AC}.$$

If, however, we are to search for some one constant more expressive of a frequency curve than the mean it seems to me that even more characteristic as a measure of dispersion than the mode would be the standard deviation. Our index would then consist of the ratio of the standard deviations. We do not know exactly why all leucocytes do not take up very nearly the same number of bacteria. But the fact is that they do not. The leucocytes, which we may for the moment regard as the prime factor, show differentiation as regards phagocytosis, and the degree of differentiation seems to depend on the degree of opsonic activity of the test sera. We are not to look on this differentiation as something of the nature of personal error or error of technique. It would appear to be an integral part of the phenomenon of phagocytosis. We should look then to a measure of differentiation or variability as giving us the best indication of the degree of activity which we are endeavouring to estimate quantitatively. For this purpose the standard deviation, if the argument is sound, would be a good one to employ. It is quite conceivable that within certain limits the standard deviation of the frequency distribution for normal serum might be very constant, in which case it might not be necessary to put up a normal serum control on each occasion. It is doubtful, however, whether one would get in this way significant differences for the tubercle bacillus, although it might work with other organisms. In the following tables I give the calculated means and standard deviations together with the frequency distributions and the corresponding indices for two organisms.

Counts were made of 100 leucocytes at a time, and these component distributions are here recorded in order to give an idea of the variability of means, standard deviations, etc. with such small samples. The two organisms used were the *Streptococcus foecalis* and the *Bacillus friedlander.*

I. *Normal Serum. Streptococcus foecalis.*

B[*]	*L* (1)[*]	*L* (2)	*L* (3)	*L* (4)	*L* (5)	Totals
0	4	6	8	5	9	32
1	10	3	11	5	7	36
2	9	12	10	8	10	49
3	6	6	13	7	13	45
4	14	14	10	13	20	71
5	6	9	8	6	2	31
6	9	8	5	7	9	38
7	6	7	9	8	4	34
8	11	7	8	14	2	42
9	6	8	2	4	5	25
10	3	6	2	9	2	22
11	2	4	2	1	5	14
12	6	2	4	4	4	20
13	0	2	2	1	2	7
14	4	3	2	1	3	13
15	1	1	0	2	1	5
16	1	1	1	2	1	6
17	0	0	0	1	1	2
18	2	1	1	1	—	5
19	—	—	1	0	—	1
20	—	—	1	0	—	1
21	—	—	—	0	—	0
22	—	—	—	1	—	1
Totals	100	100	100	100	100	500
Mean S. D.	6·17 4·24	6·16 3·99	5·42 4·44	6·65 4·38	5·38 4·21	5·96 4·28

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[*] *B*=Number of Bacteria ingested. *L* (1), *L* (2) etc.=successive frequency distributions of ingesting leucocytes.

II. *Immune Serum. Streptococcus foecalis.*

B	*L* (1)	*L* (2)	*L* (3)	*L* (4)	*L* (5)	Totals
0	4	4	1	3	5	17
1	4	4	4	3	3	18
2	7	9	9	5	3	33
3	3	7	3	6	10	29
4	6	6	9	2	4	27
5	11	5	5	8	5	34
6	7	6	3	4	5	25
7	8	5	5	3	7	28
8	4	2	4	6	6	22
9	5	11	4	10	7	37
10	4	4	4	11	4	27
11	6	6	4	2	2	20
12	4	2	11	5	6	28
13	4	5	8	3	4	24
14	3	8	1	6	5	23
15	2	0	6	5	1	14
16	5	2	4	2	3	16
17	1	0	0	7	1	9
18	1	1	3	0	2	7
19	0	3	1	2	3	9
20	2	0	2	1	4	9
21	3	3	2	1	1	10
22	1	1	0	0	2	4
23	0	1	0	1	1	3
24	0	2	0	1	3	6
25	1	0	3	0	0	4
26	0	2	1	1	1	5
27	0	0	0	1	2	3
28	1	0	0	0	—	1
29	0	0	0	0	—	0
30	0	1	2	0	—	3
31	1	—	0	0	—	1
32	0	—	0	0	—	0
33	0	—	0	0	—	0
34	1	—	0	0	—	1
35	0	—	1	0	—	1
36	0	—	—	0	—	0
37	0	—	—	0	—	0
38	0	—	—	0	—	0
39	0	—	—	0	—	0
40	0	—	—	0	—	0
41	1	—	—	0	—	1
42	—	—	—	1	—	1
Totals	100	100	100	100	100	500
Mean S. D.	9·58 7·53	9·30 6·81	10·52 7·19	9·95 6·43	10·19 7·11	9·97 7·09

Indices on totals: Average = 1·67.
„ „ Zero = 1·88.
„ „ S. D. = 1·66.

I. *Normal Serum. B. mallei.*

B	*L* (1)	*L* (2)	*L* (3)	*L* (4)	*L* (5)	Totals
0	5	8	13	10	13	49
1	12	12	15	11	13	63
2	17	15	15	15	12	74
3	19	20	18	19	16	92
4	11	18	16	8	15	68
5	10	16	8	15	13	62
6	11	4	7	7	10	39
7	8	3	4	9	3	27
8	2	1	0	1	1	5
9	2	0	3	1	3	9
10	2	1	1	1	1	6
11	0	1	—	2	—	3
12	0	1	—	0	—	1
13	1	—	—	1	—	2
Totals	100	100	100	100	100	500
Mean S. D.	3·86 2·52	3·40 2·25	3·10 2·29	3·72 2·67	3·36 2·36	3·49 2·42

II. *Immune Serum. B. mallei.*

B	*L* (1)	*L* (2)	*L* (3)	*L* (4)	*L* (5)	*L* (6)	*L* (7)	*L* (8)	*L* (9)	*L* (10)	Totals
0	3	2	3	1	4	3	3	3	7	0	29
1	5	5	4	3	1	1	5	3	6	6	39
2	8	9	2	5	1	3	6	6	7	1	48
3	5	8	8	9	9	8	6	6	12	6	77
4	9	11	9	7	9	8	6	9	7	8	83
5	10	10	15	13	14	11	6	14	7	6	106
6	15	12	17	14	9	13	15	10	7	16	128
7	10	10	9	11	13	7	10	7	8	12	97
8	10	6	8	9	14	12	10	9	10	14	102
9	4	7	6	7	5	10	8	11	9	6	73
10	5	5	5	9	5	4	10	8	3	7	61
11	3	2	5	3	6	7	5	4	4	4	43
12	1	4	3	3	4	4	1	4	4	6	34
13	5	1	1	1	1	2	1	2	3	3	20
14	3	3	4	1	1	0	1	1	2	2	18
15	1	2	1	1	2	2	3	1	2	0	15
16	1	1	—	1	1	1	3	1	1	2	12
17	1	2	—	0	0	3	0	0	1	1	8
18	1	—	—	1	0	0	1	0	—	—	3
19	—	—	—	0	1	0	—	0	—	—	1
20	—	—	—	1	—	1	—	0	—	—	2
21	—	—	—	—	—	—	—	1	—	—	1
Totals	100	100	100	100	100	100	100	100	100	100	1000
Mean S. D.	6·66 3·96	6·49 3·95	6·51 3·36	6·84 3·62	6·94 3·55	7·39 3·94	7·06 3·93	6·83 3·74	6·30 4·13	7·36 3·49	6·84 3·79

Indices on Totals: Average = 1·97. Zero = 1·68. S. D. = 1·57.

The irregularity of the distributions is a noticeable feature in the case of the *Streptococcus foecalis* as compared with the *Bacillus friedlander*. The variability is also shown by the standard deviations to be much greater for the former than the latter. The irregularity here exemplified is also to be observed in the case of the Staphylococci. It may have something to do with the method of counting of pairs or threes. A pair of cocci ought very possibly to be counted neither as 1 nor as 2 but, as I have suggested, as 1·5, this last figure being the nearest approximation to the degree of effort put forth in the phagocytosis. Again the irregularity may be indicative of nothing more than smallness of sample.

Conclusions.

1. The opsonic index is a valuable means of gauging the effect of therapeutic inoculation and is useful for diagnosis.

2. The tuberculo-opsonic index is capable of giving in routine practice definite indications of the positive or negative opsonic activity of serum for values which range outside 0·8 and 1·2.

3. Opsonic sera give a much greater degree of differentiation amongst the leucocytes in the case of organisms other than the tubercle bacillus. It is impossible to give limits of normality and abnormality in these cases, because no investigations have been made as regards the variability of normal serum action upon these bacteria.

4. It is doubtful whether the opsonic indices as calculated by different observers are comparable with one another.

I have to thank Sir Almroth Wright and his colleagues for having placed records and material freely at my disposal and for many suggestions as to some of the disturbing factors which enter into the estimation of the opsonic index. The experience of the Vaccino-Therapy Laboratory at St Mary's Hospital is now very large, and the technique of opsonic estimations has been reduced to a very fine art. Finally I owe a deep debt of gratitude to Prof. Karl Pearson for his constant supervision of the work, his suggestions of methods and controls and much help.

ON A NEW METHOD OF DETERMINING CORRELATION BETWEEN A MEASURED CHARACTER A, AND A CHARACTER B, OF WHICH ONLY THE PERCENTAGE OF CASES WHEREIN B EXCEEDS (OR FALLS SHORT OF) A GIVEN INTENSITY IS RECORDED FOR EACH GRADE OF A.

By KARL PEARSON, F.R.S.

(1) As an example of the class of cases to which the method of this paper applies I instance that we might be given the ages of candidates for a given examination, and the number of failures at each age, without the individual marks; from these data we might desire to correlate capacity and age. Or, again, we might be given the percentage of first convictions for each age group of the community, and the problem might be to determine the relation between age and the tendency to crime as judged by conviction. Or, age being put on one side, we might desire to correlate any psychical character with anthropometric characters, e.g. the cephalic index in children as a more or less marked racial character with their conscientiousness or shyness, measured by the number of shy or conscientious children at each value of the index.

If the non-measurable, or at least unmeasured character, be classed into a considerable number of groups there is no doubt that the most satisfactory method to adopt is that of the correlation ratio*. The cases we have in view here, however, are those in which no such series of groups has been made, or possibly can be made. As a rule we have, hitherto, fallen back in such cases on a fourfold table method—that is to say, we have been tacitly compelled to drop any advantage that arose from one character having a measured value. Further the result was not unique, depending to some extent on where the division of the measured character was made. The present method is unique, it involves only the discovery of two means and one standard deviation (no product moment, no second standard deviation and no complicated equation having to be worked out); it is in fact singularly brief. It is only for one determination that we have to assume that the Gaussian frequency distribution may be applied with sufficient practical accuracy. This defect (though, I think, to a minor extent) it shares with the fourfold table method; but because it does, the method will serve in the cases to which both can be applied as a useful control method.

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* Pearson: "On the Theory of Skew Correlation." *Drapers Research Memoirs* (Dulau & Co).

(2) The basal idea is so extraordinarily simple that one is inclined to believe that it must have been noticed before. I am not able, however, to refer to any previous mention of it.

I start from the hypothesis that the regression is linear. Accordingly, if a volume of the frequency be cut off from the frequency surface by a vertical plane at a given value of the variate B, the vertical through the centroid of this volume cuts the regression line. If $\bar{p}$ and $\bar{q}$ be the coordinates of this point of section measured from the means of the two variates, $\bar{p}$, $\bar{q}$ lies on the regression line and we have, σ_1 and σ_2 being the standard deviations of the two variates and r their correlation,

$$\bar{p} = r\frac{\sigma_1}{\sigma_2}\bar{q}.$$

Hence

$$r = \frac{\bar{p}/\sigma_1}{\bar{q}/\sigma_2} \quad \text{..................................(i).}$$

So far there is no assumption of a Gaussian distribution. Now $\bar{p}$ is the mean value of the A-variate, for all the pairs with specially marked B-variate; thus in the example, in the first illustration I have given above, the mean age of all candidates who passed the examination. And σ_1 is the standard deviation of the measured character and can therefore be found, e.g. in the same illustration is the age variability of *all* candidates. Thus the numerator in Equation (i) can always be found. The next point is to consider how the denominator can be discovered. Now the B-variate is not given quantitatively, but we are given the percentage of B beyond the arbitrary division, i.e. in our illustration the number out of the candidates who succeed in passing. We cannot therefore find $\bar{q}/\sigma_2$ by the usual processes of determining a mean. If, however, we assume the B-variate to follow reasonably closely a Gaussian distribution, the percentage of the B-variate gives, by means of the probability-integral tables, the ratio of y/σ_2 for the distance from the mean at which the B-variate is divided, and then

$$\frac{\bar{q}}{\sigma_2} = \frac{\dfrac{N}{\sqrt{2\pi\sigma_2^2}}\displaystyle\int_y^\infty ye^{-\frac{1}{2}\frac{y^2}{\sigma_2^2}}dy}{\dfrac{N}{\sqrt{2\pi}\sigma_2}\displaystyle\int_y^\infty e^{-\frac{1}{2}\frac{y^2}{\sigma_2^2}}dy} = \frac{\dfrac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(y/\sigma_2)^2}}{\dfrac{1}{\sqrt{2\pi}}\displaystyle\int_{y/\sigma_2}^\infty e^{-\frac{1}{2}y^2}dy}.$$

Here both numerator and denominator are known as soon as y/σ_2 has been found. They are for example the z and $\frac{1}{2}(1-\alpha)$ of Sheppard's Tables (*Biometrika*, Vol. II. p. 182).

Thus by the simple hypotheses that the regression is linear, and that a fairly close value of the mean of the marked part of the B-variate can be found on the assumption that it has a normal distribution, we can readily find numerator and denominator of the value of r as given by Equation (i).

Illustration I. Relation of Anaemia to Age. I take the data from the last *Report* (1909) *of the Education Committee of the London County Council. Report of Medical Officer.* Particulars as to age and "colour of blood" are given for 325 boys at Laxon Street and 364 boys at Sirdar Road schools; and for 367 girls at the former school and 330 girls at the latter school: see p. 19. Sirdar Road, Notting Dale, is described as "one of the poorest schools of London in the midst of a migratory, criminal, and thoroughly degenerate and poverty stricken population." Laxon Street is also in a very poor district; lack of employment, improvidence and drink are exceedingly common in both districts. Either different standards of anaemia were used in the two schools, or the Laxon Street school shows far more anaemia than the Sirdar Road. The ages covered are 7 to 13. Putting "pale" and "very pale" together, the latter "blood colour" only occurring in about 5 p.c. of the children dealt with, we have

BOYS.

Ages.

Colour of Blood		*7*	*8*	*9*	*10*	*11*	*12*	*13*	Totals
Sirdar Road	Normal	34	29	27	32	31	35	41	229
	Pale ...	17	17	23	19	21	21	17	135
Totals ...		51	46	50	51	52	56	58	364
Laxon Street	Normal	20	23	34	23	25	25	24	174
	Pale ...	22	33	21	30	21	9	15	151
Totals ...		42	56	55	53	46	34	39	325

GIRLS.

Ages.

Colour of Blood		*7*	*8*	*9*	*10*	*11*	*12*	*13*	Totals
Sirdar Road	Normal	22	32	40	39	37	30	30	230
	Pale ...	4	18	17	15	12	19	15	100
Totals ...		26	50	57	54	49	49	45	330
Laxon Street	Normal	19	23	19	26	14	13	17	131
	Pale ...	34	40	28	40	33	34	27	236
Totals ...		53	63	47	66	47	47	44	367

I find the following results:

Mean age: all Boys: Sirdar Road, 10·6456
Laxon Street, 10·3092
Anaemic Boys: Sirdar Road, 10·5444
Laxon Street, 10·0430

Standard Deviation for ages: all Boys: Sirdar Road, 2·019
Laxon Street, 1·904

For the girls we have:

Mean age: all Girls: Sirdar Road, 10·6667
Laxon Street, 10·3392
Anaemic Girls: Sirdar Road, 10·8000
Normal Girls*: Laxon Street, 10·2634

Standard Deviation for ages: all Girls: Sirdar Road, 1·852
Laxon Sreet, 1·951

I will illustrate the general process on the Sirdar Road boys:

$$\begin{aligned}\bar{p}/\sigma_1 &= (10{\cdot}5444 - 10{\cdot}6456)/2{\cdot}019\\ &= -{\cdot}1012/2{\cdot}019 = -{\cdot}0512.\end{aligned}$$

Again $\frac{1}{2}(1-\alpha) = 135/364 = {\cdot}3709$, thus $\frac{1}{2}(1+\alpha) = {\cdot}6291$. In Sheppard's Tables this corresponds to $x/\sigma = {\cdot}33$ and gives for the corresponding ordinate z the value ·3778.

Hence $$\begin{aligned}\bar{q}/\sigma_2 &= z/\tfrac{1}{2}(1-\alpha) = {\cdot}3778/{\cdot}3709\\ &= 1{\cdot}0186.\end{aligned}$$

Accordingly $$\begin{aligned}r &= (\bar{p}/\sigma_1)/(\bar{q}/\sigma_2) = -{\cdot}0512/1{\cdot}0186\\ &= -{\cdot}0503.\end{aligned}$$

The negative sign indicates that anaemia in boys decreases with age, i.e. the age of the anaemic boys is less than that of all boys. The girls at Sirdar Road give a plus correlation = + ·0627. Treated in the same manner the Laxon Street boys give − ·1632. In the case of the girls at the latter school the correlation is + ·0373, the normal girls having a lower age than all girls. Thus we find:

Correlation of Age and Anaemia.

	Boys	Girls
Sirdar Road ...	− ·0503	+ ·0627
Laxon Street ...	− ·1632	+ ·0373

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* It is necessary to take the normal and not the anaemic girls as the former are in this school in the minority.

From these results we conclude that:

(i) The relation between anaemia and age (7 to 13) is not very marked.

(ii) Anaemia decreases as the boys grow older and increases as the girls grow older.

(iii) The association of anaemia with age in boys is about trebled in intensity when we pass from Sirdar Road to Laxon Street, or from a school with upwards of 30 °/₀ to one with more than 50 °/₀ of anaemic boys.

The above distributions are perhaps not quite the type we should select as most suitable for the method—the totals for each age are too nearly alike. It would probably, however, be difficult to bring out by any more exact method, *leading to a unique result*, the correlation of age and anaemia. It is obvious that a fourfold table method would present considerable diversity according to the age partition selected and would require far more labour than the 15 minutes requisite to determine a correlation-coefficient by the present method.

Illustration II. The Influence of Age on the Capacity to pass an Examination. I take here the returns for 6156 candidates for the London University Matriculation Examination from the *Calendar* for 1908–9 (pp. 953–955).

Ages of Candidate	Passed	Failed	Totals
16	583	563	1146
17	666	980	1646
18	525	868	1393
19—21	383	814	1197
22—30 (centred at *25*)	214	439	653
Over *30* (centred at *33*)	40	81	121
Totals	2411	3745	6156

The only difficulty here is the centering of the groups "22—30" and "over 30," the official statistics (as usual!) clubbing all these ages, which are really so significant for the frequency distribution, together. After some consideration, I made the mean ages of these groups 25 and 33.

The mean age of the passing candidates was 18·4280 and of the total candidates 18·7865. The S.D. of the latter being 3·2850. Thus $\bar{p} = -\cdot 3585$, $\bar{p}/\sigma_1 = -\cdot 1091$.

Further $\frac{1}{2}(1-\alpha) = \cdot 3917$, giving $\frac{1}{2}(1+\alpha) = \cdot 6083$ and $z = \cdot 3843$. Thus

$$\bar{q}/\sigma_2 = \cdot 9806, \text{ and } r = -\cdot 1113.$$

We conclude that there is a small but sensible correlation between youth and the ability to pass this examination. There are some evidences that this holds for other examinations. It would be interesting to consider this problem at greater length. While the complexity of the brain may be greater at 28 than 18, the mean weight appears to be greater at the latter age, and possibly in mere examination activity quantity may reckon more than quality in brains as well as it frequently does in marking. It might be of value to determine the partial correlation when allowance was made for the number of times the candidate had entered.

Illustration III. Conscientiousness and Cephalic Index. It is well known that the cephalic index remains fairly constant from about 2 or 3 years old onwards. It is further a marked racial character. Folklore associates with various grades of honesty the local races of this country. It seemed accordingly worth while testing whether my school records gave any association between cephalic index and conscientiousness. The children measured were divided into those whose conscientiousness was "Keen," "Dull" or "Betwixt." The latter class were about 14 °/o of the total and we divided them equally between the "keen" and "dull" classes. The result is given in the table below. We obtained for 1534 boys:

Mean Cephalic Index of the "Dull" group	78·8876.
Mean Cephalic Index of all Boys	78·8801.
Standard Deviation for all Boys	3·2929.

Conscientiousness and Cephalic Index of Boys.

Cephalic Index	CONSCIENTIOUSNESS			Cephalic Index	CONSCIENTIOUSNESS		
	Keen	Dull	Totals		Keen	Dull	Totals
66	1	—	1	*79*	127	62	189
67	—	—	—	*80*	122	58	180
68	1	1	2	*81*	99	44	143
69	4	1	5	*82*	86	24	110
70	5	1	6	*83*	55	29	84
71	7	2	9	*84*	28	21	49
72	11	6	17	*85*	18	11	29
73	26	10	36	*86*	16	7	23
74	35	18	53	*87*	8	1	9
75	58	25	83	*88*	3	—	3
76	81	40	121	*89*	—	1	1
77	110	58	168	*90*	1	1	2
78	145	64	209	*91*	2	—	2
				Totals	1049	485	1534

Further, $\left.\begin{array}{l}\frac{1}{2}(1-\alpha)=\cdot 3162\\ \frac{1}{2}(1+\alpha)=\cdot 6838\end{array}\right\}$ giving $z=\cdot 3558$.

Thus we find
$$\bar{p}/\sigma_1=+\frac{\cdot 0075}{3\cdot 2929}=+\cdot 0023,$$
$$\bar{q}/\sigma_2=1\cdot 1252.$$

And finally
$$r=+\cdot 0020.$$

This correlation therefore confirms the view already expressed*, that there exists very little relationship between psychical characters and head-measurements in man. In this case the correlation is zero, considering the probable error which must be sensible in the second place of figures.

Illustration IV. Effect of Enlarged Glands and Tonsils on the Weight of Children. The medical inspectors of the London County Council classify the state of the glands under two systems, 0 = no enlargement, and + = enlargement, and again under 1 good state, and 2 defective, and 3 bad state; the tonsils are only classed under 0 and +. I owe to Mr David Heron the following reductions for a typical L.C.C. school.

Weight	GLANDS		TONSILS		Totals
	Good	Bad	Good	Bad	
14	2	—	2	—	2
16	3	5	5	3	8
18	15	26	33	8	41
20	20	40	36	24	60
22	28	47	50	25	75
24	34	30	47	17	64
26	30	31	49	12	61
28	29	20	41	8	49
30	30	30	47	13	60
32	21	14	24	11	35
34	18	11	22	7	29
36	18	5	15	8	23
38	6	7	10	3	13
40	5	2	4	3	7
42	7	3	8	2	10
44	1	—	1	—	1
46	—	—	—	—	—
48	3	—	2	1	3
50	1	—	1	—	1
52	1	—	1	—	1
...	—	—	—	—	—
62	1	—	1	1	2
Totals	274	271	399	146	545

Mean weight of Boys in school = 27·7522.
Standard Deviation = 6·7502.
Mean weight of those with Bad Glands = 27·3737.
Mean weight of those with Bad Tonsils = 27·1212.

For glands:

$$\tfrac{1}{2}(1-\alpha) = \cdot 4972, \quad \tfrac{1}{2}(1+\alpha) = \cdot 5028, \quad z = \cdot 3988.$$

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* K. Pearson, "Relation of Intelligence to Size and Shape of the Head," *Biometrika*, Vol. v. pp. 105—146.

For tonsils:

$$\tfrac{1}{2}(1-\alpha) = {\cdot}2679, \quad \tfrac{1}{2}(1+\alpha) = {\cdot}7321, \quad z = {\cdot}3293.$$

$$\text{Correlation of } good \text{ glands and weight} = \frac{{\cdot}3785}{6{\cdot}7502} \times \frac{{\cdot}4972}{{\cdot}3988} = {\cdot}070.$$

$$\text{Correlation of } good \text{ tonsils and weight} = \frac{{\cdot}6310}{6{\cdot}7502} \times \frac{{\cdot}2679}{{\cdot}3293} = {\cdot}076.$$

Thus bad glands and tonsils are slightly, but I think significantly associated, with less weight in the child. The correlations are not, however, as large as we might *a priori* have anticipated that they would be.

Illustration V. Effect of the Employment of Mothers on the Height of their Children. I owe the data on this point to Miss E. M. Elderton, who has been working on the Glasgow School Board Returns. The first table gives the age of sons for mothers employed and not employed; the second gives the height of sons for mothers employed and not employed. I take it that the categories employed and not employed correspond to a continuous variable, which for want of a better name, I may term, stress on mother. Practically all these mothers may be said to have their household work. In some cases—apparently relatively few in Glasgow—they do other work of a variety of kinds. This extra work must increase the claims on the mother's energy, and we might expect it to show in the physical welfare of the child. This point will be fully discussed by Miss Elderton in her forthcoming memoir. I only use the data here as illustration of the new method.

Employment of Mothers and Age of Sons.

Age of Sons	Mothers, Employed	Mothers, not Employed	Totals
4·5— 5·5	48	942	990
5·5— 6·5	185	3137	3322
6·5— 7·5	247	3656	3903
7·5— 8·5	253	3947	4200
8·5— 9·5	263	3754	4017
9·5—10·5	242	3639	3851
10·5—11·5	186	3575	3761
11·5—12·5	204	3428	3632
12·5—13·5	187	3451	3638
13·5—14·5	45	1422	1467
Totals ...	1860	30951	32811

Mean age of all Boys = 9·5563.
Standard Deviation of all Boys = 2·4806.
Mean age of sons of employed mothers = 9·3790.

$$\tfrac{1}{2}(1-\alpha) = {\cdot}0567, \quad \tfrac{1}{2}(1+\alpha) = {\cdot}9433, \quad z = {\cdot}1139.$$

$$r_{ae} = -\frac{{\cdot}1773}{2{\cdot}4806} \times \frac{{\cdot}0567}{{\cdot}1139} = -{\cdot}0356.$$

Thus the mothers are somewhat more likely to be employed if their sons are younger. This probably only means that the more well-to-do parents allow their children to stay longer at school.

Employment of Mothers and Height of Sons.

Height of Sons	MOTHERS		Totals	Height of Sons	MOTHERS		Totals
	Employed	Not Employed			Employed	Not Employed	
27·5	—	1	1	*51·5*	205	3860	4065
29·5	1	3	4	*53·5*	165	3296	3461
31·5	1	8	9	*55·5*	70	2207	2277
33·5	4	41	45	*57·5*	39	1234	1273
35·5	12	154	166	*59·5*	8	518	526
37·5	46	514	560	*61·5*	2	158	160
39·5	111	1261	1372	*63·5*	—	51	51
41·5	193	2430	2623	*65·5*	—	21	21
43·5	230	3376	3606	*67·5*	1	3	4
45·5	288	3738	4026	*69·5*	—	2	2
47·5	239	3959	4198	*71·5*	—	1	1
49·5	245	4114	4359	*73·5*	—	1	1
				Totals	1860	30951	32811

Mean Height of all Boys $= 48{\cdot}4827$.

Standard Deviation of all Boys $= 5{\cdot}3043$.

Mean Height of sons of employed mothers $= 47{\cdot}0474$.

$$\tfrac{1}{2}(1-\alpha) = {\cdot}0567, \quad \tfrac{1}{2}(1+\alpha) = {\cdot}9433, \quad z = {\cdot}1139.$$

$$r_{he} = -\frac{1{\cdot}4353}{5{\cdot}3043} \times \frac{{\cdot}0567}{{\cdot}1139} = -{\cdot}1347.$$

Miss Elderton finds by the product-moment method that the correlation of age and height for these boys is

$$r_{ah} = {\cdot}8452.$$

Finding the partial correlation coefficient between employment of mother and height of sons for a constant age of offspring, we have

$$\rho_{he} = \frac{r_{he} - r_{ha}\, r_{ae}}{\sqrt{1 - r_{ha}^2}\,\sqrt{1 - r_{ae}^2}}$$
$$= -{\cdot}1958.$$

There is thus a quite sensible correlation for a given age of child between its stature and the increasing stress due to employment of mother. Miss Elderton will, however, indicate in her memoir some difficulties in the interpretation of this result.

Conclusions. The illustrations will have indicated that the new process of determining correlation can be applied to a great variety of problems. In none of the cases dealt with was the correlation very high, but this is purely a result of the material selected, which was chosen, not from any knowledge of the existence of correlation, but to indicate the type of problem to which the new method can be applied. Hitherto such problems could only be treated by the fourfold table method. The examples given show how much more expeditious is the new process, and further how it frees us from all doubt that exists in the old method as to the suitable position for the division of the measured variate. That its probable error will be less than that of the fourfold table method will I have little doubt be demonstrated, when its value has been worked out. The method is, however, so convenient and so frequently of service that I have not delayed its publication until the leisure came to determine the probable error.

ON INHERITANCE IN THE PRODUCTION OF BUTTER FAT.

By H. L. RIETZ, Ph.D., University of Illinois, Urbana, Illinois, U.S.A.

(1) *Introduction.*

It is the main object of this paper to present an investigation into inheritance in the production of butter fat. As far as I am aware, no statistical investigation has been published dealing with this question, notwithstanding its importance in a great industry and also its purely scientific interest.

Production of butter fat is one of the chief functions of the cow in the service of man. We are, in the present paper, concerned with the inheritance of this function, which is perhaps highly correlated with certain characters of the organism of the cow, but little or nothing is known of the quantitative value of such correlations.

The source of the data is the Advanced Register of the Holstein-Friesian Association of America. The performance of an individual in the production of butter fat is not constant from day to day, but varies considerably. The records taken from the above register are in pounds of butter fat made in seven consecutive days, and are given along with the age of the cow at the time of the test. The tests for the register are all made, at present, under the supervision of representatives of State Agricultural Experiment Stations. In the earlier years of the register, certain private tests were accepted. Throughout our work, we have taken only records attested by authorized officials of the Association, who are, in general, representatives of State Agricultural Experiment Stations. In beginning this work over a year ago, there were available 17 volumes representing records of 17 successive years and Volume 18 has appeared for use in the latter part of the work. However, it turns out that Volumes 1—10 give very few pairs of variates that satisfy conditions which it seems desirable to impose. For this reason, only data from Volumes 11—18 are used.

In the register, a cow often has a record when her dam has none, and in many more cases when her granddam has none. This is due in part to the fact that it is becoming more fashionable for breeders to enter their cows in the Advanced

Register, but also to the fact that there are certain entrance requirements to be satisfied. If we had records of all the offspring of certain grandparents with records, we should doubtless find many of the offspring not eligible to the Advanced Register for the latter reason.

(2) *Requirements for Admission to Advanced Registry.*

The minimum requirements of butter fat production to admit a cow to entry in the Advanced Register are as follows:

If a cow calves at two years old or under, 7·2 lbs. of butter fat in seven consecutive days. If the cow calves at three years old, 8·8 lbs. fat in seven consecutive days. If the cow calves at four years old, 10·4 lbs. fat in seven consecutive days. If the cow calves at five years old or over, 12 lbs. fat are required in seven consecutive days. If the cow calves between two and three years, or between three and four years, or between four and five years old, every day of increased age adds to the requirements for the years 0·00439 lb. butter fat.

Our data are therefore not a random sample of pure bred Holstein-Friesian cows, but a selected group which meets certain requirements. Since ancestors as well as offspring must meet these minimal requirements, our problem presents an illustration of the double selection recently dealt with by Pearson[*]. A correction for the influence of selection will be applied (§ 9) after we obtain statistical constants from the selected groups given in the register.

(3) *Correlation of Age and Production* (Table I.).

Anticipating a high correlation between age and production, we first took the year books 1902—1906 from which can be obtained easily data necessary to determine the correlation between age and production. From the means of arrays in Table I., it follows that up to five years old, the regression is almost "truly linear," and that, by dividing the table into two parts, near the five year mark, it gives two tables of nearly linear regression in each. For this reason, we attempt to describe the population by separating the data into two parts at the 4·75 year point. It results that, for the group of cows under 4·75 years old, the correlation coefficient is

$$r = 0·662 \pm 0·007.$$

For the group of cows over 4·75 years old,

$$r = 0·030 \pm 0·016$$

if the fifteen most extreme variates with respect to age are excluded from the calculation. If these fifteen are included

$$r = 0·004 \pm 0·026.$$

From these results, we are able to assert a high correlation between age and production up to 4·75 years, but we are unable to assert that any correlation exists between age and production of cows over 4·75 years old.

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[*] *Biometrika*, Vol. VI., Part I., pp. 111, 112.

From a slightly different table, we calculated the correlation coefficient between age and production of butter fat by including cows up to 5·25 years old, and certain additional cows that entered the records on a milk test rather than on production of butter fat. This gave

$$r = 0{\cdot}688 \pm 0{\cdot}006.$$

The correlation is thus slightly increased by including cows between 4·75 and 5·25 years old, while the regression coefficient of production on age is slightly decreased.

For the group under 4·75 years old, regression of production on age is

$$r\frac{\sigma_p}{\sigma_a} = 0{\cdot}6623\,\frac{2{\cdot}513}{0{\cdot}8752} = 1{\cdot}901 \text{ with a probable error of } 0{\cdot}021.$$

For the group under 5·25 years,

$$r\frac{\sigma_p}{\sigma_a} = 0{\cdot}6879\,\frac{2{\cdot}713}{1{\cdot}026} = 1{\cdot}819 \text{ with a probable error of } 0{\cdot}018.$$

From the variability of arrays corresponding to different ages, we make the following observation:

The coefficient of variability increases gradually from youth to three years old, and then gradually decreases to maturity. This may be expressed roughly by saying that the coefficient of variability is greatest near the mean of the total period of growth.

In reference to lack of "smoothness" in the frequency distribution with respect to age, it should be noted that at each of the ages 2·5, 3·5, 4·5, 5·5, 6·5, 7·5, 9·5, 11·5 there are fewer variates than at the year marks between which each of them lies. For example, at 4·5 there are 284 while at 4 there are 498, and at 5 years 312. This seems to show simply a decided tendency on the part of breeders to have their cows tested nearer the year marks than the half-year marks. Ages are given to the day.

(4) *Mature and immature Cows.*

From the results on correlation between age and production, it appears that the data should be divided into two parts near the age of 4 years 9 months—the cows in one division being regarded as mature, and in the other division as immature.

The important question at once suggests itself as to whether inheritance, as measured by the correlation coefficient, is the greater when offspring and parents are at the same stage in the period of growth or when they are mature.

(*a*) *Offspring and dams under four years old at time of test* (Table II.). The table represents records when the offspring and dams differ by less than one year in age at time of their respective tests, and are less than four years old. Tabulated with the records of offspring in the Advanced Register are, in general, the most remarkable records of dams and granddams. To correlate records of offspring with the most remarkable records of ancestry would make the latter appear to be better producers than the former. In the case of dams and offspring under consideration, we have examined the back records of dams, so as to correlate the records when dams and corresponding offspring are most nearly equal in age.

If M, σ, V, and r represent mean, standard deviation, coefficient of variability, and coefficient of correlation respectively, the results are:

	M	σ	V	
Offspring,	$10{\cdot}687 \pm 0{\cdot}057$,	$2{\cdot}136 \pm 0{\cdot}040$,	$19{\cdot}99 \pm 0{\cdot}38$,	$r = 0{\cdot}344 \pm 0{\cdot}023.$
Dams,	$10{\cdot}605 \pm 0{\cdot}057$,	$2{\cdot}141 \pm 0{\cdot}040$,	$20{\cdot}19 \pm 0{\cdot}38$,	

(*b*) *Offspring and dams over four years old at time of test* (Table III.). If the dividing line between mature and immature cows is drawn at 4·75 years as appears desirable from Table I., too few variates for our purpose fall into the division of mature cows. By making the dividing line at four years, the numbers in the two groups are nearly equal. In the above study of immature cows, the parent and corresponding offspring may differ in ages at the time of their respective tests by any amount less than one year; and there is no reason why a small change in the dividing line should affect the point as to whether the heredity coefficient is the larger during growth or after maturity.

For these reasons, we have, for the present purpose, classed cows above four years old as mature. The results are:

	M	σ	V	
Offspring,	$15{\cdot}316 \pm 0{\cdot}070$,	$2{\cdot}543 \pm 0{\cdot}049$,	$16{\cdot}60 \pm 0{\cdot}33$,	$r = 0{\cdot}284 \pm 0{\cdot}025.$
Dams,	$15{\cdot}368 \pm 0{\cdot}070$,	$2{\cdot}646 \pm 0{\cdot}051$,	$17{\cdot}22 \pm 0{\cdot}34$,	

(*c*) It may be noted that the group of cows under four years old yields a coefficient 0·344, while the group over four years old gives $r = 0{\cdot}284$; but the value $r = 0{\cdot}344$ is, as we shall show presently, much greater than the correlation for a group of parents and offspring of one age, say two or three years old. We have used a method of correspondence which much increases the coefficient for cows in the period of growth. For the purpose of establishing this point, we have first made a direct test based, to be sure, on only 219 pairs of variates (Table IV.). This table represents records when animals are under 2·25 years old at the time of their respective tests. This material gives

$$r = 0{\cdot}145 \pm 0{\cdot}045.$$

While the probable error is large, we shall in (*d*) present an argument to show that for offspring and dams two or three years old, it is highly probable that $r \overset{=}{<} 0{\cdot}145$. This will be shown by regarding the result $r = 0{\cdot}344$ as arising from a combination of material of two sub-groups.

(*d*) *Combination of sub-groups of a population.* To illustrate the combination of sub-groups, we first combine data from Tables II. and III. to form a table in which corresponding dams and offspring differ by less than one year in age at times of their respective tests when they are under four years old, and no regard is given to age when they are over four years old. The results are:

	M	σ	V	
Offspring,	$12{\cdot}913 \pm 0{\cdot}063$,	$3{\cdot}292 \pm 0{\cdot}045$,	$25{\cdot}46 \pm 0{\cdot}36$,	$r = 0{\cdot}651 \pm 0{\cdot}011$.
Dams,	$12{\cdot}915 \pm 0{\cdot}063$,	$3{\cdot}380 \pm 0{\cdot}045$,	$26{\cdot}17 \pm 0{\cdot}38$,	

This combined group thus yields a value of the correlation coefficient greater than the sum of the correlation coefficients of the sub-groups.

It has been shown* that whether any given mixture increases or reduces correlation depends entirely on the signs of the difference of the means of the sub-groups. What is to be expected under the conditions of our problem is easily seen; that is, when the means and standard deviations of the two systems of variates for each sub-group are substantially equal.

Let n_1, m_1, m_1', σ_1, σ_1', r_1 be the number of variates, means, standard deviations, and correlation coefficient respectively of a sub-group A_1, and n_2, m_2, m_2', σ_2, σ_2', r_2 be corresponding values for a second sub-group A_2, and n, m, m', σ, σ', r for the combined group. Then

$$n\sigma^2 = n_1\sigma_1^2 + n_2\sigma_2^2 + \frac{n_1 n_2}{n}(m_1 - m_2)^2 \quad \text{.............................(1)},$$

$$n\sigma'^2 = n_1\sigma_1'^2 + n_2\sigma_2'^2 + \frac{n_1 n_2}{n}(m_1' - m_2')^2 \quad \text{.............................(2)},$$

$$nr\sigma\sigma' = n_1\sigma_1\sigma_1' r_1 + n_2\sigma_2\sigma_2' r_2 + \frac{n_1 n_2}{n}(m_1 - m_2)(m_1' - m_2') \quad \text{.........(3)}.$$

Make $$m_1 = m_1', \quad m_2 = m_2', \quad \sigma_1 = \sigma_1', \quad \sigma_2 = \sigma_2' \quad \text{.....................(4)}.$$

The relations (4) are substantially satisfied by material in (*a*) and (*b*). From (1), (2), (3), (4),

$$r = r_1 + (1 - r_1)\frac{n_1 n_2}{n^2}\frac{(m_1 - m_2)^2}{\sigma^2} + (r_2 - r_1)\frac{n_2\sigma_2^2}{n\sigma^2} \quad \text{...............(5)},$$

$$r = r_2 + (1 - r_2)\frac{n_1 n_2}{n^2}\frac{(m_1 - m_2)^2}{\sigma^2} + (r_1 - r_2)\frac{n_1\sigma_1^2}{n\sigma^2} \quad \text{...............(6)}.$$

It may be observed, incidentally, from (5) and (6), that the correlation coefficient for the combined group formed of two sub-groups under conditions (4) is always greater than the smaller of the correlation coefficients of the sub-groups; but what we propose to determine from (5) and (6) is an upper limit for the correlation in the case of cows and dams of a fixed age. The same factors which produce the large value 0·651 in combining material of Tables II. and III. enter, to a considerable degree, into the study of cows under four years old. It appears justifiable to assume that the correlation coefficient 0·344 from Table II. is equal to or greater than the correlation coefficient which arises from combining into one table material of two tables, in one of which offspring and dams are two (1·75—2·25) and in the other three (2·75—3·25) years old. Take $r_1 \underset{>}{=} r_2$, then from (6),

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* Pearson: *Phil. Trans.*, A., Vol. 192, p. 278.

$$0·344 \underset{>}{=} r_2 + (1 - r_2) \frac{n_1 n_2}{n^2} \frac{(m_1 - m_2)^2}{\sigma^2} \text{ ....(7).}$$

Values of n_1, n_2, n, m_1, m_2 are obtained from the arrays of Table I., and we assign to σ a value 2·34 unfavourable to a small value of r_2. This gives

$$0·344 \underset{>}{=} r_2 + (1 - r_2) \frac{(983)(545)}{(1528)^2} \frac{(2·36)^2}{(2·34)^2},$$

$$r_2 \underset{<}{=} 0·145.$$

When this estimate and the value 0·145 for the group of cows under 2·25 years old are compared with the value 0·284 for mature cows, it looks as if inheritance in the production of butter fat is much greater after maturity than in youth, but we avoid drawing conclusions until the corrections for selection are applied (see § 8).

(5) *Correlation of Granddams and Offspring—mature Cows* (Tables VI. and VII.).

The requirements for entrance to the Advanced Register are stated in § 2. A cow may have more than one record, and that one is regarded as "most remarkable" which exceeds most the required production of the age at which she is entered. We shall take the most remarkable records in dealing with granddams, instead of the records when the members of a pair are most nearly equal in age at the time of their respective tests, since the laborious task of finding back records is unnecessary to determine the correlation, as I have found in treating dams and offspring for comparison with the methods previously used in this paper.

Granddams have no official records in many cases where dams and offspring have such records, and there is difficulty in securing adequate numbers from our data for determining the correlation of granddams and offspring. To overcome this difficulty, we have, in order to determine correlation for mature cows, adopted a method of transmuting production of immature cows to mature cows. For this purpose we multiply the production of any immature cow of age s by $\frac{M_5}{M_s}$, where M_s is the mean production of cows of age indicated by the subscript. A good determination of $\frac{M_5}{M_s}$ is obtained from arrays of Table I. The means, standard deviations, and coefficient of correlation obtained from these transmuted values are comparable in an average way to the statistical constants obtained from mature cows.

For the sake of numerical comparison, we have determined the correlation of dams and maternal granddams (Table V.) by transmuting to mature cows. The results are:

	M	σ	
Dams,	15·383 ± 0·073,	2·377 ± 0·052,	$r = 0·301 \pm 0·028$.
Maternal Granddams,	15·423 ± 0·077,	2·495 ± 0·055,	

These results agree satisfactorily with those drawn from Table III., where the relationship is the same as in this case.

With transmuted values, we obtain the following coefficients of correlation (Tables VI., VII.):

Maternal Granddams and Offspring, $r = 0{\cdot}138 \pm 0{\cdot}024$.

Paternal Granddams and Offspring, $r = 0{\cdot}086 \pm 0{\cdot}017$.

While it is impossible to measure the potential character of butter fat production in the male, we aim to determine the extent to which he transmits this character from his dam to his female offspring. It may be observed that the correlation coefficient of maternal granddams and offspring is 0·138 while that of paternal granddams and offspring is 0·086. It is, however, to be further observed in this connection that the table of paternal granddams contains a considerable number of scattered extreme variates. If these were omitted the correlation coefficient would be appreciably increased. Furthermore, aside from entrance requirements, there is a strong tendency to select paternal granddams of high production so that selection may enter as a factor. For these reasons, we hesitate to attach significance to the difference $0{\cdot}138 - 0{\cdot}086 = 0{\cdot}052$ although possibly it indicates prepotency of females over males in the transmission of this character.

(6) *Selection of Sires.*

In breeding problems, the artificial selection of sires is comparable to assortative mating in man. The question thus arises: To what extent is the production of butter fat by any cows correlated with the production by dams of sires to whom the cows are mated? In answer to this question, we obtain the result (Table VIII.)

$$r = 0{\cdot}140 \pm 0{\cdot}025$$

for the correlation in question.

(7) *Further Correlations for immature Cows.*

In beginning this work, I expected to apply in a straightforward way the theory of multiple correlation by carrying the work back to granddams, and at the same time to take the age element into account. Then, in the usual notation, the regression equation could be written in the form

$$x_1 = a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 + a_{16}x_6 + a_{17}x_7 + a_{18}x_8,$$

where x_1, x_3, x_5, x_7 refer to production of offspring, dams, maternal granddams, and paternal granddams respectively; and x_2, x_4, x_6, x_8 refer to ages at times of respective tests of offspring, dams, maternal granddams, and paternal granddams. In Tables IX. to XII. is collected material for the correlation of first records of offspring with most remarkable records of ancestors when no regard is given to age except that all animals are under five years old. From Table IX., we obtain for the correlation of offspring and dams

$$r_{13} = 0{\cdot}132 \pm 0{\cdot}023.$$

Although the upper limit for immature cows is thus extended to five years, the data are too meagre to give reliable results in the determination of the correlation of granddams and granddaughters tested during the period of growth. However, the material is collected in Tables X. to XII. as it may be of interest in showing the poor records of the granddaughters of some fancy granddams, and the effect of these extreme variates on the correlation coefficient. These figures naturally lead to the question for the breeder as to whether the probability of progress with a considerable number of good animals is not greater than with an equal investment in fancy animals.

From Tables X. and XI.,

$$r_{15} = 0{\cdot}062 \pm 0{\cdot}046,$$

$$r_{17} = 0{\cdot}005 \pm 0{\cdot}026.$$

If we exclude the fancy paternal granddams at and above 22,

$$r_{17} = 0{\cdot}091 \pm 0{\cdot}026.$$

The variates excluded arise from only six paternal granddams; this shows how adversely a few extreme paternal granddams may influence the correlation coefficient when these ancestors are weighted with their offspring. On the question of the selection of sires, we have (Table XII.)

$$r_{37} = 0{\cdot}082 \pm 0{\cdot}048.$$

(8) *Corrections for Effects of Selection on the foregoing Results.*

The entrance requirements of the Advanced Register are stated in § 2. The material used in our study of heredity is subject to the form of double selection treated by Pearson*. To make the application, let s be the standard deviation of the group of cows which meets the requirements of admission to the Advanced Register, σ the standard deviation of pure bred Holstein-Friesian cows from which those which meet the requirements are selected, s_1 the standard deviation of dams which meet the requirements, σ_1 the standard deviation of cows from which the selection is made. In our material, a cow may meet the requirements when her dam does not, or a dam may meet the requirements when her offspring do not meet them. We can enter a pair in the correlation table when and only when both variates meet the minimal requirements.

Let $\mu = \frac{s}{\sigma}$, $\mu_1 = \frac{s_1}{\sigma_1}$, then the formula

$$R = r\frac{\mu\mu_1}{\sqrt{1 - r^2(1-\mu^2)}\sqrt{1-r^2(1-\mu_1^2)}} \quad \text{.........................(1)}$$

gives the means of correcting the foregoing heredity coefficients for selection, where R is the correlation coefficient for the selected group, and r for the total group.

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* *Biometrika*, Vol. vi., pp. 111, 112.

Neglecting other forms of selection than that of meeting requirements for entry,

$$s = s_1, \ \ \sigma = \sigma_1, \ \ \mu = \mu_1.$$

Then from (1)

$$r = \frac{-\mu^2 \pm \sqrt{\mu^4 + 4R^2(1-\mu^2)}}{2R(1-\mu^2)} \quad \text{........................(2).}$$

The value of s for cows of various ages is known from Table I., but it is necessary to evaluate σ in order to obtain μ. To determine σ we make use of the fact that the frequency distributions which give s are truncated. The requisite theory for obtaining the standard deviation of a normal distribution from a truncated portion of it has been given by Pearson*.

Let N, n be the number in the total group and in the "tail" respectively, h the distance from the mean of the total distribution at which the distribution is truncated, ν_1' the deviation of the mean of the selected group from the minimal requirement, Σ the standard deviation of the tail about its own mean,

$$\mu'_p = \int_{h'}^{\infty} x^p e^{-\frac{1}{2}x^2}\, dx,$$

where $h' = \frac{h}{\sigma}$. Then

$$\frac{n}{N} = \mu_0', \quad \sigma = \frac{\nu_1' \mu_0'}{\mu_1' - h'\mu_0'} = \psi_2 \nu_1' \quad \text{........................(3)}$$

are the formulae to be used for our purpose. Further relations are given for the determination of N from the tail, but, bearing on the application of these formulae, it is pointed out that the effect of random sampling on the frequency at the stump of the tail is very influential in modifying the results. It is obvious that, in our problem, the frequencies near the stump are likely to be less than what we should obtain from the truncated portion of a random sample of the total group, for the reason that a cow is likely to be left out when she can barely meet the requirements. Hence, instead of trying to determine n/N from the "tail," we determine this ratio by direct observation, as we have the means of making a fair estimate of the number of cows N in milk from which the n are selected. The basis of the method is found in the fact that the Holstein-Friesian Association maintain a Herd Book in which animals are registered, in general, when they are calves. Substantially all the female calves that are born are registered by such owners as use the Advanced Registry. By thus going to the Herd Book, we find the number of the female calves registered together with the age of calf and the Herd Book number of the dam, so that we can find the age of the dam. As a cow is in milk for each calf registered, we double the number of female calves registered to obtain an estimate of the number of cows in milk. We have excluded a few owners who buy and sell extensively, as the transfer of animals might prove a source of error. We have searched the records for the year May 15, 1906 to May 15, 1907, and the results may be exhibited in the following form:

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* *Loc. cit.* pp. 63—68.

	0—2·5 years	2·5—3·5 years	3·5—4·5 years	4·5—	Total under 4·5 years
Cows in milk	618	494	430	1416	1542
Number in Advanced Registry	413	346	269	620	1028
$\mu_0 = \frac{n}{N}$	0·67	0·70	0·63	0·44	0·67

With these estimates for $\mu_0 = n/N$, we can obtain σ for various classes of cows from data in Table I.

(*a*) *Value of* σ *for mature cows.*

$$\nu_1' = 14{\cdot}90 - 12{\cdot}00 = 2{\cdot}90,$$

$$\mu_0 = 0{\cdot}44,$$

$$h' = 0{\cdot}151,$$

$$\psi_2 = 1{\cdot}342,$$

$$\sigma = 3{\cdot}89.$$

(*b*) *Value of* σ *for immature cows.* For immature cows the requirements increase uniformly with age from two years old to five years old according to the relation

$$y = 1{\cdot}6\,x + 4,$$

where x is age in years and y is production in pounds. If the total frequency surface with respect to age and production is

$$z = f(x, y)$$

the selected portion is separated from the rest of the surface by the plane $y = 1{\cdot}6\,x + 4$. The theory for obtaining the constants of the total surface from such a portion of it has not, so far as I am aware, been developed. Perhaps it can be developed, without great difficulty, for the normal surface, but our distribution as to age is so far from normal that I think a method of dealing with arrays of the correlation table will give more reliable results than to assume the distribution normal.

(*c*) *Cows under* 2·25 *years.* For cows two years old or under, the requirement is 7·2 and for one 2·25 years old it is 7·6. For this small range in age we shall accept the requirement for age 2·125, that is 7·4, as locating the stump of the tail. From the combination of arrays in Table I., for cows under 2·25,

$$\Sigma = 1{\cdot}48, \text{ and } m = 9{\cdot}33,$$

$$\mu_0 = \frac{n}{N} = 0{\cdot}67,$$

$$h' = -0{\cdot}44,$$

$$\nu_1' = 9{\cdot}33 - 7{\cdot}4 = 1{\cdot}93,$$

$$\sigma = 1{\cdot}99.$$

(*d*) *Cows 3 years old* (2·5—3·5).

$$\Sigma = 2{\cdot}21,$$

$$\nu_1' = 11{\cdot}75 - 8{\cdot}8 = 2{\cdot}95,$$

$$\mu_0 = \frac{n}{N} = 0{\cdot}70,$$

$$h' = -0{\cdot}525,$$

$$\sigma = 2{\cdot}89.$$

(*e*) *Cows 4 years old* (3·5—4·0).

$$\Sigma = 2{\cdot}10,$$

$$\nu_1' = 2{\cdot}84,$$

$$h' = -0{\cdot}33,$$

$$\sigma = 3{\cdot}06.$$

(*f*) *Corrected standard deviation for the entire group of cows under* 4·5 *years old.* This standard deviation I obtain from the standard deviations of arrays up to 4·5. That is to say, we pool together the arrays with their corrected means and standard deviations. This gives for all cows under 4·5 years old

$$\sigma_i = 2{\cdot}94.$$

(*g*) *Ratio of variability of selected group to that of total group.* Let μ represent the ratio, then μ has the following values:

For group over 4·5 years old, $\mu = 0{\cdot}57$.

For group under 2·25 years old, $\mu = 0{\cdot}75$.

For group under 4·5 years old, $\mu = 0{\cdot}85$.

(*h*) *Values of corrected correlations.* By means of (2) we now give the values of some corrected correlation coefficients.

Dams and offspring:

For mature cows, $r = 0{\cdot}63$ from $R = 0{\cdot}284$ of selected group.

For cows under 2·25 years old, $r = 0{\cdot}25$ from $R = 0{\cdot}145$.

For cows under 5 years old without regard to age in selecting pairs, $r = 0{\cdot}18$ from $R = 0{\cdot}132$ for selected group.

Granddams and offspring:

For maternal granddams (mature), $r = 0{\cdot}38$ from $R = 0{\cdot}138$.

For paternal granddams (mature), $r = 0{\cdot}25$ from $R = 0{\cdot}086$.

Correlation between dams and dams of sires with which they are mated (mature), $r = 0{\cdot}39$ from $R = 0{\cdot}14$.

With the formula for the selection of one character only, we obtain 0·695 as the corrected correlation of age and production from 0·662.

It should be understood that these corrected values are to be regarded as approximate results only. In finding the percentage of cows that meet the standards, I think perhaps our method gives values for these percentages rather too small than too large, owing to the fact that breeders are likely to keep from the test animals that can barely meet the requirements. If this be the case, our corrected values of standard deviation and correlation are somewhat larger than they should be.

(9) *Mature and immature cows.* To return to the consideration of mature and immature cows (§ 4), it appears that, both before and after applying the correction for selection, the coefficient of heredity is greater for mature than for very young cows. We shall next make use of the partial correlation coefficient to obtain a sort of average value for the correlation coefficient during the period of growth. Following the usual notation, let the subscripts 1, 2, 3, 4 mark production of offspring, age of offspring at time of test, production of dams, and age of dams at time of test respectively. Then the desired value is given by

$$\rho_{13} = \frac{r_{13}(1 - r_{24}^2) + r_{12}(r_{24}r_{34} - r_{23}) + r_{14}(r_{23}r_{24} - r_{34})}{\sqrt{1 - r_{24}^2 - r_{23}^2 - r_{34}^2 + 2r_{23}r_{34}r_{24}}\,\sqrt{1 - r_{24}^2 - r_{12}^2 - r_{14}^2 + 2r_{12}r_{14}r_{24}}}.$$

The values to be substituted in this formula are $r_{13} = 0·18$ from the material of Table IX., corrected for selection; $r_{12} = r_{34} = 0·695$ from Table I. corrected for selection; $r_{24} = 0·033$ from Table XIII.; $r_{14} = 0$, $r_{23} = 0$.

$$\rho_{13} = 0·38.$$

We have then 0·38 for a sort of average value of the correlation coefficient at fixed ages in the period of growth, 0·145 and 0·25 for uncorrected and corrected coefficients of cows under 2·25 years old, 0·28 and 0·63 for uncorrected and corrected coefficients after maturity. These results seem convincing of the fact that *inheritance in production of butter is, in general, much more pronounced after maturity than at a fixed stage in the period of growth.*

(10) *Summary of results.*

(*a*) The correlation of age and production of butter fat by cows under 4·75 years is approximately $\frac{2}{3}$, and the regression is linear.

(*b*) The correlation of age and production of cows over 4·75 years is substantially zero.

(*c*) The coefficient of variability in production increases gradually from youth to three years old, and then decreases to maturity.

(*d*) Production of butter fat is a function transmitted to a marked extent. The coefficient of heredity between cows and dams in the performance of this function is probably at least as large as the coefficient of mean parental inheritance of physical characters in man*.

(*e*) Inheritance in production is more pronounced in mature than in immature cows.

(*f*) A comparison of the results corrected for selection with the uncorrected values illustrated the great influence of selection in modifying the correlation coefficient.

(11) *Description of Tables.*

(*a*) Ages in Tables are expressed in years.

(*b*) Production is expressed in pounds of butter fat made in seven consecutive days.

(*c*) In Table II., about one-tenth of the variates are re-entries, where we mean by a re-entry that a cow has a previous record. In Table IV., one-half the variates are re-entries. In Tables IV., X.–XIII., there are given only first records of offspring.

(*d*) In some Tables, the class values are 10·75, 11·75, ... instead of integral values. This grouping arises out of the fact that we first grouped in $\frac{1}{2}$ pound groups and found these groups narrower than desirable.

(*e*) The term "offspring" as used in Tables VI., VII., X., XI. means offspring of the second generation removed, that is, granddaughters.

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* [It is worth while noticing also that whereas the parental correlation for mature cows is about ·63 that for grandparents on the average $= \frac{1}{2}(\cdot 38 + \cdot 25) = \frac{1}{2}(\cdot 63)$ or, within the limits indicated by the probable errors, may be taken as *half* the parental correlation. Considering the difficulties of correcting for selection pointed out by Dr Rietz, it is not unjustifiable to suppose these relationships are in close accordance with the values of inheritance found for man. ED.]

TABLE I (see pp. 120 and 121).

TABLE II.

Correlation of Dams and Offspring for Group under Four Years Old. Members of Corresponding Pairs less than one year different in Age.

Production of Dams in Pounds of Butter Fat.

Production of Offspring in Pounds of Butter Fat.

	7	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	Totals
7	—	1	3	2	2	—	—	—	—	—	—	—	—	—	—	8
8	1	22	14	25	13	3	4	—	—	—	—	—	—	—	—	82
9	4	25	23	23	19	12	7	4	2	1	—	—	—	—	—	120
10	4	24	28	25	28	8	7	2	6	1	—	—	—	—	—	133
11	1	12	19	15	16	17	14	6	5	—	1	—	—	—	—	106
12	3	4	11	19	15	17	5	4	4	2	—	—	—	—	—	84
13	—	5	9	9	5	7	2	5	2	1	—	1	—	—	1	47
14	—	1	2	6	3	9	8	2	3	1	1	—	—	—	—	36
15	—	1	—	—	2	2	2	—	1	—	—	—	—	—	—	8
16	—	1	—	2	2	2	—	—	1	—	—	—	—	—	—	8
17	—	1	—	—	3	—	2	1	1	—	1	1	—	—	—	10
18	—	—	—	2	—	—	—	—	—	—	—	1	—	—	—	3
19	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
20	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
21	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
Totals	13	97	109	128	108	77	51	24	26	6	3	3	—	—	1	646

TABLE III.

Correlation of Dams and Offspring. Mature Cows.

Production of Dams in Pounds of Butter Fat.

Production of Offspring in Pounds of Butter Fat.

	11	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	Totals
11	1	3	1	3	2	1	1	1	—	1	—	—	—	—	—	14
12	1	8	13	12	6	9	5	—	3	1	—	—	1	—	—	59
13	3	18	18	8	13	11	6	3	3	1	—	—	—	—	—	84
14	—	14	18	14	15	11	12	8	2	4	1	—	—	—	—	99
15	—	12	16	16	11	11	8	3	3	7	3	—	—	—	—	90
16	1	8	13	16	18	11	8	2	2	5	6	2	1	—	—	93
17	—	3	6	11	13	4	8	3	4	6	4	1	—	—	—	63
18	—	3	5	2	8	5	5	2	5	3	2	1	—	—	—	41
19	—	—	2	1	4	3	6	2	2	—	1	2	—	—	1	24
20	1	—	1	7	3	3	3	—	4	—	1	—	—	—	—	23
21	—	—	1	2	—	1	—	—	—	1	—	1	—	—	—	6
22	—	1	—	—	—	—	—	—	1	—	—	—	—	—	—	2
23	—	—	—	1	1	—	—	—	1	—	1	2	—	—	—	6
24	—	—	—	1	—	—	—	1	—	—	—	—	—	—	—	2
25	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
26	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
27	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
28	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
Totals	7	70	94	94	94	70	63	25	30	30	19	9	2	—	1	608

TABLE I. *Correlation of*

Production in Pounds

Age in Years.	*7*	*7·5*	*8*	*8·5*	*9*	*9·5*	*10*	*10·5*	*11*	*11·5*	*12*	*12·5*	*13*	*13·5*	*14*	*14·5*	*15*	*15·5*	*16*	*16·5*
1·0	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1·5	4	16	10	12	10	—	—	4	—	—	—	—	—	—	—	—	—	—	—	—
2·0	5	112	124	142	145	120	93	73	44	49	25	23	13	7	3	2	3	—	—	—
2·5	—	8	35	49	55	51	69	39	48	39	17	18	15	5	8	4	1	2	1	—
3·0	—	—	1	12	44	54	64	44	34	49	40	31	40	28	29	23	17	14	3	4
3·5	—	—	—	—	6	18	31	37	32	39	39	26	29	19	16	18	10	12	7	5
4·0	—	—	—	—	—	—	—	34	48	55	48	60	41	35	41	35	20	16	17	18
4·5	—	—	—	—	—	—	—	—	13	28	25	28	30	36	20	22	18	7	13	12
5·0	—	—	—	—	—	—	—	—	—	—	21	26	37	38	27	20	22	28	22	18
5·5	—	—	—	—	—	—	—	—	—	—	18	20	22	19	14	10	14	13	11	9
6·0	—	—	—	—	—	—	—	—	—	—	17	24	31	27	23	29	21	26	17	13
6·5	—	—	—	—	—	—	—	—	—	—	6	13	10	7	14	18	9	12	12	5
7·0	—	—	—	—	—	—	—	—	—	—	9	22	21	24	19	22	5	10	15	6
7·5	—	—	—	—	—	—	—	—	—	—	3	14	19	15	5	6	11	9	5	2
8·0	—	—	—	—	—	—	—	—	—	—	9	16	8	6	15	13	8	2	11	9
8·5	—	—	—	—	—	—	—	—	—	—	5	9	7	5	5	10	3	4	7	4
9·0	—	—	—	—	—	—	—	—	—	—	2	6	7	4	8	9	6	3	3	1
9·5	—	—	—	—	—	—	—	—	—	—	2	6	3	5	5	5	2	2	1	—
10·0	—	—	—	—	—	—	—	—	—	—	—	5	7	8	6	3	5	—	3	2
10·5	—	—	—	—	—	—	—	—	—	—	3	4	2	—	1	2	1	2	4	1
11·0	—	—	—	—	—	—	—	—	—	—	2	3	1	1	—	1	1	1	1	3
11·5	—	—	—	—	—	—	—	—	—	—	—	2	3	4	1	—	2	1	1	—
12·0	—	—	—	—	—	—	—	—	—	—	1	3	2	4	—	1	2	1	2	—
12·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	1	1
13·0	—	—	—	—	—	—	—	—	—	—	2	—	—	1	—	—	1	1	—	—
13·5	—	—	—	—	—	—	—	—	—	—	1	—	—	—	2	1	—	—	—	—
14·0	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	1	—	—	—
14·5	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—
15·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
15·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—
Totals	9	137	170	215	260	243	257	231	219	259	297	359	349	299	264	254	183	167	157	113

Age and Production.

of Butter Fat.

17	*17·5*	*18*	*18·5*	*19*	*19·5*	*20*	*20·5*	*21*	*21·5*	*22*	*22·5*	*23*	Totals	Mean of Arrays	σ of Arrays	*V* of Arrays
—	—	—	—	—	—	—	—	—	—	—	—	—	1			
—	—	—	—	—	—	—	—	—	—	—	—	—	56	8·25 ± 0·08	0·86	
—	—	—	—	—	—	—	—	—	—	—	—	—	983	9·390 ± 0·30	1·483 ± 0·023	15·80 ± 0·25
1	—	—	—	—	—	—	1	—	—	—	—	—	466	10·24 ± 0·05	1·644 ± 0·036	16·05 ± 0·36
5	4	—	—	—	3	—	—	1	1	—	—	—	545	11·75 ± 0·06	2·214 ± 0·046	18·85 ± 0·40
6	2	2	—	—	1	1	—	1	—	—	—	—	357	12·32 ± 0·07	2·095 ± 0·053	17·00 ± 0·45
6	7	6	3	2	2	1	1	1	—	1	—	—	498	13·24 ± 0·06	2·070 ± 0·045	15·63 ± 0·34
12	3	5	3	3	1	3	1	—	—	—	1	—	284	13·93 ± 0·08	2·116 ± 0·060	15·19 ± 0·43
9	13	10	5	3	4	3	—	4	—	—	2	—	312	14·84 ± 0·08	2·124 ± 0·057	14·31 ± 0·40
13	9	5	6	1	3	4	—	—	2	1	—	—	194	14·88		
11	9	3	5	5	1	4	5	3	—	2	1	—	277	14·93		
7	5	2	1	3	4	1	—	—	1	—	—	—	130	14·99		
17	4	4	5	2	4	2	2	1	1	—	—	—	195	14·96		
6	4	1	2	3	2	2	1	2	—	—	—	—	112	14·83		
8	8	—	3	—	2	2	—	—	1	—	—	—	121	14·87		
1	2	3	2	1	—	—	—	2	—	—	—	—	70	14·79		
7	2	1	1	2	2	3	2	—	—	—	1	—	70	15·01 + 0·44		
3	1	3	2	—	3	—	—	1	—	—	—	—	44	15·09 ± 0·58		
5	—	2	—	1	2	2	—	1	—	—	—	1	53	15·24 ± 0·48		
1	—	—	—	—	—	—	—	1	—	—	—	—	22	14·52 ± 0·74		
—	—	1	1	1	—	—	1	1	—	—	—	—	19	15·55 ± 1·15		
1	—	1	—	—	1	—	—	—	—	—	—	—	17	14·50		
1	—	—	—	—	—	—	1	2	—	—	—	—	20	15·05		
—	—	—	—	—	—	—	—	—	—	—	—	—	4	15·50		
—	—	—	—	—	—	—	—	—	—	—	—	—	5	13·60		
—	—	—	—	—	—	—	—	—	—	—	—	—	4	13·63		
—	—	—	—	—	—	—	—	—	—	—	—	—	3	13·33		
—	—	—	—	—	—	—	—	—	—	—	—	—	1			
—	—	—	—	—	—	—	—	—	—	—	—	—	—			
—	—	—	—	—	—	—	—	—	—	—	—	—	1			
120	73	49	39	27	35	28	15	21	6	4	5	1	4765	—	—	—

TABLE I. *Correlation of Age and Production.*

Production in Pounds of Butter Fat.

Age in Years.	7	7·5	8	8·5	9	9·5	10	10·5	11	11·5	12	12·5	13	13·5	14	14·5	15	15·5	16	16·5	17	17·5	18	18·5	19	19·5	20	20·5	21	21·5	22	22·5	23	Totals	Mean of Arrays	σ of Arrays	V of Arrays
1·0	—	**1**	—		—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—		—	—	—	—	—	—	—	—	—	—	1			
1·5	4	16	**10**	12	10	—	—	4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—		—	56	8·25 ±0·08	0·86	
2·0	5	112	124	142	145	**120**	93	73	44	49	25	23	13	7	3	2	3	—	—	—	—	—	—	—			—	—	—	—	—	—	—	983	9·390 ±0·30	1·483 ±0·023	15·80 ±0·25
2·5	—	8	35	49	55	51	**69**	39	48	39	17	18	15	5	8	4	1	2	1	—	1	—	—	—	—	—	—	1	—	—	—	—	—	466	10·24 ±0·05	1·644 ±0·036	16·05 ±0·36
3·0	—	—	1	12	44	54	64	44	34	**49**	40	31	40	28	29	23	17	14	3	4	5	4	—	—	—	3	—	—	1	1	—	—	—	545	11·75 ±0·06	2·214 ±0·046	18·85 ±0·40
3·5	—	—		—	6	18	31	37	32	39	39	**26**	29	19	16	18	10	12	7	5	6	2	2	—	—	1	1	—	1	—	—	—	—	357	12·32 ±0·07	2·095 ±0·053	17·00 ±0·45
4·0	—	—		—	—	—	—	34	48	55	48	60	41	**35**	41	35	20	16	17	18	6	7	6	3	2	2	1	1	1	—	1	—	—	498	13·24 ±0·06	2·070 ±0·045	15·63 ±0·34
4·5	—	—		—	—		—	—	13	28	25	28	30	36	**20**	22	18	7	13	12	12	3	5	3	3	1	3	1		—	—	1	—	284	13·93 ±0·08	2·116 ±0·060	15·19 ±0·43
5·0	—		—	—	—		—	—		—	21	26	37	38	27	20	**22**	28	22	18	9	13	10	5	3	4	3	—	4	—	—	2	—	312	14·84 ±0·08	2·124 ±0·057	14·31 ±0·40
5·5	—				—			—	—		18	20	22	19	14	10	**14**	13	11	9	13	9	5	6	1	3	4	—	—	2	1		—	194	14·88		
6·0	—	—			—				—	—	17	24	31	27	23	29	**21**	26	17	13	11	9	3	5	5	1	4	5	3	—	2	1	—	277	14·93		
6·5	—	—	—			—			—	—	6	13	10	7	14	18	**9**	12	12	5	7	5	2	1	3	4	1		—	1	—	—		130	14·99		
7·0	—	—	—		—	—		—		—	9	22	21	24	19	22	**5**	10	15	6	17	4	4	5	2	4	2	2	1	1	—	—	—	195	14·96		
7·5	—	—				—	—	—	—		3	14	19	15	5	6	**11**	9	5	2	6	4	1	2	3	2	2	1	2	—	—		—	112	14·83		
8·0	—	—		—			—		—		9	16	8	6	15	13	**8**	2	11	9	8	8	—	3	—	2	2	—	—	1		—	—	121	14·87		
8·5	—	—	—	—		—	—		—	—	5	9	7	5	5	10	**3**	4	7	4	1	2	3	2	1	—	—		2	—		—	—	70	14·79		
9·0			—	—	—	—		—			2	6	7	4	8	9	**6**	3	3	1	7	2	1	1	2	2	3	2	—	—	—	1	—	70	15·01 +0·44		
9·5		—	—	—	—	—		—			2	6	3	5	5	5	**2**	2	1	—	3	1	3	2		3	—	—	1	—	—		—	44	15·09 ±0·58		
10·0	—		—		—	—		—	—	—		5	7	8	6	3	**5**	—	3	2	5	—	2	—	1	2	2	—	1		—	—	1	53	15·24 ±0·48		
10·5			—		—	—	—		—	—	3	4	2	—	1	**2**	1	2	4	1	1		—	—	—	—	—	—	1	—		—	—	22	14·52 ±0·74		
11·0			—		—	—	—				2	3	1	1	—	1	1	**1**	1	3	—	—	1	1	1		—	1	1	—	—		—	19	15·55 ±1·15		
11·5	—	—	—	—	—	—	—	—	—	—	—	2	3	4	1	—	2	1	1	—	1	—	1	—		1		—	—				—	17	14·50		
12·0	—	—	—	—	—	—	—	—	—	—	1	**3**	2	4	—	1	**2**	1	2	—	1	—		—	—	—	—	1	2	—		—	—	20	15·05		
12·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	**1**	1	1	—		—	—	—	—	—	—	—	—	—		—	4	15·50		
13·0	—	—	—	—	—	—	—	—	—	—	2	—	—	**1**	—	—	1	1	—	—	—	—	—	—		—	—	—	—	—	—	—	—	5	13·60		
13·5	—	—	—	—	—	—	—	—	—	—	1	—	—	—	2	1	—	—	—	—	—	—	—	—	—	—	—		—	—	—	—	—	4	13·63		
14·0	—	—	—	—	—	—	—	—	—	—	1	—	**1**	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3	13·33		
14·5	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1			
15·0		—		—	—	—	—	—	—		—	—	—	—	—	—	—	—	—	—	—	—	—	—		—	—	—		—	—	—	—	—			
15·5	—	—		—	—	—	—	—		—	—	—		—	1	—		—	—	—	—	—	—	—		—	—	—	—	—	—	—	—	1			
Totals	9	137	170	215	260	243	257	231	219	259	297	359	349	299	264	254	183	167	157	113	120	73	49	39	27	35	28	15	21	6	4	5	1	4765	—	—	—

TABLE IV.

Correlation of Dams and Offspring when Both are Under 2·25 Years Old.

Production of Dams in Pounds of Butter Fat.

Production of Offspring in Pounds of Butter Fat.	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	Totals
7	—	1	2	4	2	—	—	—	—	—	9
8	2	15	9	10	6	1	—	—	—	—	43
9	3	18	21	5	6	2	2	1	—	—	58
10	2	17	12	12	11	—	—	—	—	1	55
11	—	6	10	2	2	5	3	—	1	—	29
12	1	1	3	2	4	1	—	—	1	—	13
13	—	4	1	—	2	1	—	—	—	—	8
14	—	—	1	2	—	—	—	—	—	—	3
15	—	—	—	—	—	—	—	—	—	—	—
16	—	—	—	1	—	—	—	—	—	—	1
Totals	8	62	59	38	33	10	5	1	2	1	219

TABLE V.

Correlation of Maternal Granddams and their Offspring. Dams.

Production of Maternal Granddams in Pounds of Butter Fat.

Production of Dams in Pounds of Butter Fat.	*10·75*	*11·75*	*12·75*	*13·75*	*14·75*	*15·75*	*16·75*	*17·75*	*18·75*	*19·75*	*20·75*	*21·75*	*22·75*	*23·75*	*24·75*	*25·75*	*26·75*	*27·75*	Totals
10·75	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	2
11·75	—	2	9	8	6	3	4	—	—	1	—	—	—	—	—	—	—	—	33
12·75	—	9	11	17	15	6	5	2	3	1	—	—	—	—	—	—	—	—	69
13·75	—	5	12	11	16	10	7	3	—	4	—	—	—	—	—	—	—	—	68
14·75	—	1	10	13	10	10	10	10	3	1	2	—	—	1	—	—	—	—	71
15·75	—	5	8	8	8	16	7	4	—	3	3	1	—	—	—	1	—	—	64
16·75	—	3	12	15	3	18	10	5	2	6	3	1	—	—	—	—	1	—	79
17·75	—	1	2	9	6	4	6	3	4	1	—	1	—	—	—	—	—	—	37
18·75	—	—	—	5	1	1	8	—	2	2	1	1	—	—	—	—	—	—	21
19·75	—	1	—	3	3	—	4	—	—	3	6	—	—	—	—	—	—	—	20
20·75	—	—	2	—	1	—	1	—	—	—	2	—	—	—	—	—	—	—	6
21·75	—	—	—	1	—	—	—	1	—	—	1	—	—	—	—	—	—	—	3
22·75	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
23·75	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
24·75	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	2
Totals	—	27	67	91	71	68	63	28	14	22	18	4	—	1	—	1	1	—	476

TABLE VI.

Correlation of Maternal Granddams and Offspring. Mature Cows.

Production of Maternal Granddams in Pounds of Butter Fat.

Production of Offspring in Pounds of Butter Fat.	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	Totals
11	—	3	4	1	1	4	1	—	2	2	—	—	—	—	—	18
12	1	12	27	16	14	7	12	4	3	6	—	1	—	—	—	103
13	2	11	22	14	12	12	10	2	2	5	—	1	1	—	—	94
14	3	14	23	25	27	24	12	5	4	6	6	—	—	—	—	149
15	3	2	18	18	18	14	7	4	7	6	3	1	—	1	—	102
16	—	11	20	26	17	24	13	3	3	7	2	—	—	—	—	126
17	—	7	5	13	5	10	7	2	6	2	1	1	—	—	—	59
18	—	2	5	7	7	3	6	7	5	1	1	1	—	—	1	46
19	1	3	3	6	6	4	3	3	—	3	—	—	—	—	—	32
20	—	—	3	2	4	6	4	1	2	1	—	3	1	—	—	27
21	—	1	—	1	6	1	1	—	—	—	—	—	—	—	—	10
22	—	—	—	2	1	—	5	—	—	—	—	—	—	—	—	8
23	—	—	2	—	—	1	—	—	1	—	—	—	—	—	—	4
24	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
25	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
26	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
27	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
28	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
29	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
Totals	10	67	132	131	119	110	81	31	35	41	13	8	2	1	1	782

TABLE VII.

Correlation of Paternal Granddams and Offspring. Mature Cows.

Production of Paternal Granddams in Pounds of Butter Fat.

Production of Offspring in Pounds of Butter Fat.	*10·75*	*11·75*	*12·75*	*13·75*	*14·75*	*15·75*	*16·75*	*17·75*	*18·75*	*19·75*	*20·75*	*21·75*	*22·75*	*23·75*	*24·75*	*25·75*	*26·75*	*27·75*	*28·75*	*29·75*	Totals
10·75	—	—	1	3	1	1	—	1	1	1	2	1	1	—	—	—	—	—	—	—	13
11·75	—	5	16	22	16	14	10	7	9	11	8	7	3	—	1	—	—	—	—	1	130
12·75	—	7	31	12	21	28	15	22	21	36	25	9	4	—	—	—	1	—	—	2	234
13·75	1	2	35	32	26	24	28	28	37	34	32	21	7	1	1	—	5	3	—	2	319
14·75	1	3	26	21	22	26	28	32	23	25	17	13	3	4	1	—	2	—	—	—	247
15·75	—	4	19	21	9	23	25	16	11	19	17	9	1	—	—	—	1	—	—	—	175
16·75	—	—	9	9	12	19	6	21	20	17	9	8	1	1	—	—	—	—	—	1	133
17·75	—	1	3	7	6	12	10	7	13	16	10	11	2	1	—	3	1	—	—	—	103
18·75	—	—	4	—	2	4	5	7	5	8	5	3	—	—	—	—	—	1	—	—	44
19·75	—	—	2	1	2	6	5	4	2	2	2	2	—	—	—	—	1	1	—	—	30
20·75	—	—	2	—	2	2	2	2	1	2	1	1	—	—	—	—	—	—	—	—	15
21·75	—	—	1	1	—	—	—	—	3	1	2	1	—	1	—	—	—	—	—	—	10
22·75	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	2
23·75	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
24·75	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	2
25·75	—	—	—	1	—	—	—	—	—	1	—	—	—	—	1	—	—	—	—	—	3
26·75	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
27·75	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
28·75	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
29·75	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	2	22	150	131	119	160	135	147	147	174	132	87	22	8	4	3	11	5	—	6	1465

TABLE VIII.

Correlation of Paternal Granddams and Dams.

Production of Paternal Granddams in Pounds of Butter Fat.

Production of Dams in Pounds of Butter Fat.	*10·75*	*11·75*	*12·75*	*13·75*	*14·75*	*15·75*	*16·75*	*17·75*	*18·75*	*19·75*	*20·75*	*21·75*	*22·75*	*23·75*	*24·75*	*25·75*	*26·75*	Totals
10·75	—	—	1	—	—	—	—	—	—	—	1	—	—	—	—	—	—	2
11·75	—	—	6	9	3	3	6	9	5	1	10	2	—	—	—	—	—	54
12·75	—	1	13	3	9	11	9	12	18	6	17	2	—	—	—	—	1	102
13·75	—	—	11	8	7	14	17	12	18	14	16	5	4	—	1	—	2	129
14·75	—	1	11	13	2	18	15	20	16	13	12	6	4	—	1	1	—	133
15·75	—	1	9	7	7	16	10	11	9	11	6	11	2	—	—	—	—	100
16·75	—	—	4	5	9	5	7	2	13	9	12	3	1	—	—	—	—	70
17·75	—	1	3	1	1	6	6	1	4	3	5	3	2	—	—	—	1	37
18·75	—	—	—	2	3	4	2	1	5	3	9	6	—	—	—	—	—	35
19·75	—	—	1	2	—	2	2	4	2	3	3	—	2	—	—	—	—	21
20·75	—	1	—	—	—	—	3	2	—	1	5	10	—	1	—	—	—	23
21·75	—	—	2	—	—	—	—	—	—	1	3	—	—	—	—	—	—	6
22·75	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
23·75	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	—	6	61	50	41	79	77	74	90	65	99	48	15	1	2	1	4	713

TABLE IX.

Correlation of Dams and Offspring. Cows under Five Years Old but no other Restriction in regard to Age.

Production of Dams in Pounds of Butter Fat.

Production of Offspring in Pounds of Butter Fat.	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	Totals
7	—	1	1	4	3	4	2	—	1	—	—	—	—	—	—	—	—	—	16
8	1	12	6	14	22	13	14	13	6	5	7	2	—	1	1	—	—	—	117
9	1	10	13	12	12	21	22	23	9	10	3	6	4	—	—	—	—	—	146
10	1	18	9	22	24	18	23	12	21	9	7	3	2	—	—	—	—	—	169
11	1	8	10	13	15	16	17	9	13	8	5	6	2	1	—	1	1	1	127
12	1	5	4	11	5	11	13	13	11	9	5	4	2	2	—	—	—	—	96
13	—	5	2	3	5	7	7	9	9	4	1	2	2	1	1	—	—	—	58
14	—	—	3	4	1	5	6	2	5	3	4	—	—	2	—	—	1	—	36
15	—	3	1	1	3	1	4	3	4	1	3	—	—	1	—	—	—	—	25
16	—	2	—	1	1	1	1	1	3	—	2	—	—	—	1	—	—	—	13
17	—	1	—	—	1	—	2	—	3	—	—	2	—	—	—	—	—	—	9
18	—	—	—	—	2	—	—	—	1	1	1	—	—	—	—	—	—	—	5
19	1	1	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	4
Totals	6	66	49	85	94	97	112	85	87	50	38	25	12	8	3	1	2	1	821

TABLE X.

Correlation of Maternal Granddams and their Offspring.

Production of Maternal Granddams in Pounds of Butter Fat.

Production of Offspring in Pounds of Butter Fat.

	7	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	Totals
7	—	—	—	1	1	—	1	—	1	—	—	—	—	—	—	4
8	1	—	3	2	4	2	2	2	3	1	1	—	—	—	—	21
9	—	3	4	6	6	6	5	1	3	2	4	1	—	—	1	42
10	—	2	6	2	6	10	6	10	6	1	1	1	1	1	—	53
11	—	2	1	2	5	2	2	3	2	4	1	1	2	—	1	28
12	—	2	3	4	1	2	3	2	—	—	3	—	1	—	—	21
13	—	2	1	5	3	2	—	—	—	3	4	—	—	—	—	20
14	—	—	1	—	—	2	2	3	1	1	1	1	—	—	—	12
15	—	—	1	1	—	1	1	1	—	1	—	—	—	—	—	6
16	—	—	—	3	1	—	1	—	—	1	—	—	—	—	—	6
17	—	—	—	1	—	—	—	1	1	1	—	—	—	—	—	4
18	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
Totals	1	11	20	27	27	27	23	23	18	15	15	4	4	1	2	218

TABLE XI.

Correlation of Paternal Granddams and their Offspring.

Production of Paternal Granddams in Pounds of Butter Fat.

Production of Offspring in Pounds of Butter Fat.

	7	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	Totals
7	—	1	—	—	1	—	2	1	2	3	2	—	—	—	—	—	1	—	—	—	—	13
8	1	6	3	8	7	6	10	5	11	10	9	2	1	—	2	4	2	—	—	—	4	91
9	1	5	4	10	4	11	13	—	17	14	18	3	1	—	2	4	3	—	—	—	1	111
10	—	8	4	14	8	5	7	9	9	9	21	5	2	1	3	2	4	—	—	—	1	112
11	1	4	2	9	6	5	10	3	5	10	16	—	5	1	2	5	6	—	—	—	1	91
12	—	5	3	8	7	4	10	2	10	5	20	6	4	1	2	4	—	—	—	—	1	92
13	—	2	4	6	4	7	7	7	6	4	21	3	5	2	1	—	1	—	—	—	—	80
14	—	3	1	4	1	—	5	6	4	5	11	3	1	1	—	—	2	—	—	—	—	47
15	1	2	1	1	2	4	2	5	2	1	3	1	2	—	—	—	—	—	—	—	—	27
16	—	—	—	2	—	1	—	1	—	2	1	—	1	—	—	—	—	—	—	—	—	8
17	—	1	1	—	1	—	—	1	—	—	1	—	3	2	1	—	—	—	—	—	—	11
18	—	1	—	—	—	—	—	—	—	—	2	—	1	—	—	—	—	—	—	—	—	4
19	—	1	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	2
20	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	2
Totals	4	39	23	62	41	43	66	40	68	63	126	23	26	8	13	19	19	—	—	—	8	691

TABLE XII.

Correlation of Paternal Granddams and Dams.

Production of Paternal Granddams in Pounds of Butter Fat.

Production of Dams in Pounds of Butter Fat.	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	Totals
7	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	1	—	—	—	—	3
8	—	—	—	4	2	1	—	1	4	—	4	—	1	—	—	—	—	—	—	—	1	18
9	—	—	—	1	—	2	1	1	1	2	4	—	—	—	1	—	—	—	—	—	2	15
10	1	1	—	1	3	2	—	1	8	2	3	2	—	—	1	1	—	—	—	—	—	26
11	—	—	—	1	—	1	1	—	3	1	10	—	—	—	2	—	1	—	—	—	1	21
12	—	—	1	3	—	4	2	—	3	1	6	2	—	—	—	—	1	—	—	—	1	24
13	—	3	—	1	1	—	—	2	5	3	2	—	2	1	—	2	1	—	—	—	2	25
14	1	—	—	2	—	1	1	2	—	1	5	3	3	—	1	1	2	—	—	—	—	23
15	—	—	—	—	—	1	2	2	—	—	—	1	1	—	—	1	1	—	—	—	—	9
16	—	1	—	2	—	1	1	2	—	2	3	1	—	—	2	—	1	—	—	—	1	17
17	—	1	1	1	—	—	—	1	—	—	—	—	—	—	—	3	—	—	—	—	—	7
18	—	—	—	—	—	—	—	—	—	—	2	1	1	—	—	—	—	—	—	—	—	4
19	—	—	—	—	—	—	1	—	—	—	2	—	—	—	—	—	—	—	—	—	—	3
27	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
Totals	2	6	2	16	6	13	9	14	24	12	41	11	8	1	7	8	8	—	—	—	8	196

TABLE XIII.

Correlation of Age of Cows at First Record and Age of Corresponding Dams at Time of their "Most Remarkable Record." All Animals under Five Years Old.

Age of Dams—Years.

Age of Offspring—Years.	*1·75*	*2·25*	*2·75*	*3·25*	*3·75*	*4·25*	*4·75*	Totals
1·75	12	30	10	25	21	25	27	150
2·25	26	70	27	53	56	82	56	370
2·75	5	11	7	11	25	22	22	103
3·25	6	9	3	16	10	19	13	76
3·75	3	9	1	13	13	10	8	57
4·25	4	5	4	6	4	7	9	39
4·75	3	2	3	6	5	2	5	26
Totals	59	136	55	130	134	167	140	821

TABLES TO FACILITATE THE COMPUTATION OF THE PROBABLE ERRORS OF THE CHIEF CONSTANTS OF SKEW FREQUENCY DISTRIBUTIONS.

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THE general theory of the probable errors of the constants of skew frequency distributions was originally given with illustrations by Pearson and Filon, *Phil. Trans.* Vol. 191, A (1898), pp. 229—311. The values there deduced depend on the form of special frequency curve adopted, and involve considerable arithmetical work for each individual case. In these frequency investigations the fundamental constants are the well-known β_1 and β_2. Every frequency character expressible in terms of β_1 and β_2 can have its probable error determined, provided we know the probable errors of β_1 and β_2 and the correlation in deviations between β_1 and β_2. General expressions for the probable errors (or the S.D.'s Σ_{β_1}, Σ_{β_2}) of β_1 and β_2 as well as the correlation $R_{\beta_1\beta_2}$ of deviations in β_1 and β_2, together with the probable error of the criterion (or its S.D. $= \Sigma_K$) were first given by Pearson, *Phil. Trans.* Vol. 198, A (1902). These involve a knowledge of β_6, β_5, β_4 and β_3[*], further constants of the distribution which can only be found if the numerical values of μ_5, μ_6, μ_7 and μ_8 have been in some way determined. Now it has been shown that with the total frequencies usual in practice these high moments are subject to very large percentage errors[†], rendering their use extremely undesirable, even if we could overcome our natural repugnance to the great labour of calculating them.

But we have to bear in mind that the *exact* value of a probable error is not usually desired. What we more often require is a rough determination of its magnitude in order that we may appreciate whether a certain quantity is or is not really significant. Accordingly it had been the practice of the Biometric Laboratory, if a frequency was only moderately a-normal, to use the values of the higher β's in determining probable errors, which would flow from the assumption of normality; if on the other hand the distribution had values of β_1 and $\beta_2 - 3$ differing a good deal from zero, to assume that the higher moments might be obtained from a skew distribution of Type III. (i.e. a distribution for which $2\beta_2 - 3\beta_1 - 6 = 0$). The justification of such hypotheses lies in the fact that if our data are to be of much value, the probable errors must themselves be small, hence in calculating these errors it is legitimate to insert into the formulae for them values for the β's that only differ from their true values by small quantities. Such insertion can only introduce second order, and therefore for our purposes usually unimportant, changes in the probable error. This point is emphasised when we remember the large percentage errors of the high β's. For example, it is usual to take the probable error of a standard deviation $\sigma = {\cdot}67449\, \sigma/\sqrt{2n}$, but its true value $= {\cdot}67449 \dfrac{\sigma}{\sqrt{2n}} (1 + \tfrac{1}{2}\eta)^{\frac{1}{2}}$; the former really results from assuming the kurtosis, $\eta = \beta_2 - 3$, to have the value zero of the normal curve. There are very few arguments made from probable error which would be seriously affected if the probable error were altered by 25 per cent. of its value, or if η took values from $-{\cdot}875$ to $1{\cdot}125$, i.e. we might give β_2 any value between $2{\cdot}125$ and $4{\cdot}125$ to get in practice a sufficiently close result.

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[*] $\beta_3 = \mu_3\mu_5/\mu_2^4$, $\beta_4 = \mu_6/\mu_2^3$, $\beta_5 = \mu_7\mu_3/\mu_2^5$, $\beta_6 = \mu_8/\mu_2^4$.

[†] Pearson: "On the General Theory of Skew Correlation and Non-Linear Regression," *Drapers' Company Research Memoirs* (Dulau & Co., 1905), p. 8.

Now the object of the present paper is to extend this idea by applying a method of determining β_3, β_4, β_5 and β_6 still more exact than the methods indicated above. It is well-known that the frequency curves in common use are deduced from the integral of

$$\frac{1}{y}\frac{dy}{dx} = \frac{ax + b}{c_0 + c_1 x + c_2 x^2},$$

and that this really assumes the condition that the coefficients of higher terms in the denominator on the right, e.g. c_3, c_4, etc., are all zero. These conditions involve a finite difference relation between the successive moments, first published in 1903[*], and enable us to determine any higher moment from the first three μ_2, μ_3 and μ_4. Such a finite difference momental equation actually exists for all probability frequency distributions of the hypergeometrical series type, which cover so wide a range of chance problems.

It is practically impossible to determine in a large percentage of cases whether the higher moments do or do not within their probable errors obey this finite difference relation, for the reason above stated, i.e. the high values of their probable errors. The present tables assume that they do; in other words β_3, β_4,

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[*] *Biometrika*, Vol. II., p. 281.

β_5 and β_6 are calculated from β_1 and β_2, on the assumption that the values of them obtained from the finite difference momental formula are sufficiently accurate to use in Pearson's formulae for Σ_{β_1}, Σ_{β_2}, $R_{\beta_1\beta_2}$ and Σ_{sk}, etc.

The following are the finite difference β-formulae used:

$$\beta_n\ (\text{even}) = (n+1)\left\{\frac{\beta_{n-1}}{2} + \left(1+\frac{\alpha}{2}\right)\beta_{n-2}\right\}\Big/\left(1-\frac{n-1}{2}\alpha\right),$$

$$\beta_n\ (\text{odd}) = (n+1)\left\{\frac{\beta_1}{2}\beta_{n-1} + \left(1+\frac{\alpha}{2}\right)\beta_{n-2}\right\}\Big/\left(1-\frac{n-1}{2}\alpha\right),$$

where $\alpha = (2\beta_2 - 3\beta_1 - 6)/(\beta_2 - 3)$.

The process of calculation adopted was as follows:

Fundamental values of β_1, β_2 were adopted; these are indicated in Table VI., and the resulting values of β_3, β_4, β_5, β_6 calculated by the above formula were then found to seven figures. These are tabulated to six figures as they may be of service for the determination of other constants as occasion arises.

The values of κ_2 for the different values of β_1 and β_2 being known, it was seen that a very simple diagram would permit of a statistician ascertaining at once from his values of β_1 and β_2 the type of his frequency distribution. In fact this diagram brings out very suggestively the normal curve "point" (G), the Types II., III., V. and VII.* "lines" and the Types I., IV. and VI. "areas" of occurrence. By aid of this diagram and a reasonable consideration of the probable errors of his β_1 and β_2 the statistician can readily determine within what limits he is justified in using any special type of frequency curve for given data.

The following values of analytic constants for the fundamental values were then found $\sqrt{N}\Sigma_{\beta_1}$ (Table I.), $\sqrt{N}\Sigma_{\beta_2}$ (Table II.), $R_{\beta_1\beta_2}$ (Table III.), and intermediate values deduced by interpolation. It is believed that these interpolated values recorded to two places of decimals only are amply sufficient for the practical uses to which these tables will be put.

From these tables were found, again using interpolation, the probable errors of the two chief desired physical constants: (i) d, the distance from mean to mode or the modal divergence, and (ii) Sk, the skewness of the distribution. In Table IV. we have the values of $\sqrt{N}\,\Sigma_d/\sigma$ and in Table V. the values of $\sqrt{N}\,\Sigma_{sk}$ provided. The actual quantities tabled in Tables I., II., IV. and V. are such that when multiplied by $\cdot 67449/\sqrt{N}$ we obtain immediately the required probable errors. The value of this numerical factor can at once, however, be extracted from Winifred Gibson's Tables for computing probable errors†.

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* It is convenient to call $y=y_0\left(1+\frac{x^2}{a^2}\right)^{-n}$ Type VII., see *Biometrika*, Vol. IV., p. 174.

† *Biometrika*, Vol. IV., p. 385.

In the actual construction of the tables fundamental values of β_1 and β_2 were taken covering the usual range of these quantities in actual practice. The tables are, however, limited by certain considerations, which are not without suggestiveness for theoretical frequency discussions. Along a certain line, indicated in our diagram, β_6 becomes infinite. But this is statistically impossible. Hence, either we never get frequency distributions having such values of β_1 and β_2, or if we do the finite difference moment formula cannot approximate in such cases to the true state of affairs, and we must introduce an additional coefficient c_3 into the denominator of the right-hand side of the fundamental differential equation. The discussion and classification of such curves are now in hand. On the diagram the areas where they are absolutely needful are marked *Heterotypic.*

It is believed that the diagrams and tables now published will prove extremely useful to the biometrician. Having determined his β_1 and β_2, he will be able to see at a glance whether his frequency distribution may be safely treated by the usual types or is heterotypic. If it be one of the usual types, he will know at once how to classify it. Next an examination of Tables I., II., III. and IV., or such examination with a short interpolation, will give him with sufficient practical accuracy the probable errors of β_1, β_2, the modal divergence (d) and the skewness (Sk). Should other and more elaborate probable errors be required, they will be deducible from Tables I., II. and III., or by using Table VI., for all the usually desired constants depend upon the quantities therein tabulated.

The two diagrams represent, (A), a practical working diagram covering the customary range of β_1 and β_2 and, (B), a diagram on a small scale showing the whole nature of the distribution of the type curves from the theoretical standpoint.

The upper part of either diagram is bounded by the line $4\beta_2 - 3\beta_1 = 0$, because it has been shown that β_2 is of necessity $> \frac{3}{4}\beta_1$*. The normal curve is represented by a point $\beta_1 = 0$, $\beta_2 = 3$ marked G on the diagram.

The transition Types III. and V. are given by values of β_1, β_2 on the line

$$2\beta_2 - 3\beta_1 - 6 = 0,$$

and on the cubic

$$\beta_1 (\beta_2 + 3)^2 = 4 (4\beta_2 - 3\beta_1)(2\beta_2 - 3\beta_1 - 6),$$

respectively.

Type II. is a particular case of Type I. for $\beta_1 = 0$, and Type VII. a particular case of Type IV. for $\beta_1 = 0$; they are thus represented by portions of the vertical running through the "normal point" G. While the upper boundary of the diagram lies in the U-curve part of Type I., to be discussed below, the lower boundary is fixed by the line $8\beta_2 - 15\beta_1 - 36 = 0$ along which the finite difference moment formula first fails. This matter needs a little consideration.

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* See Pearson: *Phil. Trans.* Vol. 186, A, pp. 343—414.

Diagram A.

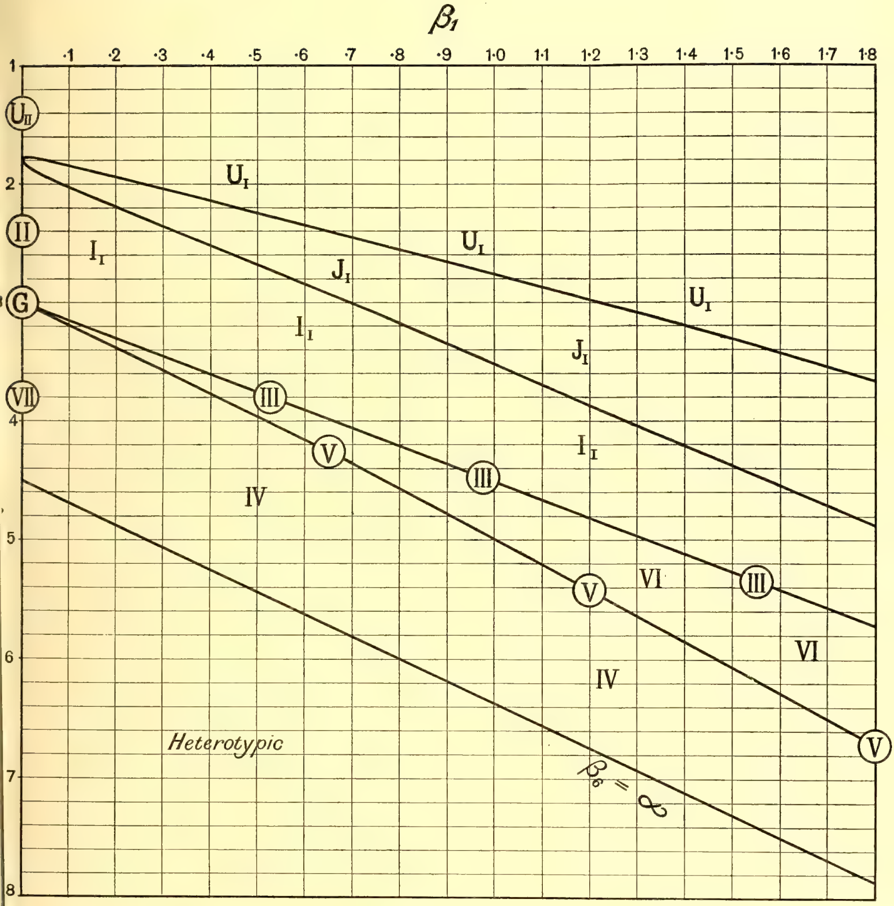
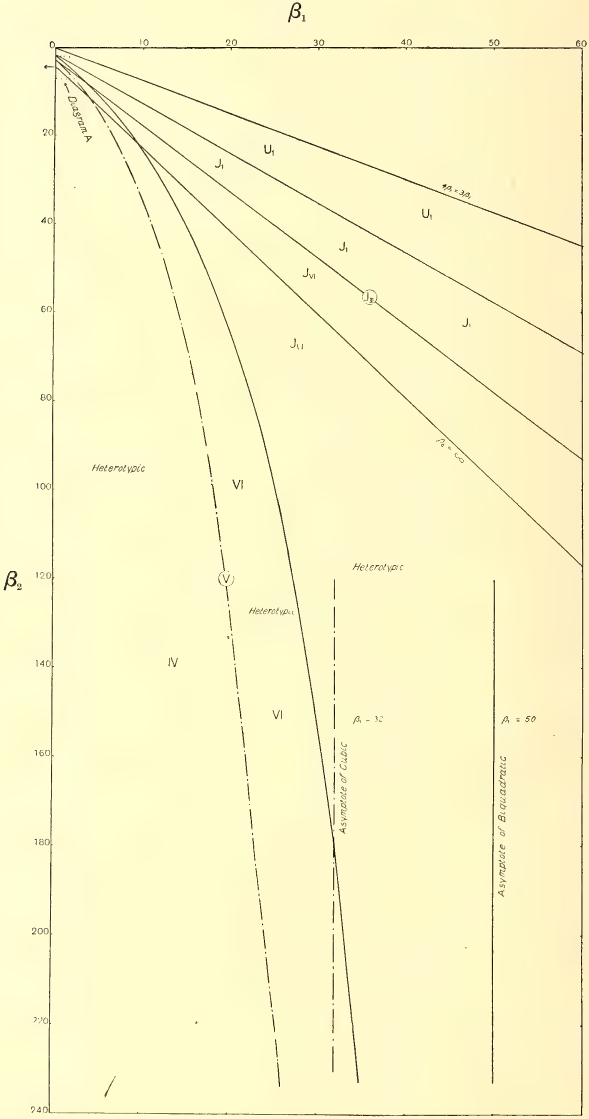

β_1
·1
·2
·3
·4
·5
·6
·7
·8
·9
1·0
1·1
1·2
1·3
1·4
1·5
1·6
1·7
1·8
1
2
4
5
6
7
8
U_{II}
II
G
VII
U_I
U_I
U_I
I_I
I_I
I_I
J_I
J_I
III
III
III
V
V
V
V
IV
IV
VI
VI
Heterotypic
$\beta_6 = \infty$

DIAGRAM B.


β_1
0
10
20
30
40
50
60
β_2
20
40
60
80
100
120
140
160
180
200
220
240
Diagram A
$4\beta_2 = 3\beta_1$
U_1
J_1
U_1
J_1
J_{VI}
J_{III}
J_1
J_{VI}
$\beta_2 = \infty$
Heterotypic
VI
V
Heterotypic
Heterotypic
IV
VI
$\beta_1 = 32$
$\beta_1 = 50$
Asymptote of Cubic
Asymptote of Biquadratic

The equation to Type IV. is

$$y = y_0 \left\{1 + \frac{x^2}{a^2}\right\}^{-\frac{1}{2}(r+2)} e^{-\nu \tan^{-1}(x/a)},$$

with the reduction formula*

$$\mu_{n+1} = \frac{n}{r-n}\left(\frac{a^2\mu_{n-1}}{\cos^2\phi} - 2a\tan\phi\,\mu_n\right),$$

where $\tan\phi = \nu/r$ and $r = 6(\beta_2 - \beta_1 - 1)/(2\beta_2 - 3\beta_1 - 6)$. It is clear that when $r = n$, μ_{n+1} becomes infinite. Now the probable errors of β_1 and β_2 require us to go as far as μ_8. Hence r must be greater than 7 if we are to use this formula, and this gives us at once the limiting line

$$8\beta_2 - 15\beta_1 - 36 = 0.$$

Of course the difference formula would also fail for μ_5, μ_6 or μ_7, if r were equal to 4, 5 or 6. But all the resulting lines lie outside the above line, which is all we need take into account. The failure of the difference formula is easily seen if we remember that

$$\frac{1}{y}\frac{dy}{dx} = \frac{a_0 + a_1 x}{c_0 + c_1 x + c_2 x^2}$$

leads at once for this type to

$$\int_{-\infty}^{+\infty} x^n (a_0 + a_1 x)\, y dx = \int_{-\infty}^{+\infty} (c_0 + c_1 x + c_2 x^2)\, x^n\, dy$$

or

$$N(a_0\mu_n + a_1\mu_{n+1}) = -N(c_0 n\mu_{n-1} + c_1(n+1)\mu_n + c_2(n+2)\mu_{n+1}) + \left[y(c_0 x^n + c_1 x^{n+1} + c_2 x^{n+2})\right]_{-\infty}^{+\infty}.$$

The difference formula above follows from supposing the term between brackets to vanish at the limits.

But this it will not do unless

$$\left[\frac{x^{n+2}}{(a^2 + x^2)^{\frac{1}{2}(r+2)}}\right]_{x=\infty} = 0,$$

or if a be finite, unless r be $> n$, or to apply present results $r > 7$.

Of course for any real data μ_8 may become large, but it cannot actually become infinite. Fairly good fits—owing to the agreement of the first four moments—may be found even near the line $8\beta_2 - 15\beta_1 - 36 = 0$, but if we want to get the probable errors of β_1 and β_2 in this neighbourhood, it is best to calculate the higher moments and β_3, β_4, β_5 and β_6 from the actual data. Outside this line we have marked the area as *Heterotypic,* because theoretically we need to introduce further terms into the denominator of our expression, i.e. to use c_3.

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* See *Biometrika*, Vol. II., p. 281.

We now pass to a consideration of the subtypes of Type I. The equation is*

$$y = y_0 \left(1 + \frac{x}{a_1}\right)^{m_1} \left(1 - \frac{x}{a_2}\right)^{m_2},$$

where m_1 and m_2 are roots of the quadratic

$$m^2 - m(r-2) + \epsilon - r + 1 = 0$$

and

$$r = 6(\beta_2 - \beta_1 - 1)/(3\beta_1 - 2\beta_2 + 6),$$

$$\epsilon = \frac{r^2}{4 + \frac{1}{4}\beta_1 (r+2)^2/(r+1)}.$$

Now m_1 and m_2 will either be both positive or both negative if $\epsilon - r + 1$ is positive, or the curve $\epsilon - r + 1 = 0$ separates the area of J-curves or modeless curves from the area of modal curves (I_1 curves) and the area of anti-modal curves or U-curves.

$\epsilon - r + 1 = 0$ is the biquadratic

$$\beta_1 (8\beta_2 - 9\beta_1 - 12)/(4\beta_2 - 3\beta_1) = (10\beta_2 - 12\beta_1 - 18)^2/(\beta_2 + 3)^2.$$

This biquadratic was traced by expressing it in the form:

$$\beta_1 = 4(1 + 2\alpha)^2 (2 + \alpha)/(2 + 3\alpha),$$

and then finding β_1 for a series of values of α and determining β_2 from the equation

$$\alpha = (2\beta_2 - 3\beta_1 - 6)/(\beta_2 + 3).$$

Within the loop of the biquadratic all curves are J-curves, and the term "skewness" loses its essential meaning. Within this area, it will be noticed, our tables do not give the probable error of the skewness or of the modal divergence. Above this loop and up to the line $4\beta_2 - 3\beta_1 = 0$, we are in the range of U-curves and the skewness signifies the ratio to the standard deviation of the distance from mean to anti-mode. Below the loop we are in the customary Type I. area with m_1 and m_2 both positive.

Our second Diagram, B, shows what becomes of the biquadratic limiting the J-shaped curves. It first meets the Type III. line, and at this point Type III. curves become J-curves† and cease to have a true mode distinct from the asymptote value. The biquadratic then passes into the Type VI. area and Type VI. curves become J-curves beyond this. It never crosses, however, into

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* Pearson: *Phil. Trans.* Vol. 186, A, pp. 367—371.

† Pearson: *Phil. Trans.* Vol. 186, A, p. 374 and Plate 9, Fig. 5.

Type IV. area, for the cubic which bounds Type IV. asymptotes to the vertical line $\beta_1 = 32$, and the biquadratic asymptotes to the vertical line $\beta_1 = 50$ as shown on the diagram.

Of course much of Diagram B extends beyond the values of β_1 and β_2 that we are familiar with in actual frequency distributions. For practical purposes Diagram A drawn to a much larger scale suffices, and guides the statistician to the appropriate type and to the probable errors. Nevertheless Diagram B will indicate many points of much theoretical interest, and serves to show where failure in curve fitting is likely to arise.

TABLE I.

Values of $\sqrt{N}\,\Sigma_{\beta_1}$.

β_1

β_2	*0·00*	*0·05*	*0·10*	*0·15*	*0·20*	*0·25*	*0·30*	*0·35*	*0·40*	*0·45*	*0·50*	*0·55*	*0·60*	*0·65*	*0·70*	*0·75*
2·0	0·00	0·58	0·93	1·15	1·37	1·57	1·77	1·97	2·17	2·38	2·58	2·80	3·02	3·24	3·46	3·71
2·1	0·00	0·59	0·95	1·12	1·30	1·50	1·70	1·90	2·10	2·29	2·48	2·69	2·91	3·12	3·34	3·57
2·2	0·00	0·60	0·97	1·13	1·30	1·48	1·67	1·86	2·05	2·22	2·41	2·61	2·81	3·01	3·22	3·44
2·3	0·00	0·62	0·99	1·15	1·32	1·49	1·67	1·84	2·02	2·19	2·36	2·55	2·74	2·93	3·12	3·32
2·4	0·00	0·64	1·02	1·20	1·37	1·54	1·70	1·86	2·02	2·18	2·34	2·51	2·68	2·85	3·03	3·22
2·5	0·00	0·66	1·05	1·26	1·45	1·61	1·76	1·91	2·05	2·19	2·33	2·49	2·65	2·81	2·97	3·14
2·6	0·00	0·69	1·10	1·34	1·54	1·72	1·86	2·00	2·13	2·25	2·37	2·50	2·64	2·78	2·92	3·08
2·7	0·00	0·73	1·15	1·42	1·64	1·83	1·96	2·09	2·22	2·32	2·42	2·53	2·65	2·77	2·90	3·05
2·8	0·00	0·77	1·22	1·51	1·75	1·94	2·07	2·20	2·32	2·41	2·50	2·60	2·70	2·80	2·93	3·05
2·9	0·00	0·81	1·30	1·61	1·87	2·06	2·20	2·33	2·44	2·53	2·62	2·70	2·79	2·89	2·99	3·09
3·0	0·00	0·87	1·40	1·73	2·01	2·20	2·34	2·47	2·57	2·67	2·76	2·84	2·92	3·00	3·09	3·18
3·1	0·00	0·94	1·53	1·86	2·17	2·35	2·51	2·64	2·75	2·85	2·94	3·00	3·08	3·15	3·23	3·30
3·2	0·00	1·02	1·67	2·02	2·33	2·52	2·71	2·84	2·95	3·05	3·14	3·22	3·27	3·33	3·40	3·46
3·3	0·00	1·12	1·82	2·20	2·50	2·71	2·92	3·06	3·18	3·28	3·37	3·44	3·50	3·55	3·60	3·65
3·4	0·00	1·24	1·99	2·38	2·68	2·93	3·14	3·30	3·43	3·53	3·63	3·70	3·75	3·79	3·83	3·87
3·5	0·00	1·37	2·16	2·57	2·89	3·17	3·39	3·56	3·69	3·81	3·91	3·98	4·02	4·06	4·10	4·12
3·6	0·00	1·50	2·33	2·78	3·11	3·43	3·65	3·84	3·99	4·12	4·22	4·29	4·33	4·37	4·40	4·41
3·7	0·00	1·64	2·50	2·99	3·36	3·70	3·93	4·14	4·31	4·44	4·54	4·61	4·66	4·70	4·72	4·74
3·8	0·00	1·78	2·67	3·20	3·62	3·97	4·23	4·46	4·64	4·77	4·87	4·95	5·00	5·05	5·07	5·09
3·9	0·00	1·93	2·86	3·43	3·89	4·25	4·54	4·79	4·97	5·11	5·23	5·32	5·38	5·43	5·46	5·48
4·0	0·00	2·10	3·07	3·69	4·17	4·55	4·87	5·13	5·32	5·48	5·62	5·72	5·79	5·84	5·88	5·89
4·1	—	—	3·29	3·87	4·47	4·87	5·21	5·49	5·69	5·87	6·03	6·15	6·23	6·28	6·32	6·33
4·2	—	—	3·53	4·19	4·79	5·21	5·58	5·88	6·10	6·30	6·46	6·60	6·69	6·75	6·80	6·81
4·3	—	—	3·78	4·52	5·13	5·58	5·97	6·29	6·54	6·75	6·93	7·07	7·18	7·25	7·29	7·31
4·4	—	—	4·05	4·85	5·49	5·98	6·40	6·74	7·01	7·24	7·42	7·57	7·68	7·76	7·80	7·83
4·5	—	—	4·33	5·18	5·88	6·42	6·87	7·23	7·52	7·75	7·95	8·10	8·21	8·29	8·34	8·37
4·6	—	—	—	—	—	—	7·37	7·76	8·07	8·30	8·51	8·66	8·76	8·85	8·91	8·95
4·7	—	—	—	—	—	—	7·90	8·31	8·64	8·90	9·11	9·25	9·35	9·44	9·50	9·54
4·8	—	—	—	—	—	—	8·46	8·88	9·24	9·54	9·75	9·89	9·99	10·08	10·14	10·18
4·9	—	—	—	—	—	—	9·05	9·47	9·86	10·21	10·42	10·58	10·69	10·78	10·84	10·80
5·0	—	—	—	—	—	—	9·66	10·08	10·50	10·90	11·19	11·33	11·44	11·53	11·60	11·64
5·1	—	—	—	—	—	—	—	—	—	—	—	—	12·26	12·36	12·42	12·43
5·2	—	—	—	—	—	—	—	—	—	—	—	—	13·10	13·26	13·29	13·29
5·3	—	—	—	—	—	—	—	—	—	—	—	—	13·98	14·15	14·18	14·18
5·4	—	—	—	—	—	—	—	—	—	—	—	—	14·91	15·05	15·10	15·11
5·5	—	—	—	—	—	—	—	—	—	—	—	—	15·90	15·98	16·05	16·07
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE I.—(*continued*).

Values of $\sqrt{N}\,\Sigma_{\beta_1}$.

β_1															
0·80	*0·85*	*0·90*	*0·95*	*1·00*	*1·05*	*1·10*	*1·15*	*1·20*	*1·25*	*1·30*	*1·35*	*1·40*	*1·45*	*1·50*	β_2
3·96	4·21	4·47	4·73	5·00	5·27	5·55	5·83	6·12	6·41	6·71	7·01	7·31	7·62	7·94	*2·0*
3·80	4·03	4·27	4·53	4·80	5·07	5·34	5·62	5·90	6·18	6·48	6·77	7·07	7·37	7·69	*2·1*
3·66	3·88	4·11	4·36	4·63	4·88	5·15	5·42	5·69	5·96	6·25	6·54	6·84	7·14	7·45	*2·2*
3·52	3·74	3·96	4·20	4·46	4·71	4·96	5·22	5·48	5·75	6·02	6·31	6·61	6·91	7·21	*2·3*
3·41	3·62	3·83	4·05	4·29	4·54	4·78	5·03	5·28	5·55	5·82	6·10	6·38	6·68	6·97	*2·4*
3·32	3·51	3·71	3·92	4·15	4·38	4·61	4·85	5·10	5·36	5·62	5·89	6·16	6·45	6·74	*2·5*
3·25	3·42	3·60	3·80	4·01	4·23	4·45	4·68	4·92	5·17	5·43	5·68	5·94	6·22	6·51	*2·6*
3·20	3·35	3·51	3·69	3·89	4·10	4·32	4·54	4·76	5·00	5·24	5·48	5·73	6·00	6·28	*2·7*
3·18	3·32	3·47	3·63	3·80	4·00	4·21	4·41	4·62	4·84	5·07	5·30	5·53	5·79	6·06	*2·8*
3·19	3·32	3·45	3·60	3·75	3·92	4·11	4·30	4·49	4·70	4·91	5·12	5·34	5·59	5·85	*2·9*
3·27	3·38	3·49	3·61	3·74	3·87	4·03	4·21	4·39	4·58	4·78	4·98	5·19	5·42	5·68	*3·0*
3·38	3·47	3·57	3·67	3·77	3·89	4·02	4·16	4·31	4·48	4·66	4·85	5·05	5·28	5·53	*3·1*
3·53	3·60	3·68	3·76	3·84	3·93	4·03	4·15	4·28	4·43	4·59	4·75	4·92	5·15	5·40	*3·2*
3·70	3·75	3·81	3·88	3·95	4·02	4·10	4·19	4·28	4·42	4·56	4·69	4·84	5·04	5·28	*3·3*
3·90	3·93	3·97	4·03	4·08	4·14	4·20	4·26	4·34	4·45	4·56	4·66	4·78	4·97	5·18	*3·4*
4·14	4·17	4·19	4·22	4·26	4·30	4·34	4·39	4·45	4·52	4·60	4·68	4·79	4·93	5·12	*3·5*
4·42	4·44	4·45	4·47	4·49	4·51	4·54	4·57	4·62	4·67	4·74	4·78	4·81	4·95	5·09	*3·6*
4·74	4·75	4·76	4·76	4·77	4·78	4·79	4·81	4·84	4·87	4·90	4·92	4·95	5·04	5·13	*3·7*
5·10	5·10	5·09	5·08	5·08	5·07	5·07	5·06	5·06	5·08	5·09	5·11	5·14	5·18	5·22	*3·8*
5·49	5·48	5·46	5·44	5·42	5·40	5·37	5·35	5·33	5·32	5·32	5·33	5·34	5·35	5·37	*3·9*
5·89	5·88	5·86	5·83	5·80	5·76	5·72	5·69	5·65	5·62	5·60	5·58	5·57	5·57	5·59	*4·0*
6·33	6·32	6·30	6·26	6·21	6·16	6·11	6·06	6·02	5·98	5·94	5·91	5·88	5·86	5·86	*4·1*
6·80	6·79	6·76	6·71	6·65	6·60	6·54	6·48	6·42	6·36	6·31	6·27	6·24	6·21	6·18	*4·2*
7·30	7·28	7·25	7·19	7·13	7·07	7·01	6·93	6·87	6·80	6·74	6·67	6·62	6·57	6·53	*4·3*
7·83	7·80	7·76	7·71	7·65	7·58	7·51	7·44	7·37	7·28	7·20	7·12	7·05	6·98	6·92	*4·4*
8·38	8·36	8·32	8·28	8·21	8·14	8·07	7·99	7·90	7·81	7·71	7·61	7·51	7·42	7·34	*4·5*
8·96	8·95	8·91	8·86	8·79	8·72	8·64	8·55	8·45	8·35	8·24	8·13	8·00	7·90	7·80	*4·6*
9·57	9·57	9·53	9·47	9·40	9·33	9·24	9·14	9·04	8·93	8·82	8·69	8·55	8·42	8·31	*4·7*
10·20	10·23	10·16	10·10	10·05	9·97	9·88	9·77	9·67	9·55	9·42	9·28	9·14	9·00	8·87	*4·8*
10·91	10·92	10·87	10·80	10·74	10·66	10·57	10·44	10·32	10·18	10·04	9·90	9·76	9·63	9·50	*4·9*
11·66	11·65	11·61	11·55	11·48	11·39	11·29	11·17	11·04	10·90	10·77	10·62	10·46	10·30	10·14	*5·0*
12·45	12·43	12·38	12·32	12·24	12·14	12·03	11·91	11·78	11·64	11·50	11·34	11·15	10·96	10·82	*5·1*
13·28	13·25	13·20	13·13	13·04	12·92	12·80	12·67	12·54	12·40	12·24	12·07	11·88	11·72	11·54	*5·2*
14·16	14·12	14·07	13·98	13·87	13·76	13·63	13·47	13·35	13·20	13·02	12·84	12·66	12·48	12·30	*5·3*
15·09	15·06	15·00	14·90	14·78	14·65	14·51	14·36	14·22	14·05	13·81	13·67	13·48	13·29	13·11	*5·4*
16·06	16·02	15·96	15·87	15·76	15·63	15·49	15·33	15·17	15·00	14·81	14·61	14·40	14·18	13·97	*5·5*
—	—	17·02	16·91	16·79	16·67	16·51	16·34	16·18	15·95	15·70	15·50	15·30	15·07	14·84	*5·6*
—	—	18·14	17·99	17·88	17·75	17·58	17·40	17·23	16·94	16·70	16·47	16·26	16·04	15·77	*5·7*
—	—	19·34	19·13	19·02	18·87	18·69	18·48	18·26	17·98	17·74	17·50	17·26	17·01	16·76	*5·8*
—	—	20·57	20·36	20·20	20·03	19·84	19·62	19·39	19·11	18·84	18·59	18·32	18·05	17·78	*5·9*
—	—	21·86	21·65	21·45	21·25	21·03	20·79	20·54	20·29	20·02	19·76	19·47	19·18	18·90	*6·0*
—	—	—	—	—	—	22·36	22·18	21·92	21·61	21·31	20·97	20·61	20·30	20·13	*6·1*
—	—	—	—	—	—	23·77	23·61	23·32	23·00	22·63	22·22	21·82	21·50	21·29	*6·2*
—	—	—	—	—	—	25·33	25·09	24·74	24·38	24·00	23·55	23·13	22·78	22·50	*6·3*
—	—	—	—	—	—	26·95	26·64	26·27	25·86	25·43	26·00	24·52	24·12	23·82	*6·4*
—	—	—	—	—	—	28·61	28·18	27·73	27·30	26·89	26·46	26·06	25·65	25·24	*6·5*
—	—	—	—	—	—	—	—	—	—	—	—	27·67	27·21	26·75	*6·6*
—	—	—	—	—	—	—	—	—	—	—	—	29·40	28·90	28·35	*6·7*
—	—	—	—	—	—	—	—	—	—	—	—	31·15	30·61	29·94	*6·8*
—	—	—	—	—	—	—	—	—	—	—	—	33·02	32·41	31·72	*6·9*
—	—	—	—	—	—	—	—	—	—	—	—	34·89	34·16	33·59	*7·0*

TABLE II.

Values of $\sqrt{N}\,\Sigma_{\beta_2}$.

β_1

β_2	*0·00*	*0·05*	*0·10*	*0·15*	*0·20*	*0·25*	*0·30*	*0·35*	*0·40*	*0·45*	*0·50*	*0·55*	*0·60*	*0·65*	*0·70*	*0·75*
2·0	1·41	1·60	1·74	1·93	2·11	2·28	2·44	2·60	2·77	2·94	3·12	3·29	3·46	3·65	3·86	4·05
2·1	1·57	1·76	1·89	2·00	2·10	2·20	2·35	2·51	2·68	2·86	3·05	3·24	3·44	3·64	3·84	4·04
2·2	1·75	1·94	2·07	2·16	2·20	2·28	2·40	2·53	2·68	2·85	3·04	3·23	3·43	3·63	3·83	4·03
2·3	1·95	2·16	2·28	2·35	2·42	2·49	2·58	2·67	2·78	2·92	3·09	3·27	3·45	3·64	3·84	4·03
2·4	2·18	2·39	2·53	2·60	2·72	2·82	2·90	2·96	3·02	3·10	3·22	3·35	3·50	3·68	3·86	4·03
2·5	2·46	2·68	2·83	2·97	3·09	3·19	3·27	3·31	3·32	3·36	3·47	3·53	3·63	3·75	3·88	4·03
2·6	2·78	3·03	3·24	3·38	3·52	3·60	3·66	3·69	3·69	3·70	3·75	3·78	3·83	3·87	3·95	4·07
2·7	3·17	3·48	3·71	3·87	3·98	4·03	4·08	4·12	4·11	4·09	4·07	4·06	4·06	4·06	4·07	4·15
2·8	3·64	4·02	4·26	4·42	4·52	4·58	4·60	4·59	4·57	4·52	4·44	4·39	4·34	4·32	4·31	4·34
2·9	4·22	4·65	4·94	5·11	5·20	5·22	5·18	5·13	5·07	4·99	4·90	4·80	4·70	4·63	4·60	4·61
3·0	4·90	5·48	5·76	5·89	5·95	5·93	5·86	5·76	5·65	5·53	5·41	5·30	5·20	5·12	5·05	5·00
3·1	5·75	6·41	6·72	6·88	6·90	6·82	6·70	6·54	6·38	6·22	6·07	5·92	5·79	5·69	5·60	5·53
3·2	6·77	7·55	7·90	8·00	7·97	7·83	7·63	7·42	7·21	7·03	6·86	6·70	6·53	6·39	6·26	6·14
3·3	8·00	8·83	9·22	9·30	9·22	9·02	8·80	8·53	8·29	8·05	7·83	7·60	7·38	7·20	7·02	6·84
3·4	9·37	10·28	10·68	10·76	10·67	10·46	10·20	9·91	9·62	9·31	9·01	8·73	8·44	8·18	7·92	7·66
3·5	10·85	11·75	12·31	12·52	12·46	12·25	11·95	11·60	11·24	10·86	10·45	10·03	9·63	9·26	8·90	8·54
3·6	12·67	13·74	14·40	14·78	14·53	14·21	13·80	13·38	12·95	12·55	12·10	11·60	11·06	10·54	10·02	9·55
3·7	14·78	15·98	16·78	17·09	16·93	16·53	16·05	15·58	15·09	14·61	14·08	13·49	12·74	12·02	11·36	10·80
3·8	17·50	18·83	19·83	20·03	19·78	19·36	18·76	18·22	17·64	16·98	16·25	15·30	14·42	13·60	12·88	12·27
3·9	20·80	22·50	23·68	23·81	23·34	22·67	21·98	21·14	20·29	19·45	18·58	17·54	16·50	15·54	14·77	14·06
4·0	24·74	26·83	28·47	28·05	27·24	26·29	25·25	24·18	23·03	22·02	21·01	20·01	19·04	18·12	17·23	16·36
4·1	—	—	35·00	34·17	32·88	31·36	29·77	28·13	26·60	25·12	23·82	22·64	21·54	20·53	19·56	18·62
4·2	—	—	43·3	41·4	39·2	37·2	35·2	33·2	31·2	29·2	27·4	26·0	24·7	23·4	22·3	21·3
4·3	—	—	55·3	51·6	48·0	44·6	41·2	38·6	36·2	33·8	31·8	30·1	28·5	26·9	25·6	24·3
4·4	—	—	72·7	66·0	59·7	54·1	49·5	45·7	42·1	39·2	36·8	34·8	32·9	31·0	29·2	27·7
4·5	—	—	96·5	82·7	72·7	65·3	59·8	54·7	50·8	47·2	44·0	41·0	38·5	36·2	34·1	32·0
4·6	—	—	—	—	—	—	75·0	68·0	62·2	56·9	52·2	48·2	45·1	42·2	39·6	37·2
4·7	—	—	—	—	—	—	101·3	87·2	76·8	68·3	62·0	56·9	52·7	49·1	45·9	42·8
4·8	—	—	—	—	—	—	140·0	115·2	96·2	82·6	72·7	66·1	60·9	56·7	52·9	49·3
4·9	—	—	—	—	—	—	204·5	150·8	122·3	102·5	89·1	80·2	72·4	66·6	61·4	56·7
5·0	—	—	—	—	—	—	325·7	206·0	154·2	126·8	110·1	96·9	86·6	78·1	71·2	65·6
5·1	—	—	—	—	—	—	—	—	—	—	—	—	103·6	94·4	85·9	78·0
5·2	—	—	—	—	—	—	—	—	—	—	—	—	130·4	116·4	104·0	91·0
5·3	—	—	—	—	—	—	—	—	—	—	—	—	175·2	144·8	124·4	109·6
5·4	—	—	—	—	—	—	—	—	—	—	—	—	224·4	178·0	151·0	132·6
5·5	—	—	—	—	—	—	—	—	—	—	—	—	340·8	246·0	195·3	163·2
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE II.—(*continued*).

Values of $\sqrt{N}\,\Sigma_{\beta_2}$.

β_1

0·80	*0·85*	*0·90*	*0·95*	*1·00*	*1·05*	*1·10*	*1·15*	*1·20*	*1·25*	*1·30*	*1·35*	*1·40*	*1·45*	*1·50*	β_2
4·24	4·43	4·62	4·81	5·00	5·19	5·38	5·56	5·75	5·94	6·12	6·30	6·49	6·67	6·84	*2·0*
4·23	4·41	4·59	4·77	4·96	5·15	5·34	5·53	5·72	5·90	6·08	6·27	6·47	6·65	6·83	*2·1*
4·22	4·39	4·56	4·74	4·93	5·12	5·31	5·50	5·69	5·87	6·05	6·24	6·44	6·63	6·82	*2·2*
4·20	4·36	4·53	4·71	4·90	5·08	5·27	5·46	5·65	5·84	6·02	6·21	6·41	6·61	6·80	*2·3*
4·19	4·35	4·51	4·69	4·87	5·05	5·23	5·42	5·61	5·80	5·99	6·18	6·38	6·58	6·78	*2·4*
4·18	4·34	4·50	4·67	4·85	5·03	5·21	5·39	5·58	5·77	5·96	6·15	6·35	6·54	6·74	*2·5*
4·20	4·35	4·50	4·67	4·84	5·01	5·20	5·36	5·54	5·72	5·91	6·11	6·30	6·49	6·68	*2·6*
4·26	4·38	4·52	4·68	4·84	5·01	5·18	5·34	5·51	5·67	5·85	6·05	6·25	6·44	6·62	*2·7*
4·40	4·50	4·60	4·72	4·86	5·03	5·19	5·34	5·49	5·65	5·83	6·02	6·21	6·39	6·58	*2·8*
4·63	4·67	4·73	4·82	4·93	5·05	5·20	5·35	5·50	5·66	5·82	6·00	6·18	6·36	6·54	*2·9*
4·98	4·97	4·99	5·03	5·10	5·18	5·28	5·39	5·52	5·66	5·82	5·98	6·15	6·33	6·51	*3·0*
5·47	5·42	5·38	5·34	5·36	5·37	5·41	5·48	5·58	5·70	5·83	5·97	6·14	6·32	6·52	*3·1*
6·03	5·92	5·83	5·75	5·67	5·62	5·60	5·62	5·68	5·78	5·90	6·03	6·18	6·34	6·52	*3·2*
6·67	6·51	6·35	6·22	6·09	6·00	5·90	5·94	5·92	5·95	6·01	6·12	6·25	6·37	6·53	*3·3*
7·41	7·17	6·95	6·77	6·61	6·48	6·29	6·26	6·24	6·22	6·22	6·26	6·34	6·47	6·61	*3·4*
8·22	7·92	7·64	7·38	7·17	6·99	6·84	6·72	6·63	6·57	6·54	6·53	6·56	6·61	6·71	*3·5*
9·14	8·80	8·51	8·23	7·98	7·70	7·53	7·40	7·22	7·09	6·99	6·98	6·95	6·92	6·93	*3·6*
10·34	9·94	9·58	9·25	8·96	8·66	8·36	8·14	7·90	7·75	7·61	7·51	7·42	7·34	7·23	*3·7*
11·77	11·29	10·82	10·37	9·98	9·62	9·31	9·03	8·73	8·51	8·29	8·11	7·94	7·78	7·60	*3·8*
13·42	12·85	12·31	11·79	11·30	10·86	10·41	10·02	9·64	9·34	9·03	8·77	8·52	8·30	8·10	*3·9*
15·58	14·84	14·10	13·42	12·79	12·20	11·64	11·13	10·65	10·21	9·83	9·51	9·20	8·92	8·67	*4·0*
17·72	16·85	16·01	15·21	14·44	13·70	13·00	12·34	11·73	11·17	10·67	10·24	9·87	9·58	9·32	*4·1*
20·2	19·2	18·3	17·3	16·4	15·5	14·7	14·0	13·3	12·6	12·0	11·5	11·0	10·5	10·3	*4·2*
23·1	22·0	20·9	19·8	18·7	17·6	16·7	15·8	15·0	14·2	13·5	12·8	12·3	11·8	11·3	*4·3*
26·3	25·0	23·8	22·5	21·3	20·1	19·0	18·0	17·1	16·1	15·3	14·6	13·9	13·2	12·6	*4·4*
30·1	28·4	26·8	25·3	23·9	22·6	21·4	20·3	19·3	18·3	17·3	16·4	15·6	14·8	14·1	*4·5*
34·7	32·5	30·5	28·8	27·3	25·6	24·2	22·9	21·7	20·6	19·5	18·4	17·5	16·7	16·1	*4·6*
40·0	37·4	35·0	32·8	30·9	29·2	27·6	26·1	24·7	23·3	22·0	20·9	19·8	18·8	18·1	*4·7*
46·1	43·1	40·3	37·7	35·3	33·2	31·4	29·7	28·0	26·4	25·0	23·6	22·3	21·2	20·3	*4·8*
52·4	48·8	46·8	43·1	40·2	37·8	35·6	33·6	31·6	29·8	28·1	26·6	25·1	23·8	22·7	*4·9*
60·6	56·1	52·3	48·8	45·5	42·6	40·0	37·6	35·4	33·4	31·5	29·8	28·1	26·6	25·2	*5·0*
71·2	65·1	60·6	56·5	52·6	49·1	45·8	43·1	40·5	38·0	35·6	33·6	31·7	30·0	28·4	*5·1*
83·0	76·4	70·5	65·4	60·6	56·3	52·5	49·3	46·3	43·4	40·4	38·0	35·6	33·6	31·7	*5·2*
98·8	89·6	81·9	75·6	70·2	65·0	60·2	56·2	52·5	48·9	45·5	42·6	39·9	37·4	35·2	*5·3*
118·4	105·2	96·0	87·6	80·4	74·0	68·3	63·4	58·8	54·7	51·0	47·7	44·5	41·5	38·9	*5·4*
141·4	124·0	111·2	99·6	91·2	84·0	77·4	71·2	65·7	61·2	56·9	52·9	49·4	46·2	43·5	*5·5*
—	—	131·2	117·4	105·2	96·0	87·3	79·3	72·8	67·8	63·2	58·6	54·8	51·4	48·8	*5·6*
—	—	160·0	142·4	126·4	113·4	102·2	93·0	84·4	77·3	71·1	65·6	60·8	57·2	54·7	*5·7*
—	—	199·2	175·8	154·8	134·2	119·6	107·0	97·2	88·4	80·6	74·4	69·4	64·9	61·5	*5·8*
—	—	266·0	221·6	192·8	163·6	142·8	128·0	114·6	104·0	94·6	86·0	79·6	74·4	70·2	*5·9*
—	—	378·1	284·0	231·5	198·2	171·6	151·5	136·2	123·8	112·8	103·4	94·8	87·5	81·4	*6·0*
—	—	—	—	—	—	206·3	186·3	167·5	150·0	134·2	121·5	111·0	101·8	92·8	*6·1*
—	—	—	—	—	—	264	232	205	280	160	141	128	116	107	*6·2*
—	—	—	—	—	—	350	297	251	216	188	164	148	132	122	*6·3*
—	—	—	—	—	—	510	376	308	263	225	196	172	152	138	*6·4*
—	—	—	—	—	—	889	524	387	313	264	229	200	177	161	*6·5*
—	—	—	—	—	—	—	—	—	—	—	—	237	204	184	*6·6*
—	—	—	—	—	—	—	—	—	—	—	—	286	249	220	*6·7*
—	—	—	—	—	—	—	—	—	—	—	—	363	305	268	*6·8*
—	—	—	—	—	—	—	—	—	—	—	—	485	392	333	*6·9*
—	—	—	—	—	—	—	—	—	—	—	—	747	510	416	*7·0*

TABLE III.

Values of $R_{\beta_1\beta_2}$.

β_1

β_2	0·00	0·05	0·10	0·15	0·20	0·25	0·30	0·35	0·40	0·45	0·50	0·55	0·60	0·65	0·70	0·75
2·0	0·00	·570	·706	·770	·823	·863	·894	·917	·935	·949	·960	·968	·976	·983	·989	·992
2·1	0·00	·557	·685	·755	·798	·838	·870	·895	·914	·936	·948	·959	·969	·977	·983	·986
2·2	0·00	·551	·672	·728	·771	·814	·847	·874	·896	·919	·934	·948	·960	·968	·975	·980
2·3	0·00	·550	·663	·719	·765	·799	·829	·859	·880	·900	·919	·935	·948	·957	·965	·971
2·4	0·00	·551	·660	·712	·752	·787	·814	·843	·867	·886	·905	·920	·936	·945	·954	·962
2·5	0·00	·554	·659	·706	·745	·776	·805	·834	·858	·878	·893	·908	·924	·933	·941	·949
2·6	0·00	·557	·662	·706	·742	·773	·799	·825	·851	·871	·883	·898	·913	·921	·930	·938
2·7	0·00	·557	·668	·710	·744	·773	·800	·825	·846	·863	·876	·889	·902	·911	·920	·928
2·8	0·00	·556	·674	·716	·750	·779	·803	·826	·842	·858	·871	·883	·893	·901	·910	·919
2·9	0·00	·550	·680	·724	·760	·787	·810	·830	·844	·857	·868	·878	·887	·895	·903	·912
3·0	0·00	·542	·684	·738	·774	·796	·816	·835	·847	·857	·867	·875	·883	·890	·898	·906
3·1	0·00	·534	·687	·744	·781	·808	·825	·840	·850	·858	·867	·874	·882	·889	·897	·903
3·2	0·00	·524	·688	·746	·786	·811	·830	·842	·852	·860	·868	·875	·882	·889	·896	·902
3·3	0·00	·512	·688	·747	·788	·814	·832	·845	·855	·863	·870	·876	·882	·888	·895	·901
3·4	0·00	·501	·686	·748	·790	·816	·833	·848	·858	·865	·872	·878	·883	·889	·895	·900
3·5	0·00	·490	·681	·747	·790	·815	·833	·849	·860	·867	·873	·879	·884	·890	·895	·900
3·6	0·00	·477	·676	·745	·788	·813	·832	·850	·860	·867	·874	·880	·886	·891	·896	·900
3·7	0·00	·462	·670	·741	·784	·810	·831	·848	·859	·867	·874	·881	·887	·892	·897	·901
3·8	0·00	·450	·662	·736	·779	·803	·828	·845	·858	·866	·874	·882	·888	·893	·898	·901
3·9	0·00	·438	·654	·720	·770	·796	·822	·841	·856	·866	·875	·882	·889	·894	·899	·903
4·0	0·00	·422	·645	·713	·760	·788	·816	·837	·853	·865	·873	·881	·888	·894	·899	·903
4·1	—	—	·630	·702	·748	·780	·807	·830	·849	·862	·871	·880	·887	·892	·897	·901
4·2	—	—	·608	·682	·733	·770	·793	·822	·842	·857	·867	·877	·884	·890	·894	·899
4·3	—	—	·580	·658	·712	·753	·784	·811	·832	·848	·860	·871	·878	·885	·890	·897
4·4	—	—	·540	·628	·688	·732	·770	·796	·819	·837	·851	·863	·872	·880	·887	·894
4·5	—	—	·481	·590	·657	·709	·749	·780	·804	·824	·841	·853	·865	·874	·882	·890
4·6	—	—	—	—	—	—	·716	·754	·784	·808	·828	·842	·856	·868	·877	·886
4·7	—	—	—	—	—	—	·674	·723	·759	·788	·812	·830	·846	·860	·870	·879
4·8	—	—	—	—	—	—	·615	·681	·727	·761	·791	·815	·834	·849	·861	·872
4·9	—	—	—	—	—	—	·532	·620	·680	·728	·766	·795	·818	·835	·850	·862
5·0	—	—	—	—	—	—	·362	·534	·628	·687	·731	·767	·798	·822	·837	·851
5·1	—	—	—	—	—	—	—	—	—	—	—	—	·768	·799	·820	·837
5·2	—	—	—	—	—	—	—	—	—	—	—	—	·730	·768	·799	·820
5·3	—	—	—	—	—	—	—	—	—	—	—	—	·679	·729	·769	·799
5·4	—	—	—	—	—	—	—	—	—	—	—	—	·608	·686	·736	·774
5·5	—	—	—	—	—	—	—	—	—	—	—	—	·496	·601	·674	·724
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE III.—(*continued*).

Values of $R_{\beta_1\beta_2}$.

β_1

0·80	*0·85*	*0·90*	*0·95*	*1·00*	*1·05*	*1·10*	*1·15*	*1·20*	*1·25*	*1·30*	*1·35*	*1·40*	*1·45*	*1·50*	β_2
·993	·995	·997	·999	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	*2·0*
·989	·991	·994	·996	·998	·998	·999	·999	·999	1·000	1·000	1·000	1·000	1·000	1·000	*2·1*
·983	·986	·989	·992	·995	·996	·997	·998	·998	·999	1·000	1·000	1·000	1·000	1·000	*2·2*
·976	·980	·984	·988	·992	·993	·994	·995	·997	·998	·999	·999	·999	1·000	1·000	*2·3*
·968	·973	·978	·983	·987	·989	·991	·993	·995	·996	·998	·998	·999	·999	1·000	*2·4*
·958	·965	·972	·977	·982	·985	·988	·990	·992	·994	·996	·997	·998	·999	1·000	*2·5*
·947	·956	·964	·970	·976	·980	·984	·986	·988	·991	·993	·995	·997	·998	·999	*2·6*
·937	·947	·957	·963	·968	·973	·977	·980	·983	·986	·989	·992	·995	·997	·998	*2·7*
·928	·939	·949	·955	·960	·965	·970	·974	·978	·981	·985	·989	·992	·994	·996	*2·8*
·921	·932	·942	·947	·952	·957	·963	·968	·972	·976	·980	·984	·988	·990	·992	*2·9*
·915	·923	·931	·937	·943	·948	·954	·960	·966	·971	·975	·979	·983	·986	·988	*3·0*
·909	·915	·922	·929	·936	·942	·947	·953	·959	·965	·970	·974	·978	·981	·984	*3·1*
·907	·912	·918	·924	·930	·936	·941	·946	·952	·958	·963	·968	·973	·977	·980	*3·2*
·906	·909	·914	·919	·925	·930	·935	·940	·946	·951	·956	·961	·966	·971	·975	*3·3*
·905	·908	·912	·916	·920	·925	·930	·935	·940	·945	·950	·954	·958	·964	·974	*3·4*
·904	·907	·910	·914	·918	·922	·926	·931	·936	·940	·944	·948	·952	·958	·965	*3·5*
·904	·907	·910	·914	·918	·921	·924	·928	·932	·935	·938	·942	·946	·952	·959	*3·6*
·905	·907	·910	·914	·917	·920	·923	·927	·930	·933	·935	·937	·940	·946	·953	*3·7*
·905	·908	·911	·914	·917	·920	·922	·925	·928	·930	·932	·934	·936	·941	·948	*3·8*
·906	·909	·911	·914	·917	·919	·921	·924	·927	·929	·931	·933	·935	·939	·944	*3·9*
·906	·909	·912	·914	·917	·919	·921	·923	·926	·928	·930	·932	·934	·936	·940	*4·0*
·905	·908	·911	·914	·917	·919	·921	·923	·925	·927	·930	·931	·932	·933	·934	*4·1*
·905	·907	·910	·913	·916	·919	·921	·923	·924	·926	·929	·929	·930	·930	·929	*4·2*
·903	·906	·910	·913	·916	·918	·920	·922	·924	·926	·929	·928	·928	·927	·924	*4·3*
·900	·904	·908	·912	·916	·918	·920	·922	·923	·926	·928	·927	·927	·925	·922	*4·4*
·897	·902	·906	·910	·915	·918	·920	·922	·923	·926	·928	·927	·926	·923	·920	*4·5*
·893	·898	·903	·908	·913	·916	·919	·920	·922	·925	·927	·926	·925	·923	·920	*4·6*
·887	·894	·900	·905	·910	·913	·917	·919	·921	·924	·926	·925	·925	·923	·922	*4·7*
·881	·890	·896	·901	·906	·910	·914	·917	·920	·923	·925	·926	·926	·925	·925	*4·8*
·874	·884	·890	·895	·901	·907	·911	·915	·919	·922	·925	·926	·927	·927	·928	*4·9*
·863	·875	·883	·889	·896	·903	·908	·913	·918	·922	·925	·927	·928	·930	·932	*5·0*
·851	·864	·875	·882	·890	·898	·905	·911	·917	·922	·925	·928	·931	·933	·936	*5·1*
·837	·852	·866	·875	·884	·892	·901	·909	·916	·921	·924	·928	·933	·937	·941	*5·2*
·820	·839	·853	·865	·876	·885	·895	·904	·913	·918	·923	·929	·935	·940	·945	*5·3*
·798	·818	·837	·853	·867	·877	·888	·898	·908	·915	·921	·928	·935	·941	·947	*5·4*
·764	·792	·817	·837	·854	·867	·880	·890	·900	·910	·918	·925	·933	·940	·947	*5·5*
—	—	·789	·815	·835	·852	·868	·880	·890	·904	·911	·917	·926	·935	·944	*5·6*
—	—	·750	·786	·811	·835	·854	·869	·880	·892	·901	·909	·917	·927	·938	*5·7*
—	—	·701	·748	·783	·811	·835	·852	·866	·879	·890	·897	·905	·915	·928	*5·8*
—	—	·640	·700	·748	·781	·810	·828	·846	·861	·875	·883	·892	·901	·913	*5·9*
—	—	·544	·639	·703	·746	·778	·802	·825	·842	·857	·867	·879	·886	·893	*6·0*
—	—	—	—	—	—	·741	·769	·796	·820	·837	·852	·866	·872	·873	*6·1*
—	—	—	—	—	—	·691	·727	·762	·792	·815	·836	·852	·858	·856	*6·2*
—	—	—	—	—	—	·628	·678	·724	·761	·790	·818	·838	·845	·842	*6·3*
—	—	—	—	—	—	·526	·606	·675	·724	·763	·793	·818	·831	·834	*6·4*
—	—	—	—	—	—	·354	·526	·619	·680	·726	·761	·791	·814	·831	*6·5*
—	—	—	—	—	—	—	—	—	—	—	—	·761	·790	·832	*6·6*
—	—	—	—	—	—	—	—	—	—	—	—	·721	·760	·837	*6·7*
—	—	—	—	—	—	—	—	—	—	—	—	·670	·727	·845	*6·8*
—	—	—	—	—	—	—	—	—	—	—	—	·600	·683	·857	*6·9*
—	—	—	—	—	—	—	—	—	—	—	—	·468	·602	·876	*7·0*

TABLE IV.

Values of $\frac{\sqrt{N}}{\sigma}\Sigma_d$.

β_1

β_2	0·00	0·05	0·10	0·15	0·20	0·25	0·30	0·35	0·40	0·45	0·50	0·55	0·60	0·65	0·70	0·75	0·80
2·0	3·54	—	—	—	—	—	—	—	—	—	3·03	2·44	2·10	1·80	1·58	1·42	1·30
2·1	2·15	4·36	—	—	—	—	—	—	—	—	—	—	—	3·10	2·53	2·16	1·88
2·2	1·87	2·75	9·65	—	—	—	—	—	—	—	—	—	—	—	3·91	3·17	2·60
2·3	1·64	1·86	3·00	—	—	—	—	—	—	—	—	—	—	—	—	—	3·78
2·4	1·46	1·58	2·07	—	—	—	—	—	—	—	—	—	—	—	—	—	—
2·5	1·35	1·46	1·67	2·08	2·87	4·04	5·21	—	—	—	—	—	—	—	—	—	—
2·6	1·28	1·37	1·58	1·98	2·60	3·42	4·43	6·72	—	—	—	—	—	—	—	—	—
2·7	1·25	1·30	1·50	1·83	2·34	2·98	3·75	5·06	7·48	—	—	—	—	—	—	—	—
2·8	1·23	1·28	1·43	1·71	2·11	2·60	3·17	4·12	5·28	7·45	—	—	—	—	—	—	—
2·9	1·22	1·27	1·38	1·60	1·90	2·27	2·69	3·20	3·84	4·78	6·65	—	—	—	—	—	—
3·0	1·23	1·26	1·34	1·51	1·73	1·98	2·29	2·63	3·06	3·58	4·28	5·18	6·43	8·24	10·89	—	—
3·1	1·25	1·27	1·32	1·44	1·58	1·76	2·00	2·23	2·54	2·94	3·42	3·93	4·52	5·50	6·76	8·66	—
3·2	1·27	1·28	1·30	1·38	1·48	1·60	1·75	1·92	2·13	2·37	2·72	3·12	3·54	4·21	5·07	6·22	8·00
3·3	1·29	1·29	1·28	1·32	1·39	1·47	1·55	1·68	1·83	2·03	2·27	2·57	2·90	3·39	4·04	4·66	5·53
3·4	1·30	1·29	1·28	1·29	1·31	1·37	1·45	1·54	1·63	1·79	2·00	2·24	2·51	2·88	3·36	3·86	4·50
3·5	1·31	1·30	1·29	1·27	1·25	1·30	1·37	1·45	1·54	1·66	1·83	2·03	2·26	2·55	2·89	3·18	3·61
3·6	1·32	1·31	1·30	1·26	1·22	1·26	1·32	1·40	1·50	1·61	1·74	1·89	2·08	2·31	2·56	2·86	3·24
3·7	1·31	1·31	1·31	1·26	1·22	1·25	1·30	1·37	1·46	1·57	1·69	1·82	1·97	2·14	2·34	2·62	2·95
3·8	1·30	1·31	1·32	1·28	1·25	1·27	1·32	1·38	1·46	1·55	1·65	1·76	1·88	2·03	2·20	2·43	2·69
3·9	1·29	1·33	1·35	1·33	1·30	1·32	1·36	1·41	1·48	1·56	1·64	1·73	1·84	1·96	2·11	2·27	2·49
4·0	1·27	1·37	1·40	1·39	1·39	1·40	1·42	1·46	1·51	1·58	1·65	1·73	1·83	1·94	2·06	2·19	2·36
4·1	—	—	1·47	1·48	1·50	1·51	1·53	1·55	1·57	1·61	1·66	1·75	1·85	1·94	2·03	2·13	2·26
4·2	—	—	1·58	1·62	1·64	1·65	1·65	1·65	1·65	1·66	1·70	1·77	1·85	1·94	2·03	2·12	2·22
4·3	—	—	1·75	1·77	1·78	1·79	1·78	1·76	1·75	1·75	1·78	1·83	1·90	1·97	2·05	2·13	2·23
4·4	—	—	1·98	1·97	1·95	1·94	1·93	1·90	1·88	1·89	1·92	1·96	2·01	2·07	2·13	2·20	2·29
4·5	—	—	2·27	2·20	2·15	2·11	2·10	2·09	2·09	2·10	2·12	2·15	2·18	2·22	2·26	2·32	2·38
4·6	—	—	—	—	—	—	2·44	2·40	2·38	2·36	2·34	2·36	2·38	2·40	2·42	2·45	2·48
4·7	—	—	—	—	—	—	2·93	2·85	2·78	2·71	2·67	2·65	2·64	2·63	2·62	2·61	2·61
4·8	—	—	—	—	—	—	3·74	3·52	3·33	3·16	3·07	3·00	2·95	2·90	2·85	2·81	2·80
4·9	—	—	—	—	—	—	5·44	4·64	4·16	3·87	3·63	3·45	3·32	3·24	3·11	3·05	3·03
5·0	—	—	—	—	—	—	10·66	6·83	5·53	4·84	4·37	4·04	3·79	3·62	3·47	3·37	3·31
5·1	—	—	—	—	—	—	—	—	—	—	—	—	4·46	4·21	3·99	3·85	3·74
5·2	—	—	—	—	—	—	—	—	—	—	—	—	5·38	5·05	4·73	4·47	4·24
5·3	—	—	—	—	—	—	—	—	—	—	—	—	6·84	6·19	5·66	5·27	4·92
5·4	—	—	—	—	—	—	—	—	—	—	—	—	9·24	7·96	7·00	6·24	5·74
5·5	—	—	—	—	—	—	—	—	—	—	—	—	14·81	10·89	8·87	7·64	6·81
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE IV.—(*continued*).

Values of $\frac{\sqrt{N}}{\sigma}\Sigma_d$.

β_1

0·85	*0·90*	*0·95*	*1·00*	*1·05*	*1·10*	*1·15*	*1·20*	*1·25*	*1·30*	*1·35*	*1·40*	*1·45*	*1·50*	β_2
1·20	1·13	1·07	1·02	·97	·92	·87	·83	·80	·76	·72	·68	·64	·60	*2·0*
1·64	1·49	1·38	1·29	1·21	1·14	1·08	1·03	·99	·94	·90	·86	·82	·78	*2·1*
2·22	1·97	1·80	1·66	1·54	1·45	1·38	1·32	1·26	1·20	1·14	1·08	1·02	·96	*2·2*
3·10	2·63	2·46	2·15	1·98	1·85	1·75	1·67	1·58	1·50	1·41	1·32	1·24	1·16	*2·3*
—	3·72	3·20	2·81	2·56	2·36	2·22	2·09	1·96	1·84	1·73	1·62	1·51	1·39	*2·4*
—	—	—	3·94	3·40	3·08	2·87	2·66	2·47	2·29	2·12	1·96	1·80	1·64	*2·5*
—	—	—	—	—	4·32	3·82	3·48	3·14	2·86	2·60	2·34	2·10	1·91	*2·6*
—	—	—	—	—	—	—	4·82	4·20	3·66	3·18	2·78	2·46	2·19	*2·7*
—	—	—	—	—	—	—	—	5·73	4·82	3·88	3·28	2·87	2·49	*2·8*
—	—	—	—	—	—	—	—	—	6·63	4·81	3·94	3·31	2·80	*2·9*
—	—	—	—	—	—	—	—	—	—	6·15	4·80	3·79	3·12	*3·0*
—	—	—	—	—	—	—	—	—	—	—	—	4·29	3·48	*3·1*
—	—	—	—	—	—	—	—	—	—	—	—	—	3·88	*3·2*
6·80	—	—	—	—	—	—	—	—	—	—	—	—	—	*3·3*
5·38	6·55	—	—	—	—	—	—	—	—	—	—	—	—	*3·4*
4·21	4·95	6·00	7·33	9·36	12·16	—	—	—	—	—	—	—	—	*3·5*
3·74	4·34	5·12	6·03	7·17	8·80	11·52	—	—	—	—	—	—	—	*3·6*
3·34	3·80	4·32	5·01	5·78	6·65	8·28	11·12	—	—	—	—	—	—	*3·7*
3·00	3·35	3·74	4·23	4·72	5·41	6·36	8·00	10·22	—	—	—	—	—	*3·8*
2·74	3·00	3·32	3·66	4·04	4·50	5·18	6·16	7·43	9·32	—	—	—	—	*3·9*
2·55	2·77	3·02	3·29	3·60	3·98	4·50	5·12	5·92	6·91	8·00	9·23	11·08	—	*4·0*
2·42	2·60	2·79	3·00	3·27	3·61	4·06	4·61	5·20	5·90	6·80	7·86	9·48	11·58	*4·1*
2·35	2·50	2·67	2·86	3·09	3·38	3·71	4·13	4·60	5·15	5·86	6·72	7·76	9·10	*4·2*
2·35	2·48	2·62	2·77	2·96	3·21	3·49	3·82	4·18	4·59	5·14	5·82	6·66	7·59	*4·3*
2·39	2·50	2·61	2·73	2·89	3·10	3·32	3·55	3·83	4·16	4·60	5·16	5·82	6·60	*4·4*
2·45	2·53	2·63	2·74	2·87	3·02	3·20	3·39	3·61	3·87	4·21	4·66	5·18	5·86	*4·5*
2·52	2·60	2·69	2·79	2·89	3·01	3·15	3·31	3·48	3·67	3·95	4·31	4·77	5·36	*4·6*
2·64	2·69	2·77	2·85	2·93	3·02	3·13	3·25	3·39	3·55	3·78	4·10	4·50	4·99	*4·7*
2·81	2·82	2·86	2·93	3·00	3·08	3·17	3·27	3·38	3·50	3·68	3·94	4·26	4·66	*4·8*
3·02	3·03	3·04	3·08	3·12	3·16	3·22	3·30	3·39	3·50	3·63	3·81	4·07	4·35	*4·9*
3·28	3·26	3·25	3·26	3·28	3·31	3·34	3·39	3·44	3·51	3·61	3·73	3·90	4·09	*5·0*
3·64	3·55	3·51	3·49	3·47	3·47	3·48	3·49	3·52	3·56	3·61	3·68	3·77	3·90	*5·1*
4·04	3·90	3·81	3·74	3·68	3·65	3·63	3·62	3·61	3·62	3·63	3·65	3·69	3·76	*5·2*
4·60	4·35	4·18	4·05	3·95	3·88	3·81	3·75	3·72	3·70	3·69	3·68	3·69	3·70	*5·3*
5·33	4·98	4·71	4·52	4·37	4·24	4·12	4·01	3·92	3·85	3·79	3·75	3·73	3·72	*5·4*
6·21	5·74	5·36	5·08	4·86	4·66	4·48	4·32	4·20	4·09	3·99	3·92	3·87	3·82	*5·5*
—	6·69	6·27	5·83	5·49	5·19	4·94	4·73	4·56	4·42	4·30	4·18	4·11	4·04	*5·6*
—	8·11	7·48	6·82	6·32	5·90	5·55	5·27	5·03	4·84	4·68	4·54	4·43	4·35	*5·7*
—	10·18	9·11	8·12	7·45	6·85	6·35	5·95	5·62	5·37	5·16	5·00	4·87	4·76	*5·8*
—	13·53	11·44	9·84	8·71	7·94	7·32	6·82	6·45	6·13	5·85	5·61	5·43	5·29	*5·9*
—	19·95	14·26	11·92	10·48	9·38	8·55	7·89	7·38	6·95	6·62	6·33	6·10	5·90	*6·0*
—	—	—	—	—	11·64	10·26	9·31	8·62	8·03	7·53	7·15	6·87	6·60	*6·1*
—	—	—	—	—	14·83	12·55	11·19	10·24	9·40	8·64	8·08	7·68	7·36	*6·2*
—	—	—	—	—	19·65	15·85	13·69	12·21	11·01	10·02	9·19	8·65	8·22	*6·3*
—	—	—	—	—	28·03	20·85	17·09	14·56	12·84	11·45	10·45	9·69	9·11	*6·4*
—	—	—	—	—	47·99	28·04	21·30	17·44	15·07	13·20	11·90	10·83	10·07	*6·5*
—	—	—	—	—	—	—	—	—	—	—	14·2	12·9	12·4	*6·6*
—	—	—	—	—	—	—	—	—	—	—	17·2	15·6	14·7	*6·7*
—	—	—	—	—	—	—	—	—	—	—	21·8	19·6	18·3	*6·8*
—	—	—	—	—	—	—	—	—	—	—	29·4	26·0	24·1	*6·9*
—	—	—	—	—	—	—	—	—	—	—	43·0	37·8	34·9	*7·0*

TABLE V.

Values of $\sqrt{N}\,\Sigma_{sk}$.

	β_1															
β_2	*0·00*	*0·05*	*0·10*	*0·15*	*0·20*	*0·25*	*0·30*	*0·35*	*0·40*	*0·45*	*0·50*	*0·55*	*0·60*	*0·65*	*0·70*	*0·75*
2·0	3·54	—	—	—	—	—	—	—	—	—	3·41	2·80	2·43	2·12	1·89	1·72
2·1	2·15	4·20	—	—	—	—	—	—	—	—	—	—	—	3·67	3·02	2·58
2·2	1·87	2·63	9·50	—	—	—	—	—	—	—	—	—	—	—	5·57	4·10
2·3	1·64	1·78	2·88	—	—	—	—	—	—	—	—	—	—	—	—	—
2·4	1·46	1·49	2·02	—	—	—	—	—	—	—	—	—	—	—	—	—
2·5	1·35	1·41	1·62	2·02	2·80	4·06	5·08	—	—	—	—	—	—	—	—	—
2·6	1·28	1·30	1·43	1·75	2·18	2·82	3·65	4·96	—	—	—	—	—	—	—	—
2·7	1·25	1·25	1·31	1·52	1·84	2·29	2·85	3·68	4·84	—	—	—	—	—	—	—
2·8	1·23	1·22	1·24	1·36	1·59	1·89	2·21	2·70	3·36	4·36	—	—	—	—	—	—
2·9	1·22	1·20	1·20	1·28	1·43	1·63	1·86	2·19	2·60	3·33	4·30	—	—	—	—	—
3·0	1·23	1·21	1·20	1·25	1·34	1·48	1·62	1·84	2·12	2·49	3·00	3·76	4·62	6·02	8·12	—
3·1	1·25	1·22	1·21	1·23	1·27	1·36	1·50	1·67	1·88	2·16	2·50	3·06	3·76	4·72	6·10	8·08
3·2	1·27	1·23	1·22	1·22	1·23	1·29	1·40	1·53	1·70	1·90	2·17	2·58	3·17	3·97	4·87	5·83
3·3	1·29	1·25	1·23	1·21	1·21	1·24	1·33	1·44	1·58	1·74	1·94	2·27	2·73	3·28	3·88	4·53
3·4	1·30	1·27	1·24	1·21	1·20	1·23	1·29	1·38	1·49	1·61	1·78	2·04	2·36	2·75	3·18	3·63
3·5	1·31	1·29	1·25	1·21	1·20	1·22	1·28	1·35	1·43	1·54	1·68	1·87	2·09	2·37	2·68	2·98
3·6	1·32	1·30	1·26	1·22	1·20	1·22	1·26	1·32	1·40	1·50	1·61	1·75	1·91	2·13	2·39	2·65
3·7	1·31	1·31	1·28	1·24	1·20	1·23	1·27	1·32	1·39	1·47	1·56	1·67	1·80	1·98	2·19	2·40
3·8	1·30	1·32	1·30	1·27	1·23	1·25	1·28	1·33	1·38	1·46	1·54	1·63	1·75	1·88	2·04	2·22
3·9	1·29	1·34	1·33	1·30	1·28	1·28	1·30	1·34	1·39	1·45	1·53	1·61	1·71	1·82	1·96	2·12
4·0	1·27	1·36	1·38	1·36	1·35	1·35	1·36	1·39	1·43	1·48	1·56	1·63	1·72	1·81	1·92	2·04
4·1	—	—	1·46	1·45	1·45	1·44	1·44	1·46	1·50	1·54	1·60	1·67	1·74	1·82	1·91	2·03
4·2	—	—	1·58	1·57	1·57	1·56	1·55	1·56	1·59	1·62	1·67	1·72	1·78	1·85	1·93	2·02
4·3	—	—	1·75	1·74	1·73	1·71	1·70	1·69	1·70	1·72	1·76	1·81	1·86	1·92	1·98	2·06
4·4	—	—	1·95	1·94	1·92	1·90	1·88	1·85	1·86	1·87	1·90	1·93	1·97	2·01	2·06	2·12
4·5	—	—	2·26	2·19	2·13	2·09	2·07	2·06	2·05	2·06	2·07	2·08	2·11	2·15	2·19	2·23
4·6	—	—	—	—	—	—	2·48	2·42	2·37	2·35	2·33	2·31	2·31	2·32	2·33	2·35
4·7	—	—	—	—	—	—	3·11	2·93	2·82	2·73	2·65	2·60	2·57	2·54	2·52	2·50
4·8	—	—	—	—	—	—	3·78	3·53	3·35	3·21	3·08	2·97	2·89	2·82	2·76	2·71
4·9	—	—	—	—	—	—	5·48	4·66	4·17	3·87	3·63	3·44	3·30	3·17	3·07	3·01
5·0	—	—	—	—	—	—	11·12	6·96	5·52	4·82	4·36	4·02	3·77	3·58	3·45	3·37
5·1	—	—	—	—	—	—	—	—	—	—	—	—	4·45	4·16	4·00	3·87
5·2	—	—	—	—	—	—	—	—	—	—	—	—	5·38	5·02	4·72	4·49
5·3	—	—	—	—	—	—	—	—	—	—	—	—	6·84	6·18	5·66	5·26
5·4	—	—	—	—	—	—	—	—	—	—	—	—	9·24	7·76	6·80	6·22
5·5	—	—	—	—	—	—	—	—	—	—	—	—	14·80	10·67	8·87	7·71
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE V.—(*continued*).

Values of $\sqrt{N}\,\Sigma_{sk}$.

β_1

0·80	*0·85*	*0·90*	*0·95*	*1·00*	*1·05*	*1·10*	*1·15*	*1·20*	*1·25*	*1·30*	*1·35*	*1·40*	*1·45*	*1·50*	β_2
1·59	1·48	1·39	1·30	1·24	1·19	1·14	1·10	1·06	1·02	·99	·95	·91	·87	·83	*2·0*
2·20	1·95	1·80	1·68	1·58	1·52	1·47	1·42	1·37	1·32	1·26	1·20	1·15	1·10	1·05	*2·1*
3·22	2·65	2·29	2·08	1·98	1·91	1·84	1·78	1·72	1·66	1·59	1·52	1·45	1·38	1·31	*2·2*
5·23	3·80	3·04	2·75	2·53	2·40	2·30	2·21	2·12	2·03	1·94	1·85	1·76	1·67	1·58	*2·3*
—	—	4·29	3·64	3·31	3·12	2·94	2·78	2·63	2·49	2·36	2·23	2·10	1·98	1·86	*2·4*
—	—	—	—	4·77	4·27	3·84	3·55	3·29	3·06	2·85	2·66	2·49	2·32	2·15	*2·5*
—	—	—	—	—	—	5·72	5·00	4·39	3·94	3·54	3·20	2·93	2·67	2·44	*2·6*
—	—	—	—	—	—	—	—	6·25	5·20	4·46	3·88	3·42	3·08	2·74	*2·7*
—	—	—	—	—	—	—	—	—	7·05	5·68	4·78	4·03	3·54	3·05	*2·8*
—	—	—	—	—	—	—	—	—	—	7·45	6·00	4·85	4·11	3·37	*2·9*
—	—	—	—	—	—	—	—	—	—	—	7·80	6·05	4·77	3·70	*3·0*
—	—	—	—	—	—	—	—	—	—	—	—	—	5·57	4·10	*3·1*
7·00	—	—	—	—	—	—	—	—	—	—	—	—	—	4·58	*3·2*
5·30	6·24	—	—	—	—	—	—	—	—	—	—	—	—	—	*3·3*
4·16	4·71	5·60	—	—	—	—	—	—	—	—	—	—	—	—	*3·4*
3·38	3·89	4·59	5·58	6·85	8·38	11·48	—	—	—	—	—	—	—	—	*3·5*
2·95	3·38	3·99	4·66	5·50	6·48	7·55	9·92	—	—	—	—	—	—	—	*3·6*
2·67	3·05	3·49	3·97	4·52	5·22	6·00	7·36	9·42	—	—	—	—	—	—	*3·7*
2·44	2·75	3·08	3·43	3·80	4·25	4·78	5·64	7·08	9·02	—	—	—	—	—	*3·8*
2·29	2·53	2·79	3·07	3·38	3·70	4·15	4·77	5·62	6·89	8·76	—	—	—	—	*3·9*
2·20	2·38	2·58	2·82	3·06	3·33	3·69	4·14	4·77	5·50	6·42	7·40	8·57	10·12	—	*4·0*
2·16	2·31	2·47	2·65	2·85	3·09	3·34	3·72	4·15	4·61	5·14	5·84	6·80	8·44	11·00	*4·1*
2·14	2·26	2·40	2·55	2·71	2·88	3·10	3·37	3·67	4·03	4·44	5·04	5·94	7·12	8·67	*4·2*
2·16	2·27	2·38	2·50	2·65	2·80	2·96	3·14	3·37	3·65	4·01	4·55	5·28	6·18	7·21	*4·3*
2·20	2·30	2·41	2·52	2·64	2·76	2·89	3·04	3·23	3·48	3·78	4·22	4·78	5·44	6·26	*4·4*
2·29	2·35	2·44	2·53	2·63	2·75	2·88	3·02	3·20	3·40	3·65	3·97	4·40	4·91	5·56	*4·5*
2·39	2·43	2·48	2·56	2·66	2·77	2·89	3·01	3·16	3·34	3·55	3·79	4·16	4·55	5·10	*4·6*
2·52	2·54	2·58	2·63	2·71	2·80	2·90	3·01	3·14	3·29	3·47	3·67	3·96	4·28	4·72	*4·7*
2·70	2·72	2·74	2·78	2·83	2·89	2·97	3·06	3·16	3·28	3·41	3·58	3·80	4·06	4·42	*4·8*
2·98	2·97	2·96	2·97	3·00	3·04	3·08	3·14	3·21	3·30	3·40	3·53	3·68	3·88	4·16	*4·9*
3·31	3·25	3·21	3·20	3·21	3·22	3·24	3·26	3·31	3·36	3·43	3·51	3·60	3·73	3·96	*5·0*
3·75	3·64	3·55	3·49	3·45	3·42	3·41	3·41	3·43	3·44	3·46	3·51	3·57	3·65	3·78	*5·1*
4·28	4·10	3·96	3·85	3·76	3·68	3·63	3·60	3·57	3·55	3·53	3·53	3·55	3·60	3·67	*5·2*
4·93	4·69	4·48	4·29	4·13	4·02	3·92	3·84	3·76	3·68	3·63	3·60	3·58	3·59	3·62	*5·3*
5·78	5·42	5·09	4·80	4·56	4·40	4·26	4·13	4·00	3·89	3·80	3·73	3·68	3·65	3·63	*5·4*
6·94	6·32	5·82	5·40	5·07	4·84	4·64	4·46	4·30	4·17	4·06	3·95	3·87	3·80	3·75	*5·5*
—	—	6·75	6·22	5·79	5·46	5·19	4·97	4·77	4·60	4·44	4·30	4·19	4·10	4·01	*5·6*
—	—	8·15	7·30	6·73	6·26	5·91	5·61	5·34	5·10	4·90	4·73	4·59	4·47	4·38	*5·7*
—	—	10·20	8·76	7·98	7·26	6·76	6·34	5·99	5·68	5·44	5·24	5·06	4·91	4·78	*5·8*
—	—	13·53	10·83	9·66	8·71	7·90	7·28	6·78	6·40	6·10	5·84	5·62	5·44	5·28	*5·9*
—	—	19·96	14·30	12·02	10·51	9·39	8·56	7·90	7·39	6·96	6·61	6·33	6·10	5·89	*6·0*
—	—	—	—	—	—	11·64	10·26	9·31	8·56	8·03	7·53	7·15	6·82	6·54	*6·1*
—	—	—	—	—	—	14·83	12·55	11·19	10·13	9·30	8·64	8·08	7·63	7·24	*6·2*
—	—	—	—	—	—	19·65	15·85	13·69	12·21	11·01	10·02	9·19	8·55	8·01	*6·3*
—	—	—	—	—	—	28·03	20·85	17·09	14·56	12·84	11·45	10·45	9·60	8·92	*6·4*
—	—	—	—	—	—	47·99	28·04	21·30	17·44	15·07	13·20	11·90	10·83	10·07	*6·5*
—	—	—	—	—	—	—	—	—	—	—	—	14·2	12·9	12·4	*6·6*
—	—	—	—	—	—	—	—	—	—	—	—	17·2	15·6	14·7	*6·7*
—	—	—	—	—	—	—	—	—	—	—	—	21·8	19·6	18·3	*6·8*
—	—	—	—	—	—	—	—	—	—	—	—	29·4	26·0	24·1	*6·9*
—	—	—	—	—	—	—	—	—	—	—	—	43·0	37·8	34·9	*7·0*

TABLE VI (i).

Values of β_3.

	β_2										
β_1	*2·0*	*2·5*	*3·0*	*3·5*	*4·0*	*4·5*	*5·0*	*5·5*	*6·0*	*6·5*	*7·0*
0·0	0	0	0	0	0						
0·1	0·48493	0·68971	0·94286	1·25688	1·64906	2·14375					
0·2	0·91585	1·32958	1·78182	2·37049	3·10000	4·01176					
0·3	1·29873	1·85270	2·53043	3·36094	4·38305	5·65000	7·2368				
0·4	1·63902	2·34286	3·20000	4·24478	5·52277	7·09474	9·0462				
0·5	1·94118	2·78126	3·80000	5·03582	6·53846	8·37500	10·6364				
0·6	2·20909	3·17350	4·33846	5·80178	7·44706	9·51429	12·0414	15·1585			
0·7	2·44615	3·52441	4·82222	6·38287	8·26202	10·53182	13·2885	16·6624			
0·8	2·65532	3·83820	5·25714	6·95698	8·99462	11·44375	14·4000	17·9932			
0·9	2·83917	4·12064	5·64828	7·47438	9·65454	12·26250	15·3940	19·1758	23·7791		
1·0	3·00000	4·36842	6·00000	7·94121	10·24999	13·00000	16·2857	20·2308	25·0000		
1·1	3·13980	4·59081	6·31613	8·36246	10·78796	13·66538	17·0877	21·1750	26·0857	32·0328	
1·2	3·26038	4·78812	6·60000	8·74286	11·27443	14·26667	17·8105	22·0225	27·0546	33·1082	
1·3	3·36330	4·96250	6·85454	9·08619	11·71461	14·81071	18·4633	22·7851	27·9217	34·0635	
1·4	3·45000	5·11589	7·08235	9·39582	12·11304	15·30345	19·0536	23·4727	28·7000	34·9164	42·3613
1·5	3·52174	5·25000	7·28571	9·67501	12·47368	15·75000	19·5864	24·0937	29·4000	35·6786	43·1538

TABLE VI (ii).

Values of β_4.

	β_2										
β_1	*2·0*	*2·5*	*3·0*	*3·5*	*4·0*	*4·5*	*5·0*	*5·5*	*6·0*	*6·5*	*7·0*
0·0	5·00000	8·92856	15·0000	23·7288	31·0000						
0·1	5·27356	9·41054	15·7973	25·7430	41·7660	69·3682					
0·2	5·44361	9·75086	16·2648	26·4018	42·5000	69·4796					
0·3	5·53293	9·86224	16·4907	26·6520	42·5613	68·5776	114·4732				
0·4	5·55998	9·91072	16·5385	26·6144	42·1807	67·0888	109·4534				
0·5	5·53802	9·87751	16·4545	26·3742	41·5076	65·2679	104·4652				
0·6	5·47791	9·81734	16·2732	26·1077	40·6453	63·2707	99·6442	162·125			
0·7	5·38824	9·63895	16·0200	25·5026	39·6623	61·1946	95·0525	151·253			
0·8	5·27513	9·45991	15·7143	24·9478	38·6061	59·1016	90·7143	141·707			
0·9	5·14437	9·25645	15·3706	24·3462	37·5099	57·0279	86·6331	133·240	210·995		
1·0	5·00000	9·02746	15·0000	23·7495	36·3971	55·0000	82·8022	125·664	195·000		
1·1	4·84537	8·78075	14·6111	23·0744	35·2835	53·0316	79·2091	118·839	181·299	286·374	
1·2	4·68319	8·53522	14·2105	22·4107	34·1811	51·1309	75·8392	112·653	169·394	261·436	
1·3	4·51562	8·27700	13·8032	21·7535	33·0971	49·3447	72·6772	107·016	158·930	240·845	
1·4	4·34440	8·01454	13·3931	21·1002	32·0367	47·5471	69·7076	101·850	149·643	223·304	343·147
1·5	4·17097	7·75000	12·9832	20·4546	31·0037	45·9038	66·9117	97·112	141·333	208·129	313·704

TABLE VI (iii).

Values of β_5.

β_2

β_1	2·0	2·5	3·0	3·5	4·0	4·5	5·0	5·5	6·0	6·5	7·0
0·0	0	0	0	0	0						
0·1	1·99086	4·39480	9·3207	19·9714	45·9387	128·529					
0·2	3·59438	8·03374	16·5960	34·7825	76·6000	193·361					
0·3	4·86677	10·68765	22·2196	45·7142	97·2263	228·104	668·284				
0·4	5·85929	12·85477	26·5187	53·7090	111·0237	246·506	650·398				
0·5	6·51704	14·51540	29·7545	59·4655	120·0543	255·295	614·633				
0·6	7·17383	15·7892	32·1362	63·9266	125·6629	258·147	581·205	1618·635			
0·7	7·56616	16·6546	33·8306	66·2045	128·8283	257·225	550·107	1368·373			
0·8	7·81963	17·2668	34·9714	67·8804	130·2010	253·872	521·257	1196·612			
0·9	7·95777	17·6667	35·6658	68·7533	130·2587	248·937	495·375	1068·877	2769·42		
1·0	8·00000	17·8291	36·0000	69·0644	129·3434	243·000	469·637	968·318	2280·00		
1·1	7·96281	17·8472	36·0437	68·7730	127·7158	236·441	446·547	886·541	1945·69	5313·80	
1·2	7·86015	17·7503	35·8535	68·1357	125·5684	229·524	425·062	818·040	1700·98	4135·56	
1·3	7·70375	17·5396	35·4754	67·2181	123·0362	222·562	405·663	759·486	1512·94	3388·18	
1·4	7·50358	17·2423	34·9467	66·0678	120·5142	214·828	386·347	708·620	1363·20	2870·08	7265·31
1·5	7·26808	16·8768	34·2983	64·7210	117·2460	207·227	368·843	663·926	1240·65	2488·62	5719·68

TABLE VI (iv).

Values of β_6.

β_2

β_1	2·0	2·5	3·0	3·5	4·0	4·5	5·0	5·5	6·0	6·5	7·0
0·0	14·0000	39·0649	105·000	290·678	868·015						
0·1	16·4616	45·7741	124·835	355·508	1243·832	10228·33					
0·2	17·7296	50·2472	132·998	369·894	1190·700	6204·69					
0·3	18·1764	51·0927	134·215	361·909	1089·739	4485·38	107697·95				
0·4	18·0667	50·2458	131·337	344·886	977·506	3471·87	25413·18				
0·5	17·5474	48·7896	126·107	323·447	877·884	2792·19	13737·63				
0·6	16·8560	46·8558	119·601	303·252	784·431	2303·07	9048·43	119230·33			
0·7	15·9787	44·3106	112·492	277·658	701·500	1934·79	6534·78	40994·77			
0·8	15·0148	41·7081	105·200	255·716	628·450	1648·52	5045·80	22660·09			
0·9	14·0113	39·0906	97·984	235·072	564·277	1420·51	4024·45	14836·90	137288·7		
1·0	13·0000	36·4119	91·000	216·137	507·894	1235·50	3286·65	10612·25	57584·9		
1·1	12·0030	33·7916	84·339	198·263	456·575	1083·04	2741·39	8135·91	33078·5	797653·2	
1·2	11·0354	31·3418	78·047	181·987	414·455	955·78	2322·13	6314·06	21891·8	155693·9	
1·3	10·1070	28·9775	72·146	167·142	375·834	848·97	1994·05	5108·55	15690·2	75009·7	
1·4	9·2240	26·7355	66·637	153·582	342·057	755·79	1726·18	4219·50	11846·9	44891·9	565740
1·5	8·3899	24·6268	61·512	141·477	310·976	676·32	1508·92	3544·82	9281·2	30280·3	180793

ON THE RELATION BETWEEN THE SYMMETRY OF THE EGG, THE SYMMETRY OF SEGMENTATION, AND THE SYMMETRY OF THE EMBRYO IN THE FROG*.

SECOND COMMUNICATION.

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PART I.

In a paper published in Volume v. of this *Journal* I gave an account of the relation between three meridional planes in the egg and embryo of the common frog, namely, the Plane of Symmetry of the unsegmented egg (as determined by the position of the grey crescent which appears after fertilization), the Plane of the First Furrow of segmentation, and the Sagittal Plane of the embryo.

The examination of the angles between these planes in a fairly large number of cases showed (1) that there was a fair tendency for the Sagittal Plane to coincide with the First Furrow since angles of small value considerably preponderated over those of large, but at the same time very little correlation between the two; (2) that there was a greater tendency for the Sagittal Plane to coincide with the Plane of Symmetry, and some correlation between the two, the value of the standard deviation from the mean (approximately 0°) being less, that of the correlation coefficient greater than in the former case; (3) that the First Furrow tended to lie either in or at right angles to the Plane of Symmetry, and that there was little correlation between the two.

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* I should like to take this opportunity of expressing the great obligations I am under to Professor Pearson for the trouble he has taken in suggesting methods of dealing with the data. I have also to thank Mr Schuster for correcting some of the arithmetic. I may say, perhaps, that the delay in publication is due to the fact, first, that the experiments have extended over three years, and secondly, that the working out of the results—especially those of Part II.—has involved a good deal of labour. In cutting the necessary sections I had assistance, thanks to a grant of money from the British Association.

The results of this investigation are presented in tabular form herewith (Table I.).

TABLE I.

	First Furrow and Sagittal Plane	Plane of Symmetry and Sagittal Plane	Plane of Symmetry and First Furrow		
			All cases	About 0° only	About 90° only
σ	40·39° ± ·65	29·75° ± ·63	48·33° ± 1·14	18·70° ± ·60	23·29° ± ·86
ρ	·138 ± ·031	·372 ± ·025	·087 ± ·032	—	—

In this experiment the eggs were placed, haphazard with regard to the direction of their axes, in rows parallel to the length of long glass slides, and the angles were measured between the various planes and lines ruled across the slides. Under these circumstances the eggs in each row become pressed against one another by their jelly-membranes, the direction of the pressure being parallel to the length of the slide; and since the axis may make any direction with the vertical, it is possible that during the half-hour which elapses before the eggs turn over into their normal position with the axes vertical and the white (yolk) pole below, gravity may act upon, and cause, as in Pflüger's permanently inverted eggs, a redistribution of the contents (the heavy yolk granules sinking to the lower side, the lighter cytoplasm and pigment rising to the upper side) and so impress upon the eggs a gravitation plane of bilateral symmetry (the plane, including the egg-axis and the vertical, on each side of which the yolk descends, the cytoplasm ascends in a corresponding manner).

It is known of course that when the eggs are kept inverted this plane becomes the median plane of the embryo, that side on which the white pole is upturned being dorsal, and further that the First Furrow, while it may make any angle with this plane, tends usually to lie in, or at right angles to or at an angle of 45° to it. It is also well known that when eggs are strongly compressed at right angles to the axis the First Furrow lies in the direction of pressure, the median plane of the embryo at right angles to it (O. Hertwig). Thirdly, the direction of the incident light (day-light) may conceivably exert some influence upon the position of one or more of these planes (though Roux has indeed stated that this is not the case), and so upon the angles they make with one another.

In order to discover therefore whether any of these external agencies do affect the direction of the planes and so of the angles between them, the eggs have been examined under four principal different conditions.

I. The eggs were placed close together in longitudinal rows, with their axes horizontal, and the white pole facing the same side of the slide so that each egg had to turn through 90° to gain its normal position. The pressure therefore is parallel to, the gravitation symmetry plane at right angles to the length of the slide.

II. The eggs were closely placed as above, but with their axes vertical, and their white poles below. Since the eggs did not have to turn over the possible influence of gravity is eliminated, but the pressure remains.

III. While the eggs are placed as in I. with their axes horizontal, they are so spaced that their jellies do not touch. The pressure is eliminated, but gravity remains.

IV. By placing the eggs wide apart and with their axes vertical both these possibly disturbing factors are removed.

The direction of the incident light was also varied:—

In I. the majority of the slides were placed and kept with one end to the window, but a few were kept in darkness until segmentation had begun.

In II. the majority were placed with one end to the window, but in a few cases one side of the slide was directed towards the light.

In III. one end of the slide was certainly directed towards the light in some cases, in the remainder I am not certain of the position although I believe it was the same.

Lastly in IV. all the slides were placed with one end facing the light.

The results of the measurements of the angles between the Plane of Symmetry, the First Furrow and the Sagittal Plane under the four main conditions (the possible effect of light will be considered later) are given in the accompanying Table VI. in which are entered the values of the standard deviations and correlation coefficients for each angle under each set of conditions.

TABLES II.—V.

Frequency of Angles between Plane of Symmetry and Sagittal Plane.

Angle	Negative												Positive												Totals	*M*	σ
	180—165	*165—150*	*150—135*	*135—120*	*120—105*	*105—90*	*90—75*	*75—60*	*60—45*	*45—30*	*30—15*	*15—0*	*0—15*	*15—30*	*30—45*	*45—60*	*60—75*	*75—90*	*90—105*	*105—120*	*120—135*	*135—150*	*150—165*	*165—180*			
II.	6	4	2	4	2	4	10	18	25	37	74	180	165	86	48	35	9	12	4	3	1	2	3	7	741	·54° ±1·12	45·29° ± ·79
III.	5	1	—	2	2	4	8	22	38	56	98	173	198	92	42	30	9	3	2	3	1	—	1	2	792	−3·67° ± ·88	36·84° ± ·62
IV.	—	—	—	—	1	2	—	1	4	11	39	56	62	41	14	4	4	1	5	—	—	—	—	—	245	2·73° ±1·27	29·37° ± ·89
V.	—	—	—	—	—	—	1	8	5	13	32	59	68	36	10	4	1	3	1	—	1	—	2	1	245	·92° ±1·36	31·50° ± ·96

Plane of Symmetry and Sagittal Plane; the angle measured being between the "grey crescent" end of the former and the "dorsal lip" end of the latter.

II. Eggs close. Axes horizontal.
III. Eggs close. Axes vertical.
IV. Eggs spaced. Axes horizontal.
V. Eggs spaced. Axes vertical.

It may be pointed out that since the two ends of the First Furrow are indistinguishable from one another the range of the value of the angle between it and either the Plane of Symmetry or the Sagittal Plane does not extend beyond 90° in either direction. In the case of the other two planes however the two ends are externally unlike, the first having the grey crescent, the second the dorsal lip of the blastopore at one end. It is consequently possible to distinguish between any angle and its supplement, and the range of values may be extended on each side to 180°. This has been done in Tables II.—V. and the results are entered in brackets in the second column of standard deviations in Table VI.

TABLE VI.

	σ			ρ		
	First Furrow and Sagittal Plane (γ)	Plane of Symmetry and Sagittal Plane (β)	Plane of Symmetry and First Furrow (α)	First Furrow and Sagittal Plane ($\rho_{\alpha\beta}$)	Plane of Symmetry and Sagittal Plane ($\rho_{\alpha\gamma}$)	Plane of Symmetry and First Furrow ($\rho_{\beta\gamma}$)
I. Eggs close. Axes horizontal	38·42° ± ·70	31·86° ± ·56 (42·29° ± ·79)	41·59° ± ·84	·201 ± ·028	·263 ± ·027	·118 ± ·029
II. Eggs close. Axes vertical	33·44° ± ·44	30·17° ± ·51 (36·84° ± ·62)	39·72° ± ·61	·352 ± ·021	·276 ± ·022	·023 ± ·024
III. Eggs spaced. Axes horizontal	33·49° ± ·96	27·53° ± ·84 (29·37° ± ·89)	36·60° ± 1·11	·292 ± ·039	·399 ± ·036	·075 ± ·043
IV. Eggs spaced. Axes vertical	31·45° ± ·73	26·80° ± ·82 (31·50° ± ·96)	34·46° ± 1·07	·364 ± ·033	·451 ± ·035	·186 ± ·043

It will be clear from these tables that both "pressure" (as I will call it, though it is doubtful as we shall see, whether pressure is the real cause of the disturbance) and gravity affect the relations between the three planes, for in all three cases the standard deviation diminishes while the correlation coefficient increases when both factors are eliminated (IV.).

Taken separately however the two agencies do not modify the values of the angles between First Furrow and Sagittal Plane, and between Plane of Symmetry and Sagittal Plane in the same way. Thus it will be seen that in the first case the standard deviation is slightly less, the correlation coefficient greater when gravity alone is removed (II.) than when the pressure only is omitted (III.), while the agreement between the Plane of Symmetry and the Sagittal Plane is closer under the influence of gravity alone than of pressure alone, whether this agreement be measured by the standard deviation or by the correlation coefficient. The value of the standard deviation progressively declines in the case of the Plane of Symmetry and the First Furrow; the correlation coefficient is somewhat inexplicably large when both agencies are allowed to interfere (I.). If the values of standard deviation and correlation coefficient be considered for each of these angles separately it will be evident that, with the one exception just noticed, the former declines when the latter increases, and conversely.

There is one other point that must be noticed. It is evident from Tables II. to V. that the dorsal lip appears in the great majority of cases on the same side of the egg as the grey crescent since in these observations the measurement was of the angle between the dorsal lip, or grey crescent, and the zero point. It may however occur that these two structures appear on opposite sides of the egg, though this does not occur at all, or very little, when the axes are horizontal but the eggs spaced (IV.), and not very much when the eggs are spaced and the axes vertical (V.). It is under these conditions that the correlation between these two planes is highest. It must however be remembered that in calculating the coefficient the angles made by each of these planes with the First Furrow are compared, angles in which it is impossible, owing to the indistinguishability of the ends of the Furrow, to discriminate between any value and its supplement.

Professor Pearson has however pointed out to me that since, any two of these angles being known, the third is directly obtainable; since in fact $\alpha - \beta = \gamma$ where α, β and γ are the three angles, the three correlation coefficients ought also to be definitely related, namely, they should be the cosines of three angles which are together equal to 180°, and further the standard deviation being known of two angles and the value of the coefficient of correlation between them, the remaining standard deviation and coefficients should be obtainable by the formulae

$$\sigma_\gamma^2 = \sigma_\alpha^2 + \sigma_\beta^2 - 2\sigma_\alpha \sigma_\beta \rho_{\alpha\beta},$$

$$\rho_{\alpha\gamma} = \frac{\sigma_\alpha - \sigma_\beta \rho_{\alpha\beta}}{\sigma_\gamma},$$

$$\rho_{\beta\gamma} = \frac{\sigma_\alpha \rho_{\alpha\beta} - \sigma_\beta}{\sigma_\gamma}.$$

In order however that γ may always $= \alpha - \beta$, it is necessary that the angles should always be measured in the same sense, and this gives in certain cases a value for α (Plane of Symmetry and First Furrow) or for γ (First Furrow and Sagittal Plane) which is greater than 90°. In working out the results already tabulated these cases had always been entered as less than 90°, that is, owing to the impossibility of distinguishing between the two ends of the First Furrow, the smaller of two supplementary values was always taken. For example:

Let β (Plane of Symmetry and Sagittal Plane) be 100°,

γ (First Furrow and Sagittal Plane) be $-30°$,

then $\alpha = \beta + \gamma = 130°$, but was entered always as $-50°$.

Hence the coefficients do not possess the magnitudes they should do to satisfy the equation. At the same time where $\beta + \gamma =$ nearly 180°, there is clearly a close relation between the First Furrow and the Symmetry Plane. After some discussion I have decided to adopt the following convention suggested by Professor Pearson.

The material is divided into two series, positive and negative. In the first the Sagittal Plane and the First Furrow (up to 90°) are on the *same* side of the Plane of Symmetry, either on the right or on the left. α is then taken as positive and less than 90°, β is positive and less than 180° (the angle being measured between "grey crescent" and "dorsal lip") while γ may be positive or negative and is less than 180°.

In the second, or negative series, the Sagittal Plane and First Furrow are on *opposite* sides of the Plane of Symmetry. β is taken as positive and less than 180°, α is taken as negative and is less than 90°, while γ is the sum of the two of them, and is negative.

α and β are then correlated, and the coefficients $\rho_{\alpha\gamma}$, $\rho_{\beta\gamma}$ found by the formula.

The results are given in the tables (Tables VII. to X., (*a*) positive series, and (*b*) negative series) and summarized in Table XI.

TABLE VII.

I. *Eggs Close. Axes Horizontal.*

a. Positive series. $\rho_{\alpha\beta} = \cdot 362$.

	β												Totals
	180											0	
90	2	—	1	—	1	1	1	5	3	1	3	6	24
	1	1	—	1	—	3	4	6	6	4	5	7	38
α	—	—	—	1	—	1	4	4	7	8	8	9	42
	1	—	1	1	—	—	1	3	11	11	12	19	60
	—	—	—	—	1	1	2	—	3	10	19	30	66
0	1	—	—	2	1	—	—	—	4	8	23	43	82
Totals	5	1	2	5	3	6	12	18	34	42	70	114	312

b. Negative series. $\rho_{\alpha\beta} = -\cdot 195$.

	β												Totals
	180											0	
90	2	1	—	—	—	—	—	1	1	4	4	9	22
	—	1	—	—	—	—	5	1	1	1	1	7	17
α	—	—	—	—	2	—	1	—	2	2	3	6	16
	—	2	2	—	—	—	1	2	2	3	6	10	28
	2	—	—	—	—	—	1	—	4	3	7	20	37
0	2	—	—	—	—	2	1	2	5	10	22	47	91
Totals	6	4	2	0	2	2	9	6	15	23	43	99	211

TABLE VIII.

II. *Eggs Close. Axes Vertical.*

a. Positive series. $\rho_{\alpha\beta} = \cdot 409$.

	β												Totals
	180											0	
90	—	—	—	1	—	1	5	5	7	7	9	5	40
	1	1	—	1	—	—	1	8	10	6	5	13	46
α	—	—	—	—	—	1	1	4	13	14	20	11	64
	—	—	—	—	—	—	—	4	16	15	30	23	88
	—	—	—	—	—	—	—	—	5	16	32	39	92
0	2	—	—	—	—	—	—	4	5	10	35	106	162
Totals	3	1	0	2	0	2	7	25	56	68	131	197	492

b. Negative series. $\rho_{\alpha\beta} = -\cdot 252$.

	β												Totals
	180											0	
90	—	—	—	—	2	2	2	1	3	5	10	12	37
	1	—	—	—	—	1	1	2	2	3	2	7	19
α	1	1	—	—	1	1	—	1	1	3	2	11	22
	1	—	—	—	—	—	1	—	—	2	8	13	25
	—	—	—	—	2	—	—	1	1	6	8	31	49
0	—	—	—	1	—	—	—	—	3	9	25	94	132
Totals	3	1	0	1	5	4	4	5	10	28	55	168	284

TABLE IX.

III. *Eggs Spaced. Axes Horizontal.*

a. Positive series. $\rho_{\alpha\beta} = \cdot 359$.

	β												Totals
	180											0	
90	—	—	—	—	—	2	—	1	—	2	5	—	10
	—	—	—	—	—	1	—	1	2	1	2	5	12
	—	—	—	—	—	—	—	3	—	4	2	7	16
α	—	—	—	—	—	—	—	—	1	3	12	7	23
	—	—	—	—	—	1	—	—	1	3	19	16	40
0	—	—	—	—	—	1	—	—	1	2	16	27	47
Totals	0	0	0	0	0	5	0	5	5	15	56	62	148

b. Negative series. $\rho_{\alpha\beta} = -\cdot 237$.

	β												Totals
	180											0	
90	—	—	—	—	1	1	—	—	—	2	1	3	8
	—	—	—	—	—	—	—	—	—	—	—	4	4
	—	—	—	—	—	—	1	—	—	—	1	2	4
α	—	—	—	—	—	—	—	—	1	1	1	6	9
	—	—	—	—	—	—	—	—	—	2	4	7	13
0	—	—	—	—	—	1	—	—	2	5	17	34	59
Totals	0	0	0	0	1	2	1	0	3	10	24	56	97

TABLE X.

IV. *Eggs Spaced. Axes Vertical.*

a. Positive series. $\rho_{\alpha\beta} = \cdot 388$.

	β												Totals
	180											0	
90	—	—	—	—	—	—	—	—	1	—	2	1	4
	—	—	—	—	—	—	2	2	1	4	2	2	13
	—	—	—	—	—	—	—	1	1	—	6	7	15
α	—	—	—	—	—	—	—	2	2	7	8	7	26
	—	—	—	—	—	—	—	—	—	5	8	17	30
0	—	—	—	—	—	—	—	—	2	3	17	31	53
Totals	0	0	0	0	0	0	2	5	7	19	43	65	141

b. Negative series. $\rho_{\alpha\beta} = -\cdot 047$.

	β												Totals
	180											0	
90	—	—	—	—	—	—	—	—	1	—	2	2	5
	—	—	—	—	—	—	1	—	—	—	1	2	4
	—	—	—	—	—	—	—	—	—	—	4	3	7
α	—	—	—	—	—	—	—	—	—	1	1	8	10
	—	—	—	—	—	—	—	1	—	—	3	8	12
0	—	—	—	1	—	1	—	1	1	3	13	36	56
Totals	0	0	0	1	0	1	1	2	2	4	24	59	94

TABLE XI.

Correlations	Positive Series			Negative Series		
	$\rho_{\alpha\beta}$	$\rho_{\alpha\gamma}$	$\rho_{\beta\gamma}$	$\rho_{\alpha\beta}$	$\rho_{\alpha\gamma}$	$\rho_{\beta\gamma}$
I. Eggs close. Axes horizontal ...	·362	·334	– ·757	– ·195	·657	– ·868
II. Eggs close. Axes vertical ...	·409	·544	– ·544	– ·252	·831	– ·747
III. Eggs spaced. Axes horizontal	·359	·648	– ·480	– ·237	·826	– ·743
IV. Eggs spaced. Axes vertical ...	·388	·684	– ·405	– ·047	·757	– ·688

The results in the positive series agree fairly well with those obtained by the other method. The positive correlation between Plane of Symmetry and Sagittal Plane ($\rho_{\alpha\gamma}$) and that between First Furrow and Sagittal Plane ($\rho_{\alpha\beta}$) rise as gravity and pressure are successively removed, while the negative correlation between Plane of Symmetry and First Furrow ($\rho_{\beta\gamma}$) diminishes.

Again, except when the eggs are close and their axes horizontal, the value of $\rho_{\alpha\gamma}$ is greater than that of $\rho_{\alpha\beta}$, the relation, that is, of the Sagittal Plane to the Plane of Symmetry is closer than to the First Furrow.

In the negative series, where First Furrow and Sagittal Plane are on opposite sides of the Plane of Symmetry, there is a negative correlation ($\rho_{\alpha\beta}$) between them, and this diminishes as the effects of pressure and gravity are removed: so also does the value of the negative correlation $\rho_{\beta\gamma}$. The third coefficient $\rho_{\alpha\gamma}$ (Plane of Symmetry and Sagittal Plane) is higher when both factors are removed than when both are present, but higher still when one is present and one absent.

There are yet other ways in which the relative positions of the three planes may be compared.

In Table XII. are given the equations to the regression lines, by which the mean values of any one of the three angles can be found when the value of one of the others is given.

TABLE XII.

Regression Lines ...	Positive Series			Negative Series		
	$\alpha=$	$\beta=$	$\gamma=$	$\alpha=$	$\beta=$	$\gamma=$
I. Eggs close. Axes horizontal	$\cdot 251\beta+26{\cdot}97^\circ$ $\cdot 235\gamma+35{\cdot}54^\circ$	$\cdot 522\alpha+16{\cdot}25^\circ$ $-\cdot 765\gamma+35{\cdot}54^\circ$	$\cdot 478\alpha-16{\cdot}25^\circ$ $-\cdot 749\beta+26{\cdot}97^\circ$	$-\cdot 128\beta-26{\cdot}01^\circ$ $\cdot 333\gamma-9{\cdot}28^\circ$	$-\cdot 297\alpha+23{\cdot}63^\circ$ $-\cdot 667\gamma-9{\cdot}28^\circ$	$1{\cdot}297\alpha-23{\cdot}63^\circ$ $-1{\cdot}128\beta-26{\cdot}01^\circ$
II. Eggs close. Axes vertical	$\cdot 409\beta+22{\cdot}16^\circ$ $\cdot 500\gamma+30{\cdot}14^\circ$	$\cdot 410\alpha+13{\cdot}42^\circ$ $-\cdot 500\gamma+30{\cdot}14^\circ$	$\cdot 590\alpha-13{\cdot}42^\circ$ $-\cdot 591\beta+22{\cdot}16^\circ$	$-\cdot 301\beta-23{\cdot}12^\circ$ $\cdot 570\gamma+0{\cdot}14^\circ$	$-\cdot 211\alpha+16{\cdot}55^\circ$ $-\cdot 430\gamma+0{\cdot}14^\circ$	$1{\cdot}211\alpha-16{\cdot}55^\circ$ $-1{\cdot}301\beta-23{\cdot}12^\circ$
III. Eggs spaced. Axes horizontal	$\cdot 413\beta+21{\cdot}60^\circ$ $\cdot 610\gamma+26{\cdot}01^\circ$	$\cdot 312\alpha+13{\cdot}12^\circ$ $-\cdot 390\gamma+26{\cdot}01^\circ$	$\cdot 688\alpha-13{\cdot}12^\circ$ $-\cdot 587\beta+21{\cdot}60^\circ$	$-\cdot 281\beta-17{\cdot}36^\circ$ $\cdot 568\gamma+1{\cdot}17^\circ$	$-\cdot 200\alpha+14{\cdot}85^\circ$ $-\cdot 432\gamma+1{\cdot}17^\circ$	$1{\cdot}200\alpha-14{\cdot}85^\circ$ $-1{\cdot}281\beta-17{\cdot}36^\circ$
IV. Eggs spaced. Axes vertical	$\cdot 487\beta+18{\cdot}18^\circ$ $\cdot 680\gamma+23{\cdot}82^\circ$	$\cdot 309\alpha+12{\cdot}68^\circ$ $-\cdot 320\gamma+23{\cdot}82^\circ$	$\cdot 691\alpha-12{\cdot}68^\circ$ $-\cdot 513\beta+18{\cdot}18^\circ$	$-\cdot 052\beta-21{\cdot}57^\circ$ $\cdot 550\gamma+0{\cdot}30^\circ$	$-\cdot 042\alpha+16{\cdot}92^\circ$ $-\cdot 450\gamma+0{\cdot}30^\circ$	$1{\cdot}042\alpha-16{\cdot}92^\circ$ $-1{\cdot}052\beta-21{\cdot}57^\circ$

In the positive series the least mean value of α compatible with β being positive, 26·97°, is greater than the least mean value of β compatible with α being positive, 16·25°. In the negative series the least mean value of α compatible with β being positive is greater than the least mean value of β compatible with α being negative.

All least mean values in the positive series diminish from I. to IV.: in the negative series they vary.

In the positive series the range of γ diminishes from I. to IV.; and the mean value of $\alpha = \beta$ when $\gamma = 0$ diminishes too. The rate of change of the mean value of β is greater than that of α as γ alters in I., equal to it in II., less than it in III. and still less in IV.

In the positive series the mean value of β is of course less than that of α only while γ is positive. In the negative series it is less than that of α as long as γ does not exceed a certain small value in II. III. and IV.: in I. it will be greater than that of α as long as γ does not fall below 55°.

Lastly, in Table XIII. are given the variabilities of each angle when one other is constant.

TABLE XIII.

Variabilities	Positive Series			Negative Series		
	of α or β when γ is constant	of β or γ when α is constant	of α or γ when β is constant	of α or β when γ is constant	of β or γ when α is constant	of α or γ when β is constant
I. Eggs close. Axes horizontal	1·499	2·140	1·483	1·305	2·579	1·698
II. Eggs close. Axes vertical ...	1·368	1·490	1·489	1·224	1·863	2·226
III. Eggs spaced. Axes horizontal	1·179	1·254	1·443	0·746	1·308	1·553
IV. Eggs spaced. Axes vertical	1·057	1·066	1·337	0·976	1·344	1·494

When the eggs are close and their axes horizontal (I.) it is evident, from both the positive and negative series, that the variability of β when α is constant is greater than that of α when β is constant.

When the influence of gravity is removed (II.) the variability of β, α being constant, is equal to that of α, β being constant, in the positive series, but in the negative series that of β is less than that of α. Under other circumstances (III. and IV.) and in both positive and negative series, β is less variable than α (α or β being constant).

Again, comparing β and γ in the same way, β is slightly more variable than γ in I. (positive series) but less variable in all other cases. Lastly, α is less-variable than γ.

It must not be forgotten, however, that in this method γ will have a much wider range of variability than in that first employed by me, in which the range was restricted to 90°. The range of α is, however, limited by our convention to 90°.

Comparisons in respect of variability may therefore be more fairly instituted between β and γ than between α and either β or γ.

It will also be noticed that the variability diminishes as the interfering agents are removed.

In whatever way we compare the relations between Plane of Symmetry, First Furrow and Sagittal Plane, whether by the standard deviation (first method), the correlation coefficient (first or second method), by comparing the mean values of two when the third is known, or by comparing the variabilities, it seems clear that there is a closer relation between the Plane of Symmetry and the Sagittal Plane than between either of these and the First Furrow, at least when the disturbing effects of either gravity or pressure are removed.

Let us consider now the relation between each of these planes and the direction of pressure, of the gravitation symmetry plane (including the egg-axis and the vertical) and of the light. These relations are immediately obtainable from the observed values of the angles between the planes and the lines ruled across the slides. It is only necessary to remember that in all cases the pressure is from 90° to 90°, the gravitation plane from 0° to 180°, the white pole of the egg being towards 0°. The direction of the light, as already mentioned, differs in the several experiments, and will be noted in each.

A. Let us take first the position of the First Furrow (Table XIV. and Fig. 1).

TABLE XIV.

First Furrow.

Angle	I. Eggs close Axes horizontal		II. Eggs close Axes vertical		III. Eggs spaced Axes horizontal		IV. Eggs spaced Axes vertical
	A All cases	B Darkness	A All cases	B Light from 0°	A All cases	B Light from +90°	Light from +90°
– *0—15*	23	2	58	4	18	6	40
15—30	23	2	57	5	14	2	37
30—45	19	2	81	6	29	7	36
45—60	50	5	76	7	23	4	43
60—75	88	5	95	15	24	13	40
75—90	114	2	133	27	23	14	34
+*90—75*	102	2	148	31	29	5	29
75—60	112	8	90	15	31	10	34
60—45	57	4	62	7	22	6	29
45—30	45	3	58	5	22	5	27
30—15	26	1	56	11	25	6	27
15—0	28	2	81	12	23	8	43
Totals	687		995		283		419
M	3·92° ± ·97		·12° ± ·99		3·36° ± 1·98		3·81° ± 1·65
σ	38·16° ± ·69		46·47° ± ·71		49·32° ± 1·40		52·76° ± 1·17

The direction of pressure is from 90° to 90°. The gravitation symmetry plane is at 0°.

I. When the eggs are close and their axes horizontal the First Furrow tends to lie parallel to the length of the slide. This is the position it ought to occupy when under the influence of pressure in that direction. It is however doubtful whether the pressure is in reality responsible for this very marked effect.

II. When the eggs are close, but the axes vertical, the First Furrow still lies in the direction of pressure, but to a less extent. It would appear then that this plane tends to some extent—a slight extent, truly—to place itself at right angles to the plane of gravitation, and this supposition is borne out by the persistence of this tendency when the eggs are spaced and the pressure therefore removed, but the axes horizontal (III.), and by its disappearance when the eggs are spaced and the axes vertical (IV.).

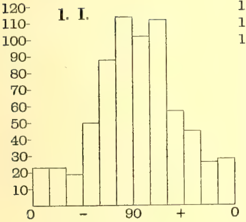
In fact the value of the standard deviation of the distribution about 90° steadily increases as first gravity only, then pressure only, and thirdly both factors are eliminated.

There seems, therefore, no escape from the conclusion that the First Furrow places itself in the direction of what I will still call the "pressure," but perpendicular to the gravitation plane. Gravity, therefore, even during the short interval that elapses between insemination and the turning over of the egg can apparently, though only to a slight extent, influence the position of the First Furrow.

In permanently inverted eggs the First Furrow, as has been mentioned, tends mainly to lie in or at right angles to or at 45° to the gravitation plane. Here only the second of these positions can be detected. It would not perhaps be going too far to attribute the difference to the continued action of gravity in the other case. The direction of division depends on the position in the egg of the fertilization spindle, since the Furrow passes in the equator of the mitotic figure. This depends in turn on the direction of elongation of the spindle, or on that of the initial division of the centrosome, and it is not difficult to understand how the centrosomes might preferably divide in, or the spindle be brought into the meridian of streaming set up by gravity in the cytoplasm. Should this streaming be more marked and lasting the two pronuclei, i.e. the equator of the spindle, not its axis, might be dragged into this plane and then division would be in the plane instead of at right angles to it.

In Table XIV. II. and Fig. 1 II. it will be seen that the frequency at 0° is high; the same peculiarity is observable in IV. I cannot find any explanation for this except by supposing that on some of the slides the rows and not merely the eggs in each row may have been in contact, and hence that there may have been a little "pressure" *across*, in addition to that along the slide.

It would seem that light exerts very little, if any, effect upon the position of the Furrow. In column I. B of Table XIV. are given the frequencies for the First Furrow when the eggs are kept in darkness. There is, as far as can be gathered from so small a number, the same tendency for the Furrow to lie in the direction of pressure as in the whole series (I. A) in which the majority were exposed to daylight falling upon them from one end of the slide.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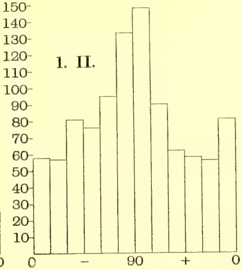
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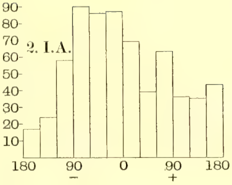
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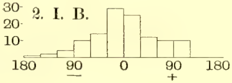
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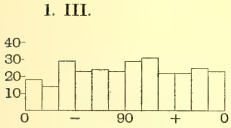
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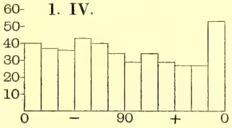
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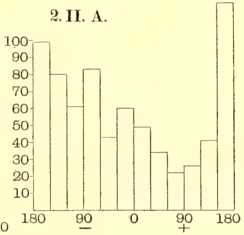
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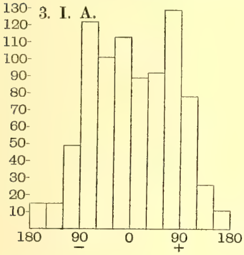
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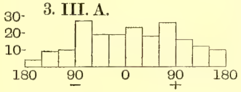
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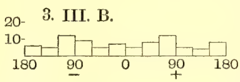
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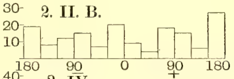
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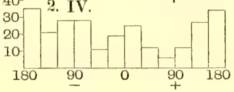
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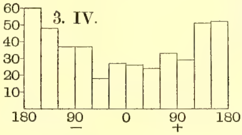
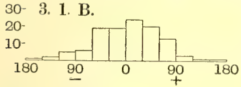
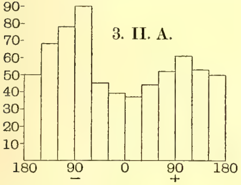
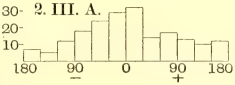
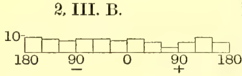
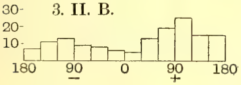

3. IV.

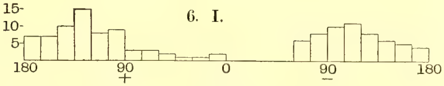

3. I. B.

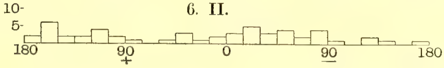

3. II. A.


2. III. A.


2, III. B.


3. II. B.


6. I.


6. II.

A comparison again of the frequencies in columns II. B and III. B, in the first of which the light is from 0°, in the second from 90°, shows that there is no greater tendency for the Furrow to lie either in or at right angles to the source of light in the one case than in the other.

We conclude, therefore, that light plays no appreciable part in determining the position of the First Furrow in the egg.

B. It is very different in the case of the Plane of Symmetry. In Table XV. are given the frequencies of the various values of the angle between the Plane of

TABLE XV. *Plane of Symmetry.*

Angle	I. Eggs close Axes horizontal		II. Eggs close Axes vertical		III. Eggs spaced Axes horizontal		IV. Eggs spaced Axes vertical
	A Light from +90°	B Darkness	A Light from +90°	B Light from 0°	A Light?	B Light from +90°	Light from +90°
– *180—165*	10	1	51	9	6	4	20
165—150	7	0	48	10	1	5	15
150—135	6	1	31	2	3	6	8
135—120	18	1	49	6	2	2	13
120—105	24	0	21	6	4	6	16
105—90	34	4	41	6	8	1	12
90—75	58	4	63	11	10	5	19
75—60	32	6	20	4	8	3	9
60—45	30	6	22	3	9	5	4
45—30	56	8	23	4	15	3	7
30—15	40	8	21	8	8	3	7
15—0	47	21	39	12	21	4	12
+ *0—15*	40	12	31	5	12	3	18
15—30	29	13	18	4	20	5	7
30—45	17	9	23	3	8	4	6
45—60	22	3	11	1	6	2	6
60—75	27	4	5	3	8	2	4
75—90	36	5	17	15	9	1	2
90—105	22	6	15	7	6	5	6
105—120	14	3	11	8	7	1	8
120—135	7	0	21	2	2	5	9
135—150	28	0	20	4	8	4	18
150—165	22	0	51	11	5	3	12
165—180	21	0	71	16	7	3	22
Total		115					
M	– ·59° ± 1·81						
σ	*28·65° ± 1·27						

The direction of pressure is from 90° to 90°. The gravitation symmetry plane from 0° to 180°, the white pole being turned towards 0°.

* Calculated on groups of 30°.

Symmetry and the lines ruled across the slides, in groups of 15°. The ranges, for the reason already given, extend on each side from 0° to 180°, the angle being measured between the middle point of the grey crescent and the zero point. In Fig. 2 are given the corresponding frequency polygons, but the groups are here just twice as large, 30°.

I. Let us consider those eggs in which the axis is horizontal, while the eggs are closely pressed and kept in darkness (I. B).

There is a striking tendency for the Plane of Symmetry to lie in the gravitation symmetry plane; and for the grey crescent to appear on that side on which the white pole was turned up, namely 0°, hardly ever on the opposite. When, however, eggs under the same conditions of pressure and gravity are exposed to day-light coming from + 90° we find (I. A), in addition to the high frequency at 0°, another about − 90°, and a minor rise in the polygon at + 90°. The rise at 0° is mainly accounted for by gravity, but the large rise at − 90°, and the lesser rise at + 90° can only be explained, as far as I am able to see, by supposing that the Plane of Symmetry has a tendency to be developed in the direction of the incident light, the grey crescent appearing preferably, if I may so put it, on that side which is turned away from the light.

II. This explanation is strengthened by the position of the grey crescent when the eggs are close, and the axes vertical (II. A). Here there is the same rise at − 90°, the same drop at + 90°, as in the case just considered, but when the light is made to fall from 0° (II. B) the drop at + 90° is replaced by a rise which is as great as that on the opposite side. This effect is not caused by the pressure, for it reappears when the eggs are spaced and their axes have been vertical from the beginning (IV.), in which the direction of light is again from + 90°. In all these cases (II. A and B, and IV.) there are also high frequencies at 0° and 180°. In II. B these are in the direction of the light, but those in II. A and IV. and the corresponding ones at − 90° and + 90° in II. B are at right angles to the light.

An anomaly which I cannot, I fear, explain is the excessive frequency (in II. A and, though to a less extent, in IV.) about 180° as compared with 0°. One might, possibly, suppose that the light was not exactly from one end, but oblique, but I doubt if this would do*.

In II. B where the light comes from 0° it is interesting to observe that this anomaly does not occur. The frequency about + 90° is practically as great as that about − 90°; the frequency however at 180°—the side turned from the light—is decidedly greater than that at 0°, which is in consonance with the other results.

III. The results recorded in column III. A and B, and in the corresponding polygons are not so unmistakable. There is, indeed, in III. B, the same evident drop at + 90°, the side from which the light comes, and there is as great a frequency at − 90°, the side turned from the light, as at 0° and at 180°, but the rise at 0° which ought to be produced by gravitation, as in I. A and B, does not appear. The number of instances (85) is perhaps too small. The effect of gravity is noticeable enough in III. A, but there is no excess in the frequency at − 90° compared with that at + 90°, only a slight rise between 0° and − 90°, which is not represented on the other side. I believe, though I am not certain, that the light came from + 90° in this case. Nevertheless we may conclude from the position of the Plane of Symmetry under all these conditions of pressure, gravity, and light in varying combination, that its direction in the egg may most certainly be affected by gravity and to some extent by light. It does not appear to be affected by pressure at all. It tends to lie in the gravitation symmetry plane, and the grey crescent appears on the side on which the white pole was upturned. It is clear, therefore, that the half-hour that passes before the egg assumes its normal position is time enough for gravity to impress a permanent effect upon the egg, and so affect the position of the Symmetry Plane. That gravity is not indispensable may, of course, be at once inferred from the appearance of the grey crescent in eggs which have never had to turn over at all.

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* For the probable explanation of this, see p. 168.

C. We have now to enquire whether and how far the egg symmetry becomes the Symmetry of the Embryo.

The frequencies are given in Table XVI., the corresponding polygons in Fig. 3.

It will be seen immediately that in some cases at least the direction of the Sagittal Plane follows very closely on that of the Plane of Symmetry.

In column I. B and Fig. 3. I. B, are given the positions of this plane in eggs which were placed closely together, had their axes horizontal, and were kept in darkness until segmentation had begun, but thereafter exposed to day-light coming from + 90°. The Sagittal Plane exhibits a very marked tendency to lie in the gravitation plane, the dorsal lip being turned towards 0°, the side on which the yolk is upturned. Its direction follows, in fact, very closely upon that previously assumed (Table XV., I. B) by the Plane of Symmetry, the standard deviation for both having practically the same value, and, as we shall presently see, the correlation between the two is under these circumstances very high, though not complete.

Turning to column I. A and Fig. 3. I. A we again find a close correspondence between the position of the Sagittal Plane and that previously occupied by the Plane of Symmetry (Table XV., I. A, and Fig. 2. I. A). The correspondence is, however, not perfect, for there is here as high, or rather a higher frequency about + 90° as about − 90°. This may be due to the light.

III. Again, when the eggs are spaced, but the axes still horizontal, there is the same high frequency at 0° and low frequency at 180° (III. A and B); in both these cases there is also a high frequency at + 90° and another at − 90°. In III. B the direction of the light is known, from + 90, and, as I have said, I believe that it had the same direction in the A series. Should this be so, the high frequency at + 90° of the Sagittal Plane as compared with the lower frequency at the same point of the Plane of Symmetry (Fig. 2. III. A) must, I think, be attributed to the action of the light. The dorsal lip would then appear as readily on the side of the egg turned towards as on that turned away from the light. The dorsal lip is formed in a region of active cell-division, and cell-division may probably be promoted by light as well as it is known to be by heat. I had previously found, as noted in the publication referred to above, that the Sagittal Plane will place itself either in or at right angles to the direction of a strong artificial light.

TABLE XVI.

Sagittal Plane.

Angle	I. Eggs close Axes horizontal		II. Eggs close Axes vertical		III. Eggs spaced Axes horizontal		IV. Eggs spaced Axes vertical
	A Light from +90°	B Darkness	A Light from +90°	B Light from 0°	A Light ?	B Light from +90°	Light from +90°
−180—165	6	0	29	4	1	4	32
165—150	9	1	21	3	3	2	28
150—135	4	2	33	4	2	2	25
135—120	11	0	38	7	7	3	23
120—105	14	2	30	7	4	6	17
105—90	35	3	48	6	6	6	20
90—75	55	1	50	8	12	6	26
75—60	67	5	40	1	15	3	11
60—45	53	12	17	5	5	1	6
45—30	48	7	28	3	14	4	12
30—15	57	9	16	2	9	3	7
15—0	56	10	23	4	10	4	20
+ *0—15*	46	15	21	1	13	2	14
15—30	43	9	16	4	10	3	12
30—45	43	13	22	7	6	3	16
45—60	49	7	22	6	12	5	8
60—75	64	10	24	7	17	9	19
75—90	65	3	28	12	9	3	14
90—105	47	2	30	14	8	5	12
105—120	31	1	31	11	8	0	17
120—135	17	1	27	6	6	2	27
135—150	9	0	26	9	6	2	24
150—165	5	0	29	9	4	5	21
165—180	6	1	21	6	6	3	31
Total		114					
M	−1·32° ± 1·81						
σ	*28·64° ± 1·28						

The direction of pressure is from 90° to 90°. The gravitation symmetry plane from 0° to 180°, the white pole being turned towards 0°.

* Calculated on groups of 30°.

II. When the eggs are close and the axes vertical the high frequency at 0° does not appear (II. A and B). The frequency at + 90°, when the eggs are illuminated from + 90° (II. A), is less than that at − 90° but not so very much less as it is in the case of the Plane of Symmetry (Table XV., II. A and Fig. 2. II. A). In the case of the eggs illuminated from 0° (II. B) no relation can be traced between the directions of the Sagittal and Symmetry Planes.

IV. Lastly, in those eggs which are freed from the influence of pressure and gravity, but exposed to light (coming from 90°) (IV.), there is a rough agreement in the position taken up by the two planes except that, as in the previous series, the frequency in the region of + 90° is increased.

Taken as a whole this third set of observations appears to indicate that the dorsal lip of the blastopore shows some tendency to be formed on that side of the egg which is turned towards the light. There is, however, no evidence that its position is affected by the pressure.

Speaking generally then we may say, as the result of the whole series of measurements, that

(1) the position of the First Furrow depends upon the direction of pressure in a very marked degree; its tendency is to coincide with that direction. In a less degree, it is dependent on the gravitation plane, since it to some extent avoids that plane, and places itself perpendicular to it. The First Furrow is not, however, influenced by the direction of incident light,

(2) the Plane of Symmetry is independent of the direction of pressure, but is very markedly dependent on the gravitation plane, with which it exhibits a strong tendency to coincide. It is also apparently influenced by light, being placed in this plane with the grey crescent turned away from the source of light,

(3) the Sagittal Plane is as independent of the pressure as is the Plane of Symmetry. It indeed follows the latter very closely, and would appear to be largely determined by it, though not completely, for while the dorsal lip exhibits nearly as great a tendency to appear on that side of the egg on which the white hemisphere was turned up as does the grey crescent, it also displays an inclination, unlike the grey crescent, for that side of the egg which is turned towards the light.

More recently (this Spring, 1909) another series of experiments has been carried out with the view of ascertaining more exactly the effect of light upon the eggs when gravity and pressure were both removed.

The eggs were all spaced and their axes were vertical.

In one lot (A) they were exposed to light (from − 90°) throughout—from the beginning of fertilization till the appearance of the dorsal lip of the blastopore.

In the second lot (B) they were kept in darkness till the beginning of segmentation. Some of these (B α) were then exposed to light coming from the same direction as before, while the remainder were kept permanently in darkness (B β).

The results of this experiment are tabulated in Tables XVII.—XXIII.

In Table XVII. the values of the standard deviations, as calculated by my method, are given. It appears that the relation between First Furrow and Sagittal Plane and between First Furrow and Plane of Symmetry is closer when the eggs are kept in darkness (B) than when they are exposed to light throughout (A). There is practically no difference, as estimated by this method, in either of these relations between the eggs exposed to light after segmentation has begun (B α) and those kept in darkness till the end (B β).

TABLE XVII.

Values of σ	First Furrow and Sagittal Plane (γ)	Plane of Symmetry and Sagittal Plane (β)	Plane of Symmetry and First Furrow (α)
A. In light throughout	38·55° ± 1·39°	30·24° ± 1·05° (38·77° ± 1·35°)	39·45° ± 1·36°
B α. In darkness till segmentation	32·30° ± ·88°	30·99° ± ·81° (41·55° ± 1·09°)	35·63° ± ·96°
B β. In darkness till dorsal lip ...	32·92° ± ·89°	32·88° ± ·88° (38·83° ± 1·03°)	37·65° ± 1·01°

The action of light on the other hand appears to improve the relation between the Plane of Symmetry and the Sagittal Plane when the range is limited to 90° on each side[*].

If, however, the range be extended to 180° on each side (the figures in brackets) then the tendency of the Sagittal Plane to lie in the Plane of Symmetry appears to be greater when the eggs are kept permanently in the light (A) or in the darkness (B β) than when the Symmetry Plane is developed in the darkness but the Sagittal Plane under the influence of light (B α).

It will be noticed that the value of σ is in each case smaller for β than for γ and for γ than for α in conformity with previous results (Tables I. and VI. IV.).

The absolute values are however larger. This must be due to some peculiarity of the eggs, or of the breeding season.

In Table XVIII. are shown the correlation coefficients (Pearson's method). The correlation between First Furrow and Sagittal Plane ($\rho_{\alpha\beta}$), and that between First Furrow and Symmetry Plane ($\rho_{\beta\gamma}$) are both improved when the eggs are kept in darkness, at least in the positive series. The correlation between Plane of Symmetry and Sagittal Plane ($\rho_{\alpha\gamma}$) hardly alters, but appears to be slightly less when the eggs are kept in the light throughout (A).

The correlations between the Plane of Symmetry and Sagittal Plane are again, always higher than those between First Furrow and Sagittal Plane.

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[*] This is due to the fact that while the grey crescent turns away from the light, the dorsal lip turns towards it, but that this difference of 180° is ignored when the range is limited to 90°.

TABLE XVIII.

Correlations	Positive Series			Negative Series		
	$\rho_{\alpha\beta}$	$\rho_{\alpha\gamma}$	$\rho_{\beta\gamma}$	$\rho_{\alpha\beta}$	$\rho_{\alpha\gamma}$	$\rho_{\beta\gamma}$
A. In light throughout	·096	·562	− ·769	− ·140	·789	− ·719
B α. In darkness till segmentation	·223	·586	− ·659	− ·328	·724	− ·839
B β. In darkness till dorsal lip ...	·282	·588	− ·610	− ·504	·863	− ·872

In the next two tables (XIX., XX.) are given the equations to the regression lines (mean values of each angle in terms of one other) and variabilities of each when one other is constant.

TABLE XIX.

	Positive Series			Negative Series		
Regression Lines ...	$\alpha=$	$\beta=$	$\gamma=$	$\alpha=$	$\beta=$	$\gamma=$
A. In light throughout	$\cdot074\beta+26\cdot30^\circ$ $\cdot361\gamma+28\cdot77^\circ$	$\cdot124\alpha+25\cdot78^\circ$ $-\cdot639\gamma+28\cdot77^\circ$	$\cdot876\alpha-25\cdot78^\circ$ $-\cdot926\beta+26\cdot30^\circ$	$-\cdot158\beta-30\cdot50^\circ$ $\cdot554\gamma-2\cdot48^\circ$	$-\cdot140\alpha+20\cdot02^\circ$ $-\cdot446\gamma-2\cdot48^\circ$	$-1\cdot140\alpha-20\cdot02^\circ$ $-1\cdot158\beta-30\cdot50^\circ$
B α. In darkness till segmentation	$\cdot307\beta+18\cdot40^\circ$ $\cdot452\gamma+25\cdot94^\circ$	$\cdot340\alpha+16\cdot58^\circ$ $-\cdot548\gamma+25\cdot94^\circ$	$\cdot660\alpha-16\cdot58^\circ$ $-\cdot693\beta+18\cdot40^\circ$	$-\cdot218\beta-19\cdot25^\circ$ $\cdot351\gamma-6\cdot13^\circ$	$-\cdot494\alpha+17\cdot57^\circ$ $-\cdot649\gamma-6\cdot13^\circ$	$1\cdot494\alpha-17\cdot57^\circ$ $-1\cdot218\beta-19\cdot25^\circ$
B β. In darkness till dorsal lip	$\cdot272\beta+18\cdot80^\circ$ $\cdot482\gamma+27\cdot12^\circ$	$\cdot292\alpha+20\cdot20^\circ$ $-\cdot518\gamma+27\cdot12^\circ$	$\cdot708\alpha-20\cdot20^\circ$ $-\cdot728\beta+18\cdot80^\circ$	$-\cdot489\beta-12\cdot32^\circ$ $\cdot490\gamma+0\cdot02^\circ$	$-\cdot520\alpha+13\cdot13^\circ$ $-\cdot510\gamma+0\cdot02^\circ$	$1\cdot520\alpha+13\cdot13^\circ$ $-1\cdot489\beta-12\cdot32^\circ$

TABLE XX.

Variabilities	POSITIVE SERIES			NEGATIVE SERIES		
	of α or β when γ is constant	of β or γ when α is constant	of α or γ when β is constant	of α or β when γ is constant	of β or γ when α is constant	of α or γ when β is constant
A. In light throughout	1·207	1·879	1·453	1·039	1·482	1·675
B α. In darkness till segmentation	1·326	1·649	1·532	1·141	2·352	1·561
B β. In darkness till dorsal lip ...	1·358	1·668	1·609	0·893	1·575	1·528

The range of γ in series B is equal to the range of γ in series III. and IV. (Table XII.): in A it is rather wider.

In all three (positive) series the mean value of $\alpha (= \beta)$ when $\gamma = 0$ is about the same. The least mean values of α and β compatible with β and α being positive (or β positive and α negative in the negative series) are about the same except in A, negative series, where that of β is less than that of α.

In the positive series β changes faster than α as γ alters.

In B α, β will be more often less than α than in B β.

The variability of β when γ is constant is always less than that of γ when β is constant (compare Table XIII. IV.), but, unexpectedly, β has a greater variability than α (except in A, negative series). α has always a less variability than γ.

It appears indeed that in this series of experiments, when the constants are calculated by Pearson's method, the relation between Plane of Symmetry and Sagittal Plane is not as close as one would have anticipated (compare the values of $\rho_{\alpha\gamma}$ in Tables XVIII. and XI. IV., and of σ_β in Tables XVII. and VI. IV.). There certainly are more instances of angles over 90° than were found in the series given in Table V.

Lastly, we have to consider the position of each of these planes separately with regard to the direction of the incident light (Tables XXI.—XXIII.).

From Table XXI., which gives the position of the First Furrow, it is clear that this plane is not affected by light. It may occupy any position, and the

TABLE XXI.

First Furrow.

Angle	A. Light (from −90°)	B. Darkness
−90—75	22	56
75—60	13	68
60—45	18	55
45—30	18	57
30—15	23	53
15—0	11	59
+ *0—15*	14	57
15—30	21	48
30—45	16	45
45—60	11	54
60—75	17	53
75—90	16	54
Totals	200	659
M	$-3{\cdot}45° \pm 2{\cdot}49$	$-2{\cdot}75° \pm 1{\cdot}38$
σ	$52{\cdot}17° \pm 1{\cdot}05$	$52{\cdot}55° \pm {\cdot}98$

standard deviation is the same in the two cases. This is the result already obtained (Table XIV. IV.).

In the case of the Plane of Symmetry (Table XXII.) there are high frequencies about 0° in both light (A) and darkness (B) and again about 180° in darkness. These high frequencies are probably due—at least I can think of no other explanation—to the manipulation. In placing the egg on the slide each one was taken up by its jelly in the forceps, being seized near the animal pole, placed first with its axis at right angles to the slide (i.e. in the direction 0° to 180°) and then turned into position with the axis vertical and the white pole below. It appears that the structure of the egg is unavoidably affected by this treatment, and a slight bilateral symmetry conferred upon it which is not without influence on the position of the Symmetry Plane. Similar high frequencies have been noticed already (Table XV.) and will be found to recur in the position of the Sagittal Plane. Obviously they cannot be attributed to the action of the light.

TABLE XXII.

Plane of Symmetry.

Angle	A. Light (from $-90°$)	B. Darkness
$-$ *180—150*	10	56
150—120	13	23
120—90	14	40
90—60	12	55
60—30	30	57
30—0	28	103
+ *0—30*	33	101
30—60	18	57
60—90	11	49
90—120	17	46
120—150	19	56
150—180	18	60
Totals	223	703

The influence of the light upon the position of the grey-crescent is indeed in this series very doubtful indeed.

The evidence that the Sagittal Plane is affected by this agency is, however, better (Table XXIII.) for when the eggs were exposed to light from the beginning of the segmentation until the appearance of the blastopore (B α) there is a slight rise in frequency between $-60°$ and $-90°$, a rise which does not occur when the eggs remain in darkness (B β). This is in agreement with the result already arrived at (Table XVI.), that the dorsal lip tends to turn towards the light.

The high frequencies about 0° must be referred, as already explained, to the manipulation of the eggs.

TABLE XXIII.

Sagittal Plane.

Angle	A. Light throughout from −90°	B α. In darkness till segmentation, then exposed to light from −90°	B β. In darkness till dorsal lip
−180—150	7	17	27
150—120	11	15	11
120—90	15	28	28
90—60	19	37	22
60—30	22	34	31
30—0	27	41	47
+ 0—30	24	43	40
30—60	10	34	28
60—90	18	26	29
90—120	14	19	22
120—150	17	25	25
150—180	15	36	25
Totals	199	355	335

The very great interest attaching, I venture to think, to these results lies in the fact that it is experimentally possible to separate those factors in the constitution of the fertilized ovum upon which the direction of cell-division depends from those which determine first the Plane of Symmetry and, next, the position of the median plane of the embryo, or the direction, if I may so express it, of differentiation. The first depends clearly on the pressure, the second on gravitation (and light). When both pressure and gravitation act upon the egg, and, as in the conditions of the above experiment, at right angles to one another, the deviations of the Plane of Symmetry and of the Sagittal Plane from the First Furrow will be large, the result actually obtained; when, on the other hand, both agencies are eliminated, the agreement of these two planes with the First Furrow is closer. Light is also a disturbing factor, and were light, as well as those small interfering agencies—such as deformation of the eggs by natural pressure in the uterus, or by handling in the course of the experiment—to be also got rid of, we might surmise that the agreement between the First Furrow and the other two planes would be considerable, provided that there were any reasons for supposing that the internal factors on which the direction of cell-division and the direction of differentiation depended, did themselves, under these circumstances, coincide. What these internal causes may be we shall have in a moment to enquire.

Meanwhile there is still one point that demands some discussion.

We have seen that the Symmetry of the Embryo—the position of the Sagittal Plane—follows very closely upon that of the previous Plane of Symmetry. It does not, however, absolutely follow, and, as we know, the correlation between the two is not complete. As far as we have seen at present the correlation is great when gravity alone acts upon the eggs, and impresses upon them a symmetry which affects the position of the grey-crescent, and then that of the Sagittal Plane. We also know that the effect of light upon the Sagittal Plane (the dorsal lip) is not the same as its effect upon the grey-crescent, and that when the eggs are grown in darkness the agreement between the positions occupied by these planes in the egg is considerable (Figs. 2 and 3, I B, Tables XV. and XVI. I B). Now it is of great interest to observe, that in these circumstances, the standard deviation of the angle between Plane of Symmetry and Sagittal Plane is low, namely, 17·64° ± 1·38 (Table XXIV.). If we except the solitary instance of an angle of 177° (which I feel sure is a mistake) there is no value greater than 60°, and only one greater than 45°. Of the 114 eggs in this series the position of the First Furrow was known, unfortunately, in only 37, but I have employed these in making the correlation table (Table XXV.). The value of the coefficient is fairly high $\rho = {\cdot}642 \pm {\cdot}040$, and would have been higher still (·736 ± ·029) had I not halved the frequency of the one instance in which the angle between First Furrow and Sagittal Plane was exactly 90°, though owing to the general trend of the correlation table it was perhaps hardly necessary to do this. Be that as it may, the correlation between Plane of Symmetry and Sagittal Plane is certainly greatest when gravity is allowed to act upon the eggs, but when light is eliminated. Under the same circumstances, the standard deviations (as calculated for this very small number) of the angles between First Furrow and Plane of Symmetry, and First Furrow and Sagittal Plane remain unaltered, or rather are slightly increased in comparison with the whole series (Table VI. I.).

TABLE XXIV.

Eggs Close. Axes Horizontal. Darkness. Plane of Symmetry and Sagittal Plane.

Negative						Positive						
90—75	*75—60*	*60—45*	*45—30*	*30—15*	*15—0*	*0—15*	*15—30*	*30—45*	*45—60*	*60—75*	*75—90*	Total
—	—	—	6	11	37+1*	38	12	8	1	—	—	114

* +177° $M = 1{\cdot}32° \pm 1{\cdot}95$ $\sigma = 17{\cdot}64° \pm 1{\cdot}38$.

Yet it may still be questioned whether, even so, the First Furrow does not exercise some independent influence upon the Embryonic Symmetry, and slight though this influence may be, it must, I think, be allowed for, at least as a possibility.

For even under these most favourable conditions, the correlation of egg-symmetry with embryonic-symmetry is not as great as might be desired. On the

TABLE XXV.

First Furrow and Plane of Symmetry.

First Furrow and Sagittal Plane.	−						+						Totals
	90						0					90	
−	—	—	—	—	—	—	—	—	—	—	—	·5	·5
	—	3	—	—	—	—	—	—	—	—	—	—	3
	—	—	2	1	—	—	—	—	—	—	—	—	3
	—	—	—	1	—	—	—	—	—	—	—	—	1
	—	—	—	—	1	1	—	—	—	—	—	—	2
	—	—	1	1	1	1·5	1	1	—	—	—	—	6·5
0 +	—	—	—	—	2	1·5	1	2	—	—	—	—	6·5
	—	—	—	—	1	—	2	2	—	—	1	—	6
	—	—	—	—	—	—	1	—	1	—	1	—	3
	—	—	—	—	—	—	1	—	—	—	—	—	1
	—	1	—	—	—	—	—	—	—	1	1	—	3
90	—	—	—	—	—	—	—	—	—	—	—	1·5	1·5
Totals	0	4	3	3	5	4	6	5	1	1	3	2	37

$\sigma = 40{\cdot}88^\circ$.

$\sigma = 42{\cdot}74^\circ$.

$\rho = {\cdot}641 \pm {\cdot}051$ or when the 90° is not halved $= {\cdot}736 \pm {\cdot}037$.

other hand there is, even under the most unfavourable circumstances, some correlation between the First Furrow and the Sagittal Plane, while under the influence of pressure alone the correlation is very fair.

This is a point to which we shall return when we have discussed the internal causes which fix the positions of the Plane of Symmetry and the First Furrow.

PART II.

As will be well remembered, it has been asserted by Roux that the position of the First Furrow of the Frog's egg depends upon the point of entrance of the spermatozoon. Roux found either (1) that the sperm-path (the track of pigment left behind it in the egg by the entering spermatozoon) was either included in, or close to and parallel to the First Furrow, or (2) that if the path was inclined at an angle to the Furrow there was an inner portion which lay parallel to, or was included in it. The path was found to be in or parallel to the Furrow when the eggs had their axes vertical from the beginning, to be inclined to it when the eggs were (permanently) prevented from assuming their normal position. In the second the outer inclined portion was distinguished as the 'penetration' path, the direction of first entrance of the spermatozoon, from the inner parallel portion, or 'copulation' path, the line, that is, of union of the two pronuclei. In the first case the two portions were regarded as lying in one and the same meridian, since the 'penetration' path is directed towards some point in the axis, and the 'copulation' path also towards the point in the axis occupied by the female pronucleus, and in all cases it was held that it was the line of union of the two pronuclei which determined the position of the (meridional) cell division. The pronuclei are situated in the equator of the fertilization spindle, and the spindle lies in a plane parallel to the equator of the egg. The direction of the axis of the spindle, and therefore of its equator, is however really determined, as many cytological studies have shown, by the direction of division of the sperm-centrosome. Since this, however, is at right angles to the line joining the two pronuclei and in a plane parallel to the equator of the egg the result is the same.

Roux further fertilized the eggs from an arbitrarily selected meridian, by applying the sperm containing water to one side of the egg by means of a fine cannulus, a camel's hair brush or a fine silk thread. The fertilization meridian so selected became the meridian of the First Furrow and, as Roux believed to be necessarily the case, the Median Plane of the embryo. Lastly, Roux believed that the grey-crescent appeared always on the side opposite to the entrance of the spermatozoon, and was in fact caused by that entrance, and that the Sagittal Plane lay in the Plane of Symmetry so produced in the egg. Since, moreover, the dorsal lip appeared upon the side of the grey-crescent, the entrance point of spermatozoon marked the future ventral side. With this plane the First Furrow—the plane of the predetermined or 'immanent' qualitative division of the segmentation nucleus —was also held to coincide. In short, this one cause was supposed to determine the Symmetry of the unsegmented egg, the Symmetry of Segmentation, and the Symmetry of the Embryo.

The subsequent statements of O. Schulze and Kopsch with regard to the sperm entrance, grey-crescent, First Furrow and Sagittal Plane are in substantial agreement with Roux's. Since, however, as we now know, the causes which determine the position of the First Furrow can be experimentally separated from those which determine the position of the other two planes, and are therefore presumably distinct, it has seemed to me to be worth while enquiring how far and in what sense it can be asserted that one cause, the entrance of the spermatozoon, is responsible for both.

I have accordingly examined by the help of sections the relation of the sperm-entrance point and sperm-path to the First Furrow in a number of eggs in which the angle between the Plane of Symmetry and the First Furrow had been previously ascertained.

As in the series of experiments already described the eggs were taken from the uterus and placed upon glass slides, fertilized, covered with water and left till the First Furrow appeared. The angles between the Plane of Symmetry and First Furrow and the lines ruled across the slide were then measured and recorded. Each egg was given a number, and killed by means of boiling water containing a little formol and preserved separately, being later cut into a series of sections. The eggs were placed on the slide under two different conditions. In one series they were closely packed in rows parallel to the length of the slide, and their axes were horizontal, directed *across* the slide and with their white poles all facing one way. In the second series they were spaced and their axes were vertical. As I could only hope to have a limited number of eggs cut into sections I thought it best to obtain as many as possible under each of these two conditions, that is with both pressure and gravity, and without both, and not to attempt to investigate the effect of each factor separately.

In the first series (I)—eggs close, axes horizontal—I have 123 eggs each cut into a series of sections, in the second (II) 55 eggs.

The sections were cut equatorially, that is, at right angles to the axis—and therefore also to the First Furrow, since this is meridional—or as nearly as possible so. The direction of cutting was not, however, always exact, and in calculating the magnitudes of the several angles, an allowance had to be made for the obliquity of the sections to the axis or to both axis and furrow.

The obliquity of the plane of sectioning to the axis was determined by finding the number of sections—the thickness of which was known—intervening between the section in which the yolk first appeared and the section in which it appeared at a point diametrically opposite. The length of the radius of the egg being known, the angle made by the axis with the plane of section may be determined, on the assumption, of course, that the egg is a sphere and that the yolk is so uniformly distributed around the axis that any plane at right angles to the latter would cut equal areas of yolk in all directions from its centre.

These assumptions are unavoidable, and I think fair. The yolk is normally distributed in that way, and I do not think that the spherical eggs become much distorted in the processes of preserving and embedding. The same assumptions have to be made in calculating the obliquity of the sections to the furrow. This was done by counting the sections intervening between the one in which one blastomere appears and that in which the other is first seen. In a section at right angles to the furrow they would of course appear simultaneously.

Making these corrections the following angles have been, indirectly, measured.

(1) The angle between the meridian which includes the point of entrance of the spermatozoon and the meridian of the First Furrow.

(2) By subtraction, the angle between the sperm-entrance meridian and the Plane of Symmetry (a meridional plane).

(3) The angle between the meridian including the end of the first part of the sperm-path ('penetration' path of Roux) and the First Furrow. The former meridian I shall call the sperm-sphere meridian because this structure appears at the inner end of the path.

(4) The angle between the sperm-sphere meridian and the Plane of Symmetry.

(5) The angle between the sperm-entrance radius and the egg axis, or the angle subtended at the centre of the egg by the distance between the animal pole

and the entrance point. The sperm always enters somewhere in the animal hemisphere and the value of this angle therefore never exceeds 90°.

(6) The angle between the sperm-entrance meridian and the lines ruled across the slide, obtained from (1) and (2) and the previously determined positions of the First Furrow and Plane of Symmetry on the slide.

(7) The angle between Sperm-path and the First Furrow. By the Sperm-path I shall always mean the first part or 'penetration' path of Roux. This is a conical streak of pigment, the base outwardly, the apex inwardly directed. It is evidently what is termed in cytological descriptions of fertilization, the entrance funnel. It is formed when the sperm enters the egg by the aggregation of a clear hyaline substance (I describe what I have observed in another Amphibian—the Axolotl) about the anterior end—apical body or acrosome—of the spermatozoon. The substance streams into the interior of the egg—the phenomenon is probably

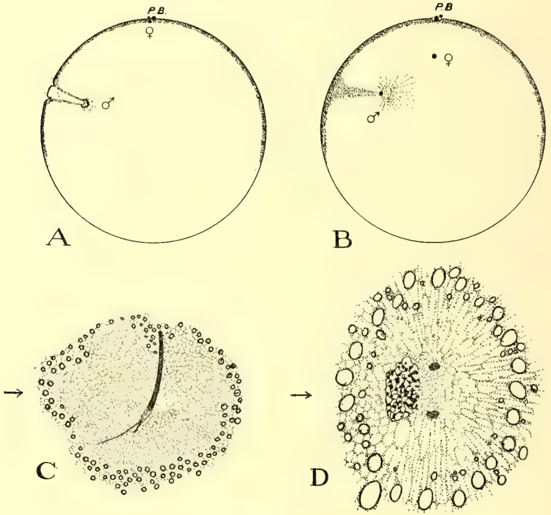

Fig. 4. A. Meridional section of Axolotl egg just after the entrance of the spermatozoon. On the left is seen the entrance-funnel and sperm-sphere, containing the spermatozoon (♂). ♀, female pronucleus. P. B. polar bodies.

B. Similar section a little later. In front of the Sperm-nucleus (♂) is the Sperm-aster. The entrance-funnel is marked by a streak of pigment ("penetration" path).

C. Enlarged view of the spermatozoon and sperm-sphere of A.

D. Enlarged view of the sperm-nucleus, and sperm-aster with the centrosome divided.

The direction of the sperm-path is shown by the arrows.

capillary—and carries with it the spermatozoon, and at the same time sweeps before it a good deal of the superficial pigment (Fig. 4 A). This pigment remains when the clear substance has disappeared as the visible sperm tract (Fig. 4 B and Fig. 5).

At the inner end of the entrance-funnel (if I may resume here very briefly what occurs in the Axolotl) a sperm-sphere—or yolk free area—appears round the middle piece (a modified centrosome), the sperm-head having meanwhile been rotated through 180° so that the middle piece is inwardly directed (Fig. 4 C). Radiations appear round the sperm-sphere which then becomes the sperm-aster, and the sperm-nucleus—developed from the sperm-head—preceded by its aster travels towards the egg-nucleus. As it does so its centrosome divides (Fig. 4 D) at right angles to the direction in which it is now moving. This direction (the 'copulation' path of Roux) may, but need not, be in the same meridional plane with the entrance funnel, and when it is, it may but need not be in the same straight line as the other, for the entrance funnel may be directed towards the

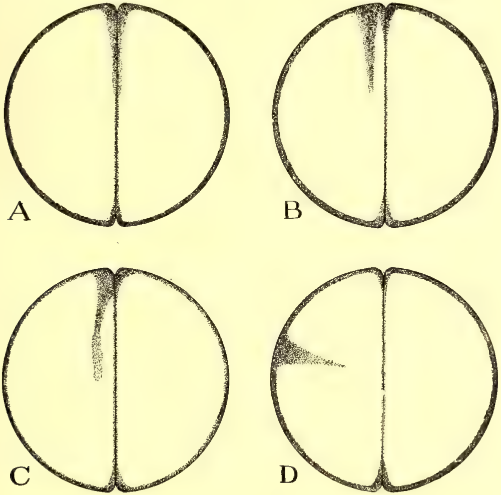

FIG. 5. Projections in an equatorial plane of the various relations of Sperm-path to First Furrow.
A. The whole path included in the Furrow.
B. The whole path parallel to the Furrow.
C. The "penetration" path turned away from, the "copulation" path parallel to the Furrow.
D. The Sperm-path inclined towards the Furrow at a large angle.

axis and yet not towards that point in the axis at which the fertilization spindle is to be developed (this point is pretty constant and about half-way between the animal pole and the centre of the egg). The funnel need not, however, even be directed towards the axis. Hence, the meridian of the 'copulation' path may make an angle with that of the 'penetration' path. As a matter of fact, I have not always in the Frog succeeded in detecting a 'copulation' path. It is generally most in evidence when, as in Fig. 5 C, the penetration path is turned away from the axis, and it is always, parallel to the furrow. In no case that I have observed did the penetration path under these circumstances make a greater angle than 30° with the First Furrow, and in all cases but one, the angle was less than 15°. We shall find that it will be necessary to divide the eggs into three classes according as the sperm-path is included in or parallel to the furrow (Fig. 5 A, B), inclined towards the furrow (Fig. 5 D) or away from it (Fig. 5 C): in the last case the path does not of course lie in a meridional plane at all.

The data necessary for the calculation of these angles are, the perpendicular distance, in the section in which it occurs, of the sperm-entrance point from the furrow, and the distance from the foot of this perpendicular to the middle point of the furrow in the same section; similar measurements for the inner end of the sperm-path ('penetration' path); the distances (the number of sections of known thickness) between the sections in which occur the entrance point, the inner end of the path and the centre of the egg; and lastly, the length of the egg radius. With the help of a little Trigonometry the value of the angles may then be found.

Let us now consider the measurements. The distributions and correlations for Plane of Symmetry and First Furrow, Sperm-entrance Meridian and First Furrow, Sperm-entrance Meridian and Plane of Symmetry, Sperm-sphere Meridian and First Furrow, and Sperm-sphere Meridian and Plane of Symmetry will be found in Tables XXVI. to XXXI. There is a summary of results in Tables XXXII. and XXXIII.

(1) The Plane of Symmetry and the First Furrow.

I tabulate these merely for the purpose of comparison with previous results.

I. When the eggs are close and the axes horizontal (XXVI. a, a', a'') the standard deviation is $\sigma = 45\cdot 99 \pm 2\cdot 00$ but there is clearly a tendency for the Furrow to be not only in but at right angles to the other plane. This is what occurred in the original series of eggs examined (described in the previous paper, and tabulated above in Table I.). The value of σ was there $48\cdot 33 \pm 1\cdot 14$. The values of the standard deviation have also been calculated about 0° and about 90° separately; they come to $\sigma = 18\cdot 63^\circ \pm 1\cdot 39$ and $\sigma = 21\cdot 48^\circ \pm 1\cdot 60$ respectively, as compared with $\sigma = 18\cdot 70^\circ \pm 0\cdot 60$ and $\sigma = 23\cdot 29^\circ \pm 0\cdot 86$ found before (Table I.). The agreement is, I think, sufficiently close.

II. When the eggs are spaced and the axes vertical, the standard deviation works out at $\sigma = 25\cdot 34^\circ \pm 1\cdot 63$. This is not so near to the values obtained previ-

TABLE XXVI. TABLE XXVII.

Angle	Eggs close. Axes horizontal			Eggs spaced. Axes vertical		
	Sperm-entrance Meridian and First Furrow XXVI. *b*	Plane of Symmetry and First Furrow XXVI. *a*	Sperm-entrance Meridian and Plane of Symmetry XXVI. *c*	Sperm-entrance Meridian and First Furrow XXVII. *b*	Plane of Symmetry and First Furrow XXVII. *a*	Sperm-entrance Meridian and Plane of Symmetry XXVII. *c*
−90°—75°	1	11	5	1	0	0
75 —60	3	3	1	1	1	0
60 —45	5	5	7	0	1	2
45 —30	5	9	12	1	2	3
30 —15	9	6	11	5	3	4
−15 —0	40	22	25	26	20	15
+ 0 —15	37	27	18	19	16	20
15 —30	5	9	12	0	4	4
30 —45	4	6	8	0	4	3
45 —60	6	5	7	1	2	2
60 —75	6	4	5	0	2	1
+75 —90	2	13	9	1	0	1
Totals	123	120	120	55	55	55
M	−1·53° ± 1·86	1·88° ± 2·83	3·63° ± 2·52	3·96° ± 1·91	3·41° ± 2·31	3·41° ± 2·33
σ	30·65° ± 1·32	45·99° ± 2·00	41·01° ± 1·79	21·02° ± 1·35	25·34° ± 1·63	25·67° ± 1·65

Plane of Symmetry and First Furrow. Eggs close. Axes horizontal.

XXVI. *a′*. About 0°		XXVI. *a″*. About 90°	
−45°—30°	9	*+45°—60°*	5
30 —15	6	*60 —75*	4
−15 —0	22	*+75 —90*	13
+ 0 —15	27	*−90 —75*	11
15 —30	9	*75 —60*	3
30 —45	6	*60 —45*	5
Total	79		41
M	−·11° ± 1·96		90·92° ± 3·36
σ	18·63° ± 1·39		21·48° ± 1·60

ously for eggs under the same conditions (Table VI., IV. *a*), when σ was $34\cdot46^\circ \pm 1\cdot07$. The difference is evidently due to the absence in the present case of any angles over 75°, but I can suggest no reason for this.

(2) The Sperm-entrance Meridian and the First Furrow. (Tables XXVI. *b*, XXVII. *b*, XXIX. *b*, XXX. *b*.)

TABLE XXVIII.

Sperm-sphere Meridian.

Angle	I. Eggs close. Axes horizontal		II. Eggs spaced. Axes vertical	
	b. Sperm-sphere Meridian and First Furrow	*c.* Sperm-sphere Meridian and Plane of Symmetry	*b.* Sperm-sphere Meridian and First Furrow	*c.* Sperm-sphere Meridian and Plane of Symmetry
− 90°—75°	3	3	0	2
75 —60	3	2	1	0
60 —45	5	5	1	2
45 —30	8	9	3	6
30 —15	12	12	15	5
15 —0	32	26	11	8
+ 0 —15	28	17	8	13
15 —30	10	12	9	9
30 —45	8	11	3	6
45 —60	2	7	0	3
60 —75	6	3	1	0
75 —90	3	13	3	1
Totals	120	120	55	55
M	14·00° ± 2·10	9·37° ± 2·51	14·33° ± 2·85	1·77° ± 2·98
σ	34·19° ± 1·48	40·73° ± 1·77	31·31° ± 2·02	32·81° ± 2·11

It is quite clear that the tendency for the First Furrow to include the point of entrance of the spermatozoon is very great, whether the eggs be under the influence of gravity and pressure or not. It is difficult indeed to determine whether these external conditions exert an unfavourable influence upon the value of the relation or not, since, where the standard deviation is less when both factors are omitted ($\sigma = 21\cdot02^\circ \pm 1\cdot35$ as compared with $\sigma = 30\cdot65^\circ \pm 1\cdot32$), the correlation is not greater as we should expect but less ($\rho = \cdot435^\circ \pm \cdot074$ as compared with $\rho = \cdot613^\circ \pm \cdot038$). In comparing these two correlation tables (XXIX. *b* and XXX. *b*) it will be seen at once that in the first case the coefficient is increased by the presence of high frequencies in the positive corners of the table (all examples of precisely 90°, I should say, have been halved), that is, of instances in which the Plane of Symmetry deviates to a great *and an equal* extent from both First Furrow and Sperm-entrance Meridian. The second correlation is clearly thrown out by three abnormal values. There is no escaping these, though I may point out that in all three cases the sperm had entered near the equator of the egg (several instances of its entering at the same large inclination to the axis occur, however, in the rest of the table) and in all three of course the value of the angle between Sperm-entrance Meridian and First Furrow is great. Should these three cases be omitted the value of ρ would rise to ·598 ± ·060, practically the same as when the eggs are in what one would have imagined to be unfavourable circumstances. That the coefficient is not greater still is due to the absence of high frequencies in the positive corners of the table, the absence, that is, of large deviations, and in the same sense, of both Sperm-entrance and First Furrow from the Plane of Symmetry. It must, indeed, be conceded that these two external agencies are not unfavourable to the relation between Sperm-entrance and First Furrow. This will become intelligible when we remember that the position of the latter is only so far affected by gravity that it tends, to some slight extent, to be at right angles to the gravitation plane, and when we see, as we shortly shall do, that 'pressure' affects the Sperm-entrance and First Furrow in precisely the same way.

TABLE XXIX.

I. Eggs close. Axes horizontal.

XXIX. *c. Correlation between Sperm-entrance Meridian and Plane of Symmetry.*

First Furrow and Sperm-entrance Meridian.

First Furrow and Plane of Symmetry.

	90		−			0			+			90	Totals
−90	—	1	1	—	1	2	1·5	—	·5	—	3	1	11
	—	—	—	1	—	1	—	—	—	—	—	1	3
	—	—	1	—	1	2	1	—	—	—	—	—	5
−	—	2	—	—	2	3	2	—	—	—	—	—	9
	—	—	—	1	—	4	—	—	—	—	1	—	6
	—	—	1	1	1	12	7	—	—	—	—	—	22
0	—	—	1	1	2	6	10	3	1	3	—	—	27
	—	—	—	—	—	3	2	2	—	1	1	—	9
	—	—	—	—	1	—	2	—	1	1	1	—	6
+	1	—	—	—	—	1	3	—	—	—	—	—	5
	—	—	—	—	1	2	1	—	—	—	—	—	4
90	—	—	1	1	—	3	6·5	—	·5	1	—	—	13
Totals	1	3	5	5	9	39	36	5	3	6	6	2	120

$\rho = {}^{\cdot}006 \pm {}^{\cdot}061.$

XXIX. *b. Correlation between Sperm-entrance Meridian and First Furrow.*

Plane of Symmetry and Sperm-entrance Meridian.

Plane of Symmetry and First Furrow.

	−						+						Totals
	90						0					90	
− 90	3·5	—	1·5	—	2	—	—	4	—	—	—	—	11
	—	1	—	—	1	—	—	—	1	—	—	—	3
−	—	—	3	—	1	1	—	—	—	—	—	—	5
	—	—	—	5	—	2	—	2	—	—	—	—	9
	1	—	—	—	3	1	—	1	—	—	—	—	6
	—	—	—	—	1	16	3	—	1	1	—	—	22
0	—	—	—	4	3	2	13	2	2	1	—	—	27
	—	—	1	1	—	—	3	3	1	—	—	—	9
+	—	—	—	1	—	2	—	—	2	—	1	—	6
	—	—	—	1	—	—	—	—	—	4	—	—	5
	—	—	—	—	—	—	—	—	—	—	3	1	4
90	·5	—	1·5	—	—	—	—	—	1	1	1	8	13
Totals	5	1	7	12	11	24	19	12	8	7	5	9	120

$\rho = {\cdot}613 \pm {\cdot}038.$

TABLE XXX.

II. Eggs spaced. Axes vertical.

XXX. *c. Correlation between Sperm-entrance Meridian and Plane of Symmetry.*

First Furrow and Sperm-entrance Meridian.

First Furrow and Plane of Symmetry.

	−						+						Totals
	90						0					90	
− 90	—	—	—	—	—	—	—	—	—	—	—	—	0
	·5	—	—	—	—	—	—	—	—	—	—	·5	1
−	—	1	—	—	—	—	—	—	—	—	—	—	1
	—	—	—	—	1	1	—	—	—	—	—	—	2
	—	—	—	—	—	1	2	—	—	—	—	—	3
	—	—	—	1	3	12	4	—	—	—	—	—	20
0	—	—	—	—	—	7	9	—	—	—	—	—	16
	—	—	—	—	—	2	2	—	—	—	—	—	4
+	·5	—	—	—	1	—	2	—	—	—	—	·5	4
	—	—	—	—	—	2	—	—	—	—	—	—	2
	—	—	—	—	—	1	—	—	—	1	—	—	2
90	—	—	—	—	—	—	—	—	—	—	—	—	0
Totals	1	1	0	1	5	26	19	0	0	1	0	1	55

$\rho = {\cdot}302 \pm {\cdot}083.$

XXX. *b.* *Correlation between Sperm-entrance Meridian and First Furrow.*

Plane of Symmetry and Sperm-entrance Meridian.

Plane of Symmetry and First Furrow.

	90		−			0			+			90	Totals
−90	—	—	—	—	—	—	—	—	—	—	—	—	0
	—	—	—	—	—	—	1	—	—	—	—	—	1
−	—	—	1	—	—	—	—	—	—	—	—	—	1
	—	—	—	1	1	—	—	—	—	—	—	—	2
	—	—	—	1	2	—	—	—	—	—	—	—	3
	—	—	—	1	1	10	7	1	—	—	—	—	20
0	—	—	—	—	—	5	11	—	—	—	—	—	16
	—	—	—	—	—	—	—	3	1	—	—	—	4
+	—	—	1	—	—	—	—	—	2	1	—	—	4
	—	—	—	—	—	—	—	—	—	1	1	—	2
	—	—	—	—	—	—	1	—	—	—	—	1	2
90	—	—	—	—	—	—	—	—	—	—	—	—	0
Totals	0	0	2	3	4	15	20	4	3	2	1	1	55

$\rho = \cdot 435 \pm \cdot 074$ $(\cdot 598 \pm \cdot 060)$.

TABLE XXXI.

XXXI. *b.* II. Eggs spaced. Axes vertical.

Correlation between Sperm-sphere Meridian and Plane of Symmetry.

First Furrow and Sperm-sphere Meridian.

First Furrow and Plane of Symmetry.

	90		−			0			+			90	Totals
−90	—	—	—	—	—	—	—	—	—	—	—	—	0
	—	—	—	—	—	—	—	—	—	—	1	—	1
−	—	1	—	—	—	—	—	—	—	—	—	—	1
	—	—	—	—	1	1	—	—	—	—	—	—	2
	—	—	—	—	—	1	1	1	—	—	—	—	3
	—	—	1	2	7	6	1	2	—	—	—	1	20
0	—	—	—	1	3	2	4	4	2	—	—	—	16
	—	—	—	—	2	—	1	1	—	—	—	—	4
+	—	—	—	—	—	—	1	1	—	—	—	2	4
	—	—	—	—	1	1	—	—	—	—	—	—	2
	—	—	—	—	1	—	—	—	1	—	—	—	2
90	—	—	—	—	—	—	—	—	—	—	—	—	0
Totals	0	1	1	3	15	11	8	9	3	0	1	3	55

$\rho = \cdot 188 \pm \cdot 088$.

XXXI. *a*. I. Eggs close. Axes horizontal.

First Furrow and Sperm-sphere Meridian.

First Furrow and Plane of Symmetry.	− 90			−		0				+		90	Totals
− 90	·5	1	—	—	1	1	2	1	—	·5	3	1	11
	—	—	—	1	—	1	—	—	—	—	—	1	3
−	—	—	—	3	—	1	1	—	—	—	—	—	5
	1	1	—	—	3	2	1	—	1	—	—	—	9
	—	—	1	—	—	2	2	—	—	—	—	1	6
	—	—	2	2	2	9	6	1	—	—	—	—	22
0	—	—	—	2	4	4	8	5	3	1	—	—	27
	—	—	1	—	—	2	1	3	—	—	2	—	9
+	—	—	—	—	1	2	—	—	2	—	1	—	6
	1	—	—	—	—	3	1	—	—	—	—	—	5
	—	—	—	—	1	2	—	—	1	—	—	—	4
90	·5	1	1	—	—	3	6	—	1	·5	—	—	13
Totals	3	3	5	8	12	32	28	10	8	2	6	3	120

$\rho = - {\cdot}048 \pm {\cdot}061.$

TABLE XXXII.

Standard Deviations.

Eggs Close. Axes Horizontal.

	Plane of Symmetry and First Furrow	Sperm-entrance and First Furrow	Sperm-entrance and Plane of Symmetry	Sperm-sphere and First Furrow	Sperm-sphere and Plane of Symmetry	Sperm-path and First Furrow
All cases ...	45·99° ± 2·00	30·65° ± 1·32	41·01° ± 1·78	34·19° ± 1·48	40·73° ± 1·77	21·47° ± ·93
Towards furrow	50·87° ± 3·39	45·15° ± 2·96	40·53° ± 2·71	—	36·66° ± 2·45	29·08° ± 1·90
In or parallel to furrow ...	43·91° ± 3·20	—	42·90° ± 3·12	—	—	—
Away from furrow	30·77° ± 2·88	8·51° ± ·78	31·05° ± 2·91	—	35·67° ± 3·34	17·66° ± 1·62

Eggs Spaced. Axes Vertical.

	Plane of Symmetry and First Furrow	Sperm-entrance and First Furrow	Sperm-entrance and Plane of Symmetry	Sperm-sphere and First Furrow	Sperm-sphere and Plane of Symmetry	Sperm-path and First Furrow
All cases ...	25·34° ± 1·63	21·02° ± 1·35	25·67° ± 1·65	31·31° ± 2·02	32·81° ± 2·11	17·94° ± 1·15
Towards furrow	32·67° ± 4·18	40·26° ± 5·13	33·33° ± 4·25	—	37·19° ± 4·74	28·90° ± 3·68
In or parallel to furrow ...	20·01° ± 2·88	—	20·01° ± 2·88	—	—	—
Away from furrow	18·12° ± 1·58	8·31° ± ·72	23·22° ± 2·02	—	33·32° ± 2·90	12·09° ± 1·05

TABLE XXXIII.

Correlations.

		Sperm-entrance and First Furrow	Sperm-entrance and Plane of Symmetry	Sperm-sphere and Plane of Symmetry	Sperm-path and Plane of Symmetry
All cases	I	·613 ± ·038	·006 ± ·061	− ·048 ± ·061	·030 ± ·061
,, ...	II	·435 ± ·074	·302 ± ·083	·188 ± ·088	·479 ± ·070
Towards and in or	I	—	—	—	—
parallel to furrow	II	—	—	—	·725 ± ·064
Towards furrow only	I	—	—	—	·086 ± ·094
,, ,,	II	·502 ± ·135	·411 ± ·148	—	·880 ± ·040

I = Eggs close. Axes horizontal. II = Eggs spaced. Axes vertical.

The Plane of Symmetry is, however, affected by gravity, and since gravity and pressure are here acting at right angles to one another, there is a marked tendency for the Plane of Symmetry to lie at 90° as well as to coincide with the Furrow. This of course accounts for the high frequencies in the positive corners of Table XXIX. *b* and the high value of ρ.

(3) The Sperm-entrance Meridian and the Plane of Symmetry.

When the eggs are close and the axes horizontal the standard deviation is high, $\sigma = 41{\cdot}01° \pm 1{\cdot}79$ (Table XXVI. *c*), the correlation negligible $\rho = {\cdot}006 \pm {\cdot}061$ (Table XXIX. *c*).

We know that gravity affects the position of the Plane of Symmetry. We should expect therefore, that the relation between the Sperm-entrance and this plane would be closer if the influence of gravity were removed. This is, as a matter of fact, the case; the standard deviation sinks to 25·67° ± 1·65, while the correlation coefficient rises to ·302 ± ·083 (Tables XXVII. *c*, XXX. *c*).

(4) The Sperm-sphere Meridian and the First Furrow (Table XXVIII. I. *b*, II. *b*).

I give the values of the standard deviations since they may have some significance, though I think this is doubtful. It will be seen that in each series (I. and II.) they are greater than is the case with the Sperm-entrance Meridian and the First Furrow. Nor is the reason for this far to seek. The Sperm-path is often parallel to the First Furrow. In such cases the angle between the furrow and the meridian including the inner end of the path must needs be greater than that between the furrow and the meridian of the outer end of the path, or entrance point.

(5) The Sperm-sphere Meridian and the Plane of Symmetry.

In view of the known deviation, in many cases, of the First Furrow from the Plane of Symmetry, there might possibly be some significant relation between the latter and the Sperm-sphere.

In Table XXVIII. I. *c* and II. *c* will be found the distributions, and the correlations in Table XXXI. *a* and *b*.

When the eggs are close and their axes are horizontal the standard deviation is high, $\sigma = 40{\cdot}73° \pm 1{\cdot}77$, and practically as high as that found in the case of Sperm-entrance and Plane of Symmetry (Table XXVI. *c*). The correlation is negligible.

When, however, the eggs are spaced and their axes vertical the standard deviation is lower, the correlation coefficient higher, $\sigma = 32{\cdot}81° \pm 2{\cdot}11$, $\rho = {\cdot}188 \pm {\cdot}088$. Judged by both constants the relation of the Sperm-sphere Meridian to the Plane of Symmetry is less than that between the latter and the Sperm-entrance. There is however some slight connection between the two.

(6) The position, on the slide, of the point of entrance of the spermatozoon, or the relation of this point to (α) the direction of 'pressure,' parallel to the length of the slide, and (β) the direction, *across* the slide, of the gravitation plane of symmetry.

When the eggs are spaced and the axes horizontal, the spermatozoon enters at any point at random, as may be gathered from the frequency polygon in Fig. 6 II. (p. 159, above.)

When however the eggs compress one another in the longitudinal rows, the axes being horizontal, the spermatozoon enters mainly on those sides of the eggs which are in contact, as is evident from Table XXXIV. and Fig. 6 I. The frequencies are much higher about 90° than elsewhere. Further, while a few eggs may be found in which the sperm has entered about 180°, hardly a single one has entered about 0°.

It was towards this side that the white pole was turned up, and, as is known, the sperm does not enter in the vegetative hemisphere.

These values have been determined from the known value, in each egg, of the angle between the entrance point and the First Furrow, and by subtraction of the angle between Sperm-entrance and Plane of Symmetry. The position of the latter on the slide is known, and hence the point of entrance of the sperm. On the assumption that this point is opposite to the grey-crescent, angles may be distinguished from their supplements, a distinction which could not otherwise be made since what is actually measured is the angle between Sperm-entrance and First Furrow after the eggs have been removed from the slide. This assumption may or may not be justified. It might be better therefore to ignore the distinction and tabulate the frequencies as in Table XXXIV. *b*, where the distribution is about 90°. It may be worth while to point out that the standard deviation of this distribution ($\sigma = 44{\cdot}13° \pm 1{\cdot}87$) is nearly the same as those of the distributions of the First Furrow given in Table XIV., I. A and II. A.

TABLE XXXIV.

Sperm Entrance Meridian and Direction of "Pressure."
Eggs close. Axes horizontal.

Difference between angles and their Supplements.

a. Regarded.				*b.* Disregarded.	
+180—165	7	*– 0— 15*	0	*– 0—15*	7
165—150	7	*15— 30*	0	*15—30*	7
150—135	10	*30— 45*	0	*30—45*	10
135—120	15	*45— 60*	0	*45—60*	15
120—105	8	*60— 75*	6	*60—75*	14
105— 90	9	*75— 90*	8	*75—90*	17
90— 75	3	*90—105*	10	*+90—75*	13
75— 60	3	*105—120*	11	*75—60*	14
60— 45	2	*120—135*	8	*60—45*	10
45— 30	1	*135—150*	6	*45—30*	7
30— 15	1	*150—165*	5	*30—15*	6
15— 0	2	*165—180*	4	*15—0*	6
Total	68	Total	58	Total	126
M =	+118·46° ± 3·18		–114·32° ± 2·68		–85·83° ± 2·65
σ =	38·94° ± 2·25		30·27° ± 1·90		44·13° ± 1·87

But however that may be, it is clear that spermatozoa enter mainly where the eggs in the rows are in contact. This must be attributed, I think, to their congregating in greater numbers in these places, and this in turn to either capillarity, or to some chemotactic stimulus exerted in excess at these points by the jelly. It cannot, I believe, be assigned to any phototactic stimulus, since it is not observable in the case of the spaced eggs.

We have already established the facts that the First Furrow tends to place itself parallel to the length of the slide, and that it very frequently includes the point of entrance of the sperm. The position of the First Furrow in compressed eggs would then appear not to be determined by the pressure, which is slight, (certainly less than that used in ordinary pressure experiments), but to be due simply to the fact that the sperm prefers to enter in between the eggs in the rows.

There is no reason that I can see for supposing that gravity, which is not eliminated in this experiment, has any share in the result.

(7) The inclination of the Sperm-radius to the Egg-axis, or the angle subtended at the centre of the circle by the distance of the point of entrance from the animal pole. This angle is never greater than 90°.

There is naturally no distinction between positive and negative angles.

I. When the eggs are close, their axes horizontal (Table XXXV. I.), the mean value of this angle is 40·65°, that is, the sperm under these conditions usually enters about half-way between the animal pole and the equator. These eggs it

TABLE XXXV.

Inclination of Sperm-radius and Egg-axis, or angle subtended at the centre of the egg by the distance of the sperm-entrance-point from the animal pole.

I. Eggs close. Axes horizontal.		II. Eggs spaced. Axes vertical.	
90—75	5	*90—75*	27
75—60	9	*75—60*	23
60—45	42	*60—45*	5
45—30	35	*45—30*	0
30—15	15	*30—15*	0
15— 0	15	*15— 0*	0
Total	123	Total	55
M=40·65°		M=73·50°	

must be remembered are laid with the white hemisphere, into which the sperm cannot enter, on one side. While the axis lies across the slide the equator of the egg is parallel to its length. The mean point of entry is (Table XXXIV. *a*) 118° on one side and 114° on the other, or 28° and 24° from the equator respectively. This may account in part for the point of entry selected by the sperm.

II. When the eggs are spaced and their axes vertical, the point selected by the spermatozoon for entrance is near the equator (Table XXXV. II.), the mean value is 73·50°. The eggs were white pole downwards, and the sperm-containing water with which they were inseminated was placed all round the eggs but did not cover them. This affords a sufficient explanation of what occurs.

As we have already seen, the correlations between Sperm-entrance Meridian and Plane of Symmetry, between Plane of Symmetry and First Furrow, and between Sperm-sphere Meridian and Plane of Symmetry are all higher when "pressure" and "gravity" are not allowed to interfere, while that between the Sperm-entrance Meridian and the First Furrow is not quite as high in the absence as in the presence of these factors (Table XXXIII.). We might almost infer from this that the relations between these planes are independent of the distance of the Sperm-entrance from the animal pole.

An inspection of the correlation tables (Table XXXVI.) will show how far this inference is justified.

I have not calculated the coefficients, but the mean values of this angle (Inclination of Sperm-radius to Egg-axis) for each class of the several planes (the means are inserted at the bottom of the tables) will show at once that the correlation is to all intents and purposes nil (I. *a*, I. *b*, I. *c*), at least, when the eggs are close and their axes horizontal.

The correlation is just as poor in the case of those eggs that were spaced and had their axes vertically placed, II. *a*, II. *b*, II. *c*. Where however the angle between the Plane of Symmetry and the First Furrow is small there is a *slight* lowering in the value of the means of the other angle (II. *c*).

TABLE XXXVI.

Correlations between Inclination of Sperm-radius to Egg-axis and the several planes.

Inclination of Sperm-radius to Egg-axis.

II. *a.*

Sperm-entrance and First Furrow.

	0°					90°	Totals
0°	—	—	—	—	—	—	0
	—	—	—	—	—	—	0
	—	—	—	—	—	—	0
	3	2	—	—	—	—	5
	19	3	1	—	—	—	23
90°	23	—	—	1	1	2	27
Totals	45	5	1	1	1	2	55

Means 74·0° 61·5° 67·5° 82·5° 82·5° 82·5°

II. *b.*

Sperm-entrance and Plane of Symmetry.

	0°					90°	Totals
0°	—	—	—	—	—	—	0
	—	—	—	—	—	—	0
	—	—	—	—	—	—	0
	4	—	—	1	—	—	5
	14	4	3	—	1	1	23
90°	17	5	3	2	—	—	27
Totals	35	9	6	3	1	1	55

Means 73·0° 76·5° 75·0° 72·0° 67·5° 67·5°

II. *c.*

Plane of Symmetry and First Furrow.

	0°					90°	Totals
0°	—	—	—	—	—	—	0
	—	—	—	—	—	—	0
	—	—	—	—	—	—	0
	4	—	1	—	—	—	5
	16	4	1	1	1	—	23
90°	16	3	4	2	2	—	27
Totals	36	7	6	3	3	0	55

Means 71·0° 73·5° 75·0° 78·0° 78·0° 0

Inclination of Sperm-radius to Egg-axis.

I. *a.*

Sperm-entrance and First Furrow.

	0°					90°	Totals
0°	14	—	—	—	1	—	15
	12	—	—	1	2	—	15
	22	1	3	5	3	1	35
	22	8	5	4	2	1	42
	3	4	—	—	1	1	9
90°	2	1	1	1	—	—	5
Totals	75	14	9	11	9	3	121

Means 36·0° 58·5° 51·0° 46·5° 37·5° 52·5°

I. *b.*

Sperm-entrance and Plane of Symmetry.

	0°					90°	Totals
0°	5	2	2	2	1	2	14
	8	2	—	1	1	3	15
	14	6	5	4	1	5	35
	14	6	9	5	3	3	40
	1	5	1	2	—	—	9
90°	1	1	3	—	—	—	5
Totals	43	22	20	14	6	13	118

Means 37·5° 46·5° 42·0° 42·0° 37·5° 33·0°

I. *c.*

Plane of Symmetry and First Furrow.

	0°					90°	Totals
0°	5	1	2	2	1	3	14
	7	2	2	1	—	3	15
	15	5	2	3	2	8	35
	18	4	6	3	3	6	40
	4	1	1	1	—	2	9
90°	—	1	2	—	1	1	5
Totals	49	14	15	10	7	23	118

Means 40·5° 43·5° 45·0° 37·5° 46·5° 37·5°

(8) The Sperm-path and the First Furrow.

As explained above the Sperm-path (outer or "penetration" path) may be parallel to the First Furrow when it is not actually included in the latter, or when it makes an angle with the Furrow it may be turned towards or away from it.

In Table XXXVII. will be found the distribution of this angle in all cases, and in each of these three variations of position, both when the eggs are affected by gravity and pressure (I.) and when they are not (II.).

TABLE XXXVII.

Sperm-path and First Furrow.

Angle	I. Eggs close. Axes horizontal.				II. Eggs spaced. Axes vertical.			
	a. All cases	*b.* Towards furrow	*c.* Away from furrow	In or parallel to furrow	*a.* All cases	*b.* Towards furrow	*c.* Away from furrow	In or parallel to furrow
− *90—75*	0	0	0	—	0	0	0	—
75—60	1	1	0	—	0	0	0	—
60—45	0	0	0	—	2	2	0	—
45—30	4	4	0	—	0	0	0	—
30—15	11	9	2	—	5	2	3	—
15—0	42	11	9	43	26	7	13	11
+ *0—15*	41	11	9		17	1	11	
15—30	8	4	4	—	3	0	3	—
30—45	9	7	2	—	1	1	0	—
45—60	6	5	1	—	0	0	0	—
60—75	1	1	0	—	1	1	0	—
75—90	0	0	0	—	0	0	0	—
Totals	123	53	27	43	55	14	30	11
M	3·47° ± 1·31	4·85° ± 2·69	6·35° ± 2·29	—	− 2·04° ± 1·63	6·44° ± 5·20	0·62° ± 1·49	—
σ	21·47° ± 0·93	29·08° ± 1·90	17·66° ± 1·62	—	17·94° ± 1·15	28·89° ± 3·68	12·09° ± 1·05	—

It will be seen that the tendency of the Sperm-path to lie in or parallel to the First Furrow is great, and greater when the external disturbing influences are removed, $\sigma = 17{\cdot}94^\circ \pm 1{\cdot}15$ as opposed to $\sigma = 21{\cdot}47^\circ \pm 0{\cdot}93$, when gravity and pressure act upon the egg. It will be noticed however that in any case the value of the standard deviation is less for this angle than for that between Sperm-entrance Meridian and First Furrow (Table XXXII.). This is due to the fact that while the angle between Sperm-path and First Furrow cannot be greater than that between Sperm-entrance Meridian and First Furrow, it may, of course, be less. Also the path may be parallel to the Furrow when the Entrance Meridian makes a considerable angle with it (Table XL.).

Further, when the path is turned away from the Furrow the standard deviation diminishes, when it is turned towards the Furrow its value increases. It is in the former cases that the inner or "copulation" path is generally seen, parallel to the Furrow, but inclined to the "penetration" path (Fig. 5 C).

It might be supposed that when the Sperm-path is turned away from the Furrow, the relations between Sperm-entrance and Sperm-sphere Meridian, on the one hand, and Plane of Symmetry and First Furrow on the other would be less close than when it is turned towards it. This however (to accept the standard deviation as a criterion) is not the case, as may be gathered at once from Tables XXXVIII. and XXXIX. Whether the eggs are close and under the influence of gravity, or not, the standard deviation, in the case of each of the angles considered, except that between the Sperm-sphere Meridian and the Plane of Symmetry, is considerably less in the former than in the latter contingency. The numbers of course are very small, but all the results point in the same direction.

When the path is parallel to or in the Furrow (Table XL.) the angles made by Sperm-sphere and Sperm-entrance Meridians with the Furrow are of course nil. The standard deviations of the other two are greater than when the path turns away from the First Furrow when the eggs are under the influence of gravity and pressure, about the same when they are not.

Nevertheless I am bound to point out that if the relation between Sperm-entrance and Symmetry Plane and First Furrow be measured by the correlation coefficient, instead of by the standard deviation (the results are given in Tables XLI. and XLII.), it is closer in those eggs in which the Sperm-path is turned towards the Furrow than it is in the whole series.

This may be readily seen from the values of ρ in Table XXXIII., first and second columns. Yet the corresponding standard deviations (Table XXXII., second and third columns, first and third rows) are also greater, instead of less, when the path is inclined towards the Furrow than in the whole series. This shows plainly enough the difficulty of interpreting results based on such small numbers.

(9) Sperm-path and Plane of Symmetry.

At present we have discovered no close correlation between the Plane of Symmetry and any one point of the Sperm-path. There is however a correlation—if calculations based on such small numbers can be trusted—between the angle made by the Sperm-path with the First Furrow and the deviation of the latter from the Plane of Symmetry, at least when the eggs are not subjected to the influence of gravity and pressure.

When the eggs are close and their axes horizontal there is no correlation (Table XLIII. *a*).

TABLE XXXVIII.

When the Sperm-path is turned towards the First Furrow.

I. Eggs close. Axes horizontal.

Angle	Sperm-entrance Meridian and First Furrow	Sperm-entrance Meridian and Plane of Symmetry	Plane of Symmetry and First Furrow	Sperm-sphere Meridian and Plane of Symmetry
− 90—75	1	1	7	1
75—60	3	1	3	0
60—45	5	5	3	5
45—30	4	8	4	6
30—15	7	7	2	7
15—0	7	7	5	9
+ *0—15*	4	2	11	4
15—30	4	8	3	8
30—45	4	4	4	5
45—60	6	4	2	3
60—75	6	2	2	2
75—90	2	2	5	1
Totals	53	51	51	51
M	+ 4·67° ± 4·18	− 2·82° ± 3·83	− 4·23 ± 4·80	− 0·44° ± 3·46
σ	45·15° ± 2·96	40·53° ± 2·71	50·87 ± 3·39	36·66° ± 2·45

II. Eggs spaced. Axes vertical.

Angle	Sperm-entrance Meridian and First Furrow	Sperm-entrance Meridian and Plane of Symmetry	Plane of Symmetry and First Furrow	Sperm-sphere Meridian and Plane of Symmetry
− 90—75	1	0	0	1
75—60	1	0	1	0
60—45	0	1	1	1
45—30	1	1	1	3
30—15	4	2	1	1
15—0	4	3	5	2
+ *0—15*	1	4	1	1
15—30	0	1	0	3
30—45	0	0	2	1
45—60	1	1	0	1
60—75	0	0	2	0
75—90	1	1	0	0
Totals	14	14	14	14
M	− 11·35° ± 7·26	− 0·36° ± 6·01	− 1·07 ± 5·90	− 8·57° ± 6·70
σ	40·26° ± 5·13	33·33° ± 4·25	32·67 ± 4·18	37·19° ± 4·74

TABLE XXXIX.

When the Sperm-path is turned away from the First Furrow.

I. Eggs close. Axes horizontal.

Angle	Sperm-entrance Meridian and First Furrow	Sperm-entrance Meridian and Plane of Symmetry	Plane of Symmetry and First Furrow	Sperm-sphere Meridian and Plane of Symmetry
−90—75	0	2	2	1
75—60	0	0	0	0
60—45	0	1	1	0
45—30	0	3	3	2
30—15	0	2	2	3
15—0	13	8	8	7
+ 0—15	13	6	5	4
15—30	1	2	4	3
30—45	0	1	0	3
45—60	0	1	1	1
60—75	0	0	0	0
75—90	0	0	0	2
Totals	27	26	26	26
M	0·84° ± 1·11	− 9·81° ± 4·11	− 2·31° ± 4·07	− 1·73° ± 4·72
σ	8·51° ± 0·78	31·05° ± 2·91	30·77° ± 2·88	35·67° ± 3·34

II. Eggs spaced. Axes vertical.

Angle	Sperm-entrance Meridian and First Furrow	Sperm-entrance Meridian and Plane of Symmetry	Plane of Symmetry and First Furrow	Sperm-sphere Meridian and Plane of Symmetry
−90—75	0	0	0	1
75—60	0	0	0	0
60—45	0	1	0	1
45—30	0	1	0	3
30—15	1	2	2	1
15—0	16	9	12	5
+ 0—15	13	11	10	8
15—30	0	3	4	4
30—45	0	1	0	4
45—60	0	1	2	2
60—75	0	1	0	0
75—90	0	0	0	1
Totals	30	30	30	30
M	−1·50° ± 1·02	− 3·49° ± 2·86	4·50° ± 2·23	− 6·00° ± 4·10
σ	8·31° ± 0·72	23·22° ± 2·02	18·12° ± 1·58	33·32° ± 2·90

TABLE XL. *When the Sperm-path is either included in or is parallel to the First Furrow.*

Angle	Eggs close. Axes horizontal		Eggs spaced. Axes vertical	
	Sperm-entrance Meridian and Plane of Symmetry	Plane of Symmetry and First Furrow	Sperm-entrance Meridian and Plane of Symmetry	Plane of Symmetry and First Furrow
− 90—75	2	2	0	0
75—60	0	0	0	0
60—45	1	1	0	0
45—30	1	2	1	1
30—15	2	2	0	0
15—0	10	9	3	3
+ 0—15	10	11	5	5
15—30	2	2	0	0
30—45	3	2	2	2
45—60	2	2	0	0
60—75	3	2	0	0
75—90	7	8	0	0
Totals	43	43	11	11
M	17·27° ± 4·41	16·22° ± 4·52	4·77 ± 4·07	4·77 ± 4·07
σ	42·90° ± 3·12	43·91° ± 3·20	20·01 ± 2·88	20·01 ± 2·88

TABLE XLI. *Correlation between Sperm-entrance Meridian and First Furrow when the Sperm-path is directed towards the First Furrow.*

Eggs spaced. Axes vertical.

Sperm-entrance Meridian and Plane of Symmetry.

First Furrow and Plane of Symmetry.

	−						+						Totals
	90						0					90	
90	—	—	—	—	—	—	—	—	—	—	—	—	0
	—	—	—	—	—	—	1	—	—	—	—	—	1
−	—	—	—	—	—	—	1	—	—	—	—	—	1
	—	—	—	—	1	—	—	—	—	—	—	—	1
	—	—	—	—	1	—	—	—	—	—	—	—	1
	—	—	—	1	—	2	1	1	—	—	—	—	5
0	—	—	—	—	—	1	—	—	—	—	—	—	1
	—	—	—	—	—	—	—	—	—	—	—	—	0
+	—	—	1	—	—	—	—	—	—	1	—	—	2
	—	—	—	—	—	—	—	—	—	—	—	—	0
	—	—	—	—	—	—	1	—	—	—	—	1	2
90	—	—	—	—	—	—	—	—	—	—	—	—	0
Totals	0	0	1	1	2	3	4	1	0	1	0	1	14

$\rho = \cdot 502 \pm \cdot 135.$

TABLE XLII.

Correlation between Sperm-entrance Meridian and Plane of Symmetry when the Sperm-path is directed towards the First Furrow.

Eggs spaced. Axes vertical.

Sperm-entrance Meridian and First Furrow.

First Furrow and Plane of Symmetry.	−						+						Totals
	90						0					90	
90	—	—	—	—	—	—	—	—	—	—	—	—	0
	·5	—	—	—	—	—	—	—	—	—	—	·5	1
−	—	1	—	—	—	—	—	—	—	—	—	—	1
	—	—	—	—	1	—	—	—	—	—	—	—	1
	—	—	—	—	—	1	—	—	—	—	—	—	1
	—	—	—	1	2	2	—	—	—	—	—	—	5
0	—	—	—	—	—	—	1	—	—	—	—	—	1
	—	—	—	—	—	—	—	—	—	—	—	—	0
+	·5	—	—	—	1	—	—	—	—	—	—	·5	2
	—	—	—	—	—	—	—	—	—	—	—	—	0
	—	—	—	—	—	1	—	—	—	1	—	—	2
90	—	—	—	—	—	—	—	—	—	—	—	—	0
Totals	1	1	0	1	4	4	1	0	0	1	0	1	14

$\rho = \cdot 411 \pm \cdot 148.$

TABLE XLIII.

Correlation between Sperm-path and Plane of Symmetry.

XLIII. *a.* *All cases.*

Eggs close. Axes horizontal.

Plane of Symmetry and First Furrow.

Sperm-path and First Furrow.	−						+						Totals
	90						0					90	
90	—	—	—	—	—	—	—	—	—	—	—	—	0
	—	—	—	—	—	1	—	—	—	—	—	—	1
	—	—	—	—	—	—	—	—	—	—	—	—	0
−	2	—	—	—	—	—	—	—	1	—	—	1	4
	1	1	3	2	—	2	—	—	—	1	—	1	11
	1	1	1	4	4	9	11	2	1	2	2	4	42
0	2	—	1	2	2	8	9	4	1	1	2	6	38
	3	—	—	—	—	1	3	1	—	—	—	—	8
+	—	—	—	1	—	1	3	1	2	—	—	1	9
	2	—	—	—	—	—	1	1	1	1	—	—	6
	—	1	—	—	—	—	—	—	—	—	—	—	1
90	—	—	—	—	—	—	—	—	—	—	—	—	0
Totals	11	3	5	9	6	22	27	9	6	5	4	13	120

$\rho = \cdot 030 \pm \cdot 061.$

XLIII. *b.* *When only those cases are considered in which the Sperm-path is directed towards the Egg-axis and First Furrow.*

Plane of Symmetry and First Furrow.

Sperm-path and First Furrow.	−						+						Totals
	90°						0°					90°	
90°	—	—	—	—	—	—	—	—	—	—	—	—	0
	—	—	—	—	—	1	—	—	—	—	—	—	1
	—	—	—	—	—	—	—	—	—	—	—	—	0
−	2	—	—	—	—	—	—	—	1	—	—	1	4
	1	1	3	2	—	—	—	—	—	1	—	1	9
	—	1	—	2	1	2	3	—	—	1	1	—	11
0°	—	—	—	—	1	2	2	1	—	—	1	2	9
	2	—	—	—	—	—	2	—	—	—	—	—	4
+	—	—	—	—	—	—	3	1	2	—	—	1	7
	2	—	—	—	—	—	1	1	1	—	—	—	5
	—	1	—	—	—	—	—	—	—	—	—	—	1
90°	—	—	—	—	—	—	—	—	—	—	—	—	0
Totals	7	3	3	4	2	5	11	3	4	2	2	5	51

$\rho = {\cdot}086 \pm {\cdot}094.$

TABLE XLIV.

Correlation between Sperm-path and Plane of Symmetry.

XLIV. *a.* *All cases.*

Plane of Symmetry and First Furrow.

Sperm-path and First Furrow.	−						+						Totals
	90°						0°					90°	
90°	—	—	—	—	—	—	—	—	—	—	—	—	0
	—	—	—	—	—	—	—	—	—	—	—	—	0
	—	1	1	—	—	—	—	—	—	—	—	—	2
−	—	—	—	—	—	—	—	—	—	—	—	—	0
	—	—	—	—	—	4	1	—	—	—	—	—	5
	—	—	—	2	1	11	6	2	1	2	1	—	26
0°	—	—	—	—	1	3	9	2	2	—	—	—	17
	—	—	—	—	1	2	—	—	—	—	—	—	3
+	—	—	—	—	—	—	—	—	—	—	1	—	1
	—	—	—	—	—	—	—	—	—	—	—	—	0
	—	—	—	—	—	—	—	—	1	—	—	—	1
90°	—	—	—	—	—	—	—	—	—	—	—	—	0
Totals	0	1	1	2	3	20	16	4	4	2	2	0	55

$\rho = {\cdot}479 \pm {\cdot}070.$

XLIV. *b.* *When only those cases are considered in which the Sperm-path is either directed towards or is parallel to the First Furrow.*

Eggs spaced. Axes vertical.

Plane of Symmetry and First Furrow.

Sperm-path and First Furrow.	−						+						Totals
	90						0					90	
90	—	—	—	—	—	—	—	—	—	—	—	—	0
	—	—	—	—	—	—	—	—	—	—	—	—	0
−	—	1	1	—	—	—	—	—	—	—	—	—	2
	—	—	—	—	—	—	—	—	—	—	—	—	0
	—	—	—	—	—	2	—	—	—	—	—	—	2
	—	—	—	2	1	6	2	—	1	—	1	—	13
0	—	—	—	—	—	—	4	—	2	—	—	—	6
	—	—	—	—	—	—	—	—	—	—	—	—	0
+	—	—	—	—	—	—	—	—	—	—	1	—	1
	—	—	—	—	—	—	—	—	—	—	—	—	0
	—	—	—	—	—	—	—	—	1	—	—	—	1
90	—	—	—	—	—	—	—	—	—	—	—	—	0
Totals	0	1	1	2	1	8	6	0	4	0	2	0	25

$\rho = \cdot 725 \pm \cdot 064$.

XLIV. *c.* *Only those cases in which the path is turned towards the Furrow.*

Eggs spaced. Axes vertical.

Plane of Symmetry and First Furrow.

Sperm-path and First Furrow.	−						+						Totals
	90						0					90	
90	—	—	—	—	—	—	—	—	—	—	—	—	0
	—	—	—	—	—	—	—	—	—	—	—	—	0
−	—	1	1	—	—	—	—	—	—	—	—	—	2
	—	—	—	—	—	—	—	—	—	—	—	—	0
	—	—	—	—	—	2	—	—	—	—	—	—	2
	—	—	—	1	1	3	—	—	1	—	1	—	7
0	—	—	—	—	—	—	1	—	—	—	—	—	1
	—	—	—	—	—	—	—	—	—	—	—	—	0
+	—	—	—	—	—	—	—	—	—	—	1	—	1
	—	—	—	—	—	—	—	—	—	—	—	—	0
	—	—	—	—	—	—	—	—	1	—	—	—	1
90	—	—	—	—	—	—	—	—	—	—	—	—	0
Totals	0	1	1	1	1	5	1	0	2	0	2	0	14

$\rho = \cdot 880 \pm \cdot 040$.

Under the contrary conditions the value of the coefficient is $\rho = \cdot 479 \pm \cdot 070$ (Table XLIV. *a*) which rises to ·725 ± ·064, and ·880 ± ·040, as first those instances are excluded in which the path turns away from the Furrow (Table XLIV. *b*) and then those also in which the path is parallel to the Furrow or in it (Table XLIV. *c*).

It may be noted that, when the eggs are close and their axes horizontal, there is still no correlation even under what appear to be the most favourable circumstances (Table XLIII. *b*).

I am well aware of the doubtful value of a correlation determined on so very few examples, but, if the results can be accepted, it is evident that when the path makes a wide angle with the Furrow, the deviation of the grey crescent from the Furrow is large too.

More direct evidence can, however, also be brought forward in support of this conclusion, for from the known values of the angle between Sperm-path and First Furrow, and First Furrow and Plane of Symmetry, the values of the angles between Sperm-path and Plane of Symmetry may be obtained. The results are given in Table XLV. in which the standard deviations are shown (the difference between negative and positive angles being neglected). In the same table the values of σ, calculated in the same way, for the angle between Sperm-path and First Furrow are given for comparison.

It is evident at once that when all cases are considered the relation of Sperm-path to First Furrow is as close as its relation to Symmetry Plane, if not closer; that when the Sperm-path is included in or parallel to the First Furrow its relation to the Symmetry Plane is as in all the cases taken together; that when the path is turned away from the Furrow its relation to the Furrow is much closer than to the Plane of Symmetry, but that when it is turned towards the Furrow the reverse is the case.

If, however, the numbers are considered to be so small that this result must be rejected, then it must be confessed that no internal factor has yet been discovered by which the direction occupied by the plane of the grey crescent is conditioned; nor am I able to see by what known internal factor it could be conditioned since, as far as I am aware, every possible cause has been tried. Provisionally, therefore, we must regard the direction of the whole ("penetration") Sperm-path as the cause which determines, in the absence of all outside interference, the Plane of Symmetry of the unsegmented egg, and so the median plane of the embryo.

The foregoing investigations have established a very close relation between the point of entrance of the spermatozoon, or the path taken by the spermatozoon in the egg, and the position of the First Furrow of segmentation. The relation is closest when the influence of gravity is removed by placing the eggs upon the slides with their axes vertical and their white poles below, but it still exists in a degree which cannot be ignored when the axes are horizontal; indeed, if it be measured by the correlation coefficient instead of by the standard deviation it is as

TABLE XLV.

Eggs spaced. Axes vertical.

All cases.

	Angle between Sperm-path and Plane of Symmetry	Angle between Sperm-path and First Furrow
0—15	33	43
15—30	13	8
30—45	7	1
45—60	1	2
60—75	1	1
Totals	55	55
	$\sigma = 13{\cdot}56^\circ \pm 0{\cdot}87^\circ$.	$\sigma = 12{\cdot}58^\circ \pm 0{\cdot}81^\circ$.

When Sperm-path turns away from Furrow.

0—15	17	24
15—30	9	6
30—45	4	0
Totals	30	30
	$\sigma = 10{\cdot}72^\circ \pm 0{\cdot}94$.	$\sigma = 6{\cdot}00^\circ \pm 0{\cdot}53^\circ$.

When Sperm-path is included in or parallel to First Furrow.

0—15	8	
15—30	0	
30—45	3	
Total	11	
	$\sigma = 13{\cdot}35^\circ \pm 1{\cdot}92$.	$\sigma = 0^\circ$.

When Sperm-path is turned towards Furrow.

0—15	8	8
15—30	4	2
30—45	0	1
45—60	1	2
60—75	1	1
Totals	14	14
	$\sigma = 10{\cdot}16^\circ \pm 1{\cdot}29$.	$\sigma = 19{\cdot}65^\circ \pm 2{\cdot}50$.

close in the latter as in the former case, if not closer. This is due to the fact that there is a slight tendency for the First Furrow to place itself at right angles to the gravitation plane, that is, parallel to the length of the slide. Since, moreover, the spermatozoa enter the egg mainly from the compressed sides, there is already a considerable tendency for the First Furrow to lie in that direction, and under the conditions of the experiment the two are summed. The pressure, as such, seems to have little influence.

In certain cases the Sperm-path is inclined away from the Furrow, none the less its inner end bends upon its outer end, and is found to be in, or parallel to the cell division.

There is on the other hand but a very slight relation between the meridian occupied by either the entrance point or the inner end of the Sperm-path and the Plane of Symmetry, even under the most favourable conditions, and when the position of the Symmetry Plane is altered, as we know it can be altered, by gravitation, the correlation is negligible. A correlation of considerable magnitude has, however, been found between the whole ("penetration") path of the sperm and the position of the Plane of Symmetry, and since the inclination of the Sperm-radius and Sperm-path have, by themselves, but little effect, the only possible conclusion is that if the position of the grey crescent is dependent upon the spermatozoon at all, it must be determined by all the meridians successively occupied by the sperm as it travels from the surface into the interior.

In this case it is evident that while gravity improves the relation between Sperm-path and First Furrow it utterly destroys the relation between Sperm-path and Plane of Symmetry. This is in exact accord with the results previously obtained. For it was shown that while gravity, during the short interval in which it operated, could very largely determine the position of the Plane of Symmetry, it had but little influence on the direction of the Furrow; while there was a considerable tendency for the Plane of Symmetry to lie in the Gravitation Plane, there was but a slight tendency for the Furrow to be situated not in but at right angles to that plane.

The internal factors which determine the direction of the deviation of cell division must be therefore distinct from those which determine the symmetry of the egg, and later, of the embryo.

Let us briefly enquire what the mechanisms may be by which these two different processes are brought about.

As far as the division of the fertilized ovum is concerned, it is not hard to understand why the Furrow should include the Sperm-entrance and path, or at least be parallel to the inner portion of the latter. The cell-division takes place in the equator of the fertilization spindle, where the spindle fibres thicken to form a cell-plate after the chromosomes have passed to opposite poles; this equatorial plane is of course at right angles to the spindle axis—the line joining the two centrosomes, or, at an earlier stage, the direction of separation of the two halves of the divided sperm-centrosome (Fig. 4 D).

This line lies in a plane parallel to the egg equator (probably owing to the manner of distribution of the yolk about the axis) and is at right angles to that uniting the male and female pronuclei, or "copulation" path of Roux.

The first part of the path ("penetration" path of Roux) is simply the entrance funnel; its formation is probably a capillary phenomenon, attributable to the aggregation about the acrosome of a watery substance withdrawn from the egg cytoplasm, and to the streaming of this substance into the interior of the egg (Fig. 4 A). The spermatozoon or sperm nucleus previously carried in by this movement then turns towards the female pronucleus, and this second portion of the path makes an angle with the first. Should the first portion have been directed towards the axis, and should the female pronucleus, as is usually the case, also lie in the axis, then both parts will lie in one and the same meridional plane; this becomes the plane of the First Furrow. But should the first part be inclined away from the axis, only the second will lie in a meridional plane, which becomes the plane of division for the same reasons. Should the divergence of the first part from the axis not have been great (in only three cases out of fifty-seven has it been found to exceed 30°, and in only one of these to be greater than 45°) the First Furrow may still practically include the point of entrance of the sperm. But there are cases in which the First Furrow deviates widely from the Sperm-path, even when the latter is directed towards it. The reason for this may very possibly be that the female pronucleus has not moved axially, but exaxially in its return from the animal pole; the meridians of Sperm-entrance and "copulation" path, or in other words the meridians of Sperm-entrance and First Furrow, would then be more or less widely separate from one another. I have, however, no direct evidence that this is what has occurred; but Roux states that in eggs taken at the end of the breeding season—after a long stay in the uterus—the female pronucleus is exaxial. The dates on which my eggs were taken were, for Series I. (close, horizontal), March 30th, for Series II. (spaced, vertical), April 4th. In Oxford the breeding-season ends usually in the third week of April.

And now let us turn to the Plane of Symmetry. This is brought about by the formation of the grey crescent, and this is due, according to Roux, to the disappearance into the interior of some of the superficial pigment of the egg. The pigment disappears over a crescentic region at the border of the pigmented area on one side of the egg. When all the pigment has gone, the crescent is white and indistinguishable in colour from the original circular white area, though its position may of course be readily detected (Fig. 7). I do not, of course, know, but I cannot help supposing that this retreat of the pigment into the inside is directly due to the streaming movement set up in the egg-cytoplasm by the sperm.

It is known from cytological descriptions that the entrance-funnel is an accumulation of a clear substance; this takes place about the apical body or acrosome of the spermatozoon as soon as the latter gets inside the surface membrane of the egg; it looks as though the clear substance were more watery than the surrounding cytoplasm, as though it were due to a local abstraction of water. Again, at the bottom of the entrance-funnel the Sperm-sphere, surrounded later by the Sperm-aster, is formed, and this consists of a yolk-free area of cytoplasm (Fig. 5 C) in which later on large vacuoles of a clear, watery looking substance, become evident. Some of these vacuoles can be seen in the Sperm-sphere of the Axolotl figured in Fig. 5 D, and I have observed a precisely similar appearance on the inner side of the Sperm-nucleus in the Frog's egg. It may be supposed in like manner that these vacuoles are due to a local abstraction of water from the cytoplasm.

On this view the acrosome and the middle piece are both hygroscopic particles, and it is interesting to remember that they are both of centrosomal origin, the one being derived from the centrosphere, the other from the centrosome of the spermatid.

This hypothesis is, it seems to me, very much strengthened by the result of a simple experiment. By placing a hygroscopic particle—a small crystal of salt or sugar for example—in a drop of the yolk of a Hen's egg, an imitation Sperm-sphere may very easily be made. A clear area from which the yolk granules have been driven away appears round the particle, and as the latter begins to dissolve it soon comes to lie in a little pool or vacuole of its own solution.

Should this interpretation of the processes observed in the egg during fertilization be correct, it would seem obvious to connect the immigration of the pigment with the streaming movements of watery substance so set up, the focus towards which the streams are directed being in that case the whole (" penetration ")

Sperm-path—the entrance-funnel and Sperm-sphere. It is with this that the direction of the Symmetry Plane appears to be correlated when gravity is not permitted to exert its influence. When gravity does operate the correlation disappears, and intelligibly so. For the movements due to the sinking of the yolk, the rising of the cytoplasm and pigment which we have known to be produced ever since Born described them in the permanently inverted eggs of Pflüger's experiment, will evidently alter the direction of the other movement directed

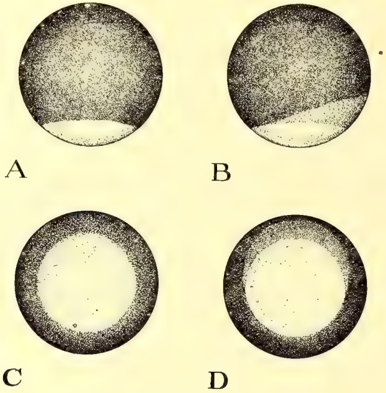

Fig. 7. The formation of the grey crescent.

In A and C it has not yet appeared.

A, B, the egg seen from the side. C, D, from the vegetative pole (centre of the white area).

towards the Sperm-path, while from what has been ascertained of the behaviour of chromatophores in the presence of light, we may also understand why this agent should modify the position of the grey crescent.

The union of the pronuclei is, however, a process independent of the direction of first entrance of the sperm, and of the formation of the Sperm-sphere, in which the vacuoles are developed before the two nuclei meet. Here then are two perfectly distinct processes, on one of which the direction of cell division depends, while the other determines the symmetry of the egg, with which is correlated the symmetry of the embryo. One of these can be readily affected by gravity and by light, the other cannot. But when all external disturbing influences are removed, when the Sperm-path is radial and the egg-nucleus is axial, then the causes which determine the symmetry of segmentation and those on which the symmetry of egg and embryo depend will be coincident.

Experimentally however they may be separated from one another. This is, indeed, no new discovery for the Frog's egg, for Pflüger showed that in forcibly inverted ova, while the First Furrow might make any angle with the plane including the egg-axis and the vertical (the "streaming meridian" of Born, or, as I have called it, the gravitation symmetry plane), the dorsal lip of the blastopore always appeared in this plane and on the side on which the white pole was turned up.

What is, I believe, new, is that the thirty minutes that pass before the egg turns over are sufficient for gravity to do its work. And this, I may remark, will explain the controversy that has been carried on for so long between Roux, Morgan and Kathariner on the one side, and Schulze and Moszcowski on the other. Roux maintained that the position of the First Furrow was determined not by gravity, but by causes residing in the structure of the egg. Schulze replied that gravity was a necessity for normal development, while Moszcowski claimed that the grey crescent, on which, as admitted and asserted by Roux, the position of the First Furrow, and later of the Sagittal Plane, depended, was brought about by gravity, quite a short time being amply sufficient for the purpose. It may be regarded as quite certain that the eggs kept in any constant state of slow rotatory motion (by being placed on a vertical wheel or in any other way) will produce normal tadpoles; and with regard to the second point, Morgan and Kathariner have shown that eggs kept in motion during insemination and later, still develope the grey crescent. This experiment is paralleled by placing the eggs on the slide with the axis vertical; in them the crescent appears. But it is interesting to notice that though gravity is certainly not indispensable, it may still modify the position of the Symmetry Plane.

The grey crescent of the Frog's egg is no solitary example of the change of structure that may be brought about by fertilization. The same conversion of a radial into a bilateral symmetry may be observed in other cases, and the same determination of the First Furrow by the point of entrance of the spermatozoon.

In the Sea-urchin *Toxopneustes* Wilson and Matthews were able to show that after the extrusion of the polar bodies at the animal pole of what may be called the primary egg-axis, the female pronucleus wandered in the cytoplasm to any position till met by the male pronucleus. The segmentation nucleus formed of their union then took up an excentric position near the centre, so determining a new egg-axis and a new animal pole. This axis became the axis of cleavage, the meridian of the First Furrow being determined by the point of entrance of the spermatozoon.

In the Ascidian *Cynthia* according to Conklin, the immature oocyte has a radial symmetry about an axis determined by the excentric position of the germinal vesicle; there is a peripheral layer of yellow cytoplasm surrounding a central grey yolk. Upon the entrance of the spermatozoon near the vegetative pole the nucleus breaks down and from it a clear substance is formed. Some of this remains at the animal pole as a small patch containing the female pronucleus; the rest flows, with the peripheral yellow substance, to the vegetative pole, where it lies in a shallow layer over the latter.

The symmetry is still radial. It soon, however, becomes bilateral, for the yellow and clear substances stream up on one side, the future posterior side, to the equator of the egg, the sperm being carried along in the latter to meet the female nucleus, which has meanwhile descended from the animal pole. The two together pass into the centre, along with the clear substance, and the fertilization spindle is formed at right angles to their path. The First Furrow, and later the Sagittal Plane, lie therefore in the plane of egg symmetry which is established during fertilization.

Again, in the Mollusc *Dentalium* we know from Wilson's account that when the sperm enters, the centrally placed germinal vesicle breaks down and its substance becomes confluent with a small clear area at the animal pole, a larger clear area at the vegetative pole. Later the brick-red yolk is interposed between animal and vegetative hyaline areas, the latter being soon extruded as the polar lobe.

A few other instances may be cited. In the Nematode *Diplogaster* it is the point of union of the two pronuclei which determines which end of the elliptical egg—the anterior ovarian end at which the polar bodies are formed, or the posterior oviducal end at which the sperm enters—shall become the animal and ectodermal end (Ziegler). In another Nematode *Strongylus* Wandolleck has shown that the yolk, originally uniformly distributed, becomes placed at one end, and the egg therefore telolecithal when fertilization occurs. Similarly in Cirrhipedes (Groom), in the Snail *Physa* (Kostanecki and Wierzejski), and in Turbellarians (Lang). In Ctenophora in the same way the micromere-forming substance becomes aggregated at one pole (Agassiz), and in Teleostei the blastodisc is formed by the local accumulation of the periblast. Lastly may be mentioned the polar rings of Leeches and Oligochaets (Whitman, Vejdovský, Foot).

The phenomenon is therefore of *fairly* widespread occurrence, though it must be remembered that there are cases in which the bilateral symmetry of the egg is fixed, apparently unalterably, in the unfertilized egg, for instance, in Cephalopods and many Insects, and secondly, that in parthenogenesis, the determination of symmetry, if not pre-existent, must be due to other causes.

But however the definite arrangement of cytoplasmic substances, on which the bilateral symmetry of the egg depends, may be brought about, modern experimental research has made it increasingly clear that it is upon these substances that at any rate the first differentiation of the embryo depends. Thus, to select only one or two examples, the Plane of Symmetry in the Frog becomes the Sagittal Plane of the embryo, in the absence of external interferences, the animal pole being approximately at the anterior end, the grey crescent on the dorsal side.

In *Dentalium* the polar lobe contains the material for the apical organ and trunk region of the larva; removal of the lobe entails the absence of both these structures.

Derangement of the micromeres brings about a multiplication of the sense organ of the Ctenophor embryo (Fischel); an isolated vegetative blastomere of a Nemertine will give rise to a larva with an archenteron but without an apical organ, an animal blastomere to one with an apical organ but no archenteron (Zeleny), and in Sea-urchins vegetative blastomeres gastrulate more readily than do animal cells (Driesch).

By the removal of one of the two posterior cells—containing the yellow stuff—of *Cynthia*, an embryo is produced which is complete anteriorly, with nerve plate, notochord and anterior mesenchyme, but possesses posterior mesenchyme and muscles upon one side only.

The substance which in these cases is necessarily associated with the formation of some particular organ, need not be, however, originally present in what will become its ultimate situation. Thus the stuff for the apical organ of *Dentalium* is at first situated in the polar lobe, subsequently migrating, between the first and second divisions, into the animal hemisphere. Though the organ-forming substances may be said to be in a sense preformed, they are not therefore necessarily prelocalized.

Speaking generally, a very definite relation may be noticed between the symmetry of the egg and that of the embryo, even where experiment has not, as yet, shown the necessary connection between this or that substance, and this or that organ. In a very large number of cases the animal pole is anterior, while the blastopore closes at the vegetative pole (Turbellarians, Annelids, Mollusca other than Cephalopods). In Cephalopods the animal pole is dorsal, the more convex side of the bilateral egg anterior, and a similar relation obtains in many Insect ova. In *Cynthia* the animal pole is ventral, the yellow side posterior. In *Ascaris megalocephala*, the plane in which the first four cells all lie is sagittal, while the vegetative germ-cell slips round to what will be the posterior side. The relation between the cytoplasmic organization of the egg and the symmetry of the future embryo may be taken then to be a well established fact. It does not, however, at all follow that the symmetry of segmentation will necessarily coincide with these other two symmetries.

According to the terms of Roux's original "Mosaik-Theorie" the process of cell division in a segmenting ovum was qualitative, not, however, entirely because the parts of the cytoplasm were unlike, for the cytoplasm was imagined to be practically "isotropic" or equipotential in all its parts, but because the dissimilar units or determinants of the nucleus were supposed to be gradually sundered from one another by successive divisions. Later on, of course, these nuclear determinants were held to incite in the cytoplasm to which they were distributed the various processes of differentiation.

This hypothesis of qualitative nuclear division has, on a number of grounds which need not now be recalled, been shown to be untenable, and has been given up by its author himself. It may, however, well be urged that since the "anisotropy" of the cytoplasm has now been demonstrated, cell, though not nuclear division is still the qualitative process of the "Mosaik-Theorie," that it is still an essential factor in differentiation.

But even this view must be abandoned.

In the Frog the First Furrow bears no necessary relation either to the Plane of Symmetry or to the Sagittal Plane. When outside interference is removed it is true that the tendency of all three towards coincidence increases, but by means of these same external agents it is possible experimentally to separate those internal factors which determine cell division from those which decide which meridian of the egg shall be occupied by the embryo. Further, by means of pressure (O. Hertwig, Born) the character of segmentation may be altered, but the embryos are nevertheless completely normal.

In the Newt (Spemann) the First Furrow is sometimes in the Sagittal, sometimes in the Transverse Plane.

In Teleostei (Morgan) the First Furrow may make any angle with the median plane of the embryo.

In Sea-urchins it is not easy to say what the exact relation of embryonic and egg-axes may be, since the egg is colourless and almost isolecithal, and for that reason impossible to orient except either by the position of the germinal vesicle and polar bodies or by that of the segmentation axis (intersection of the first two divisions). As we have seen, these two axes do not coincide, and it is not certainly known which determines the embryonic axis, though it has been stated that the First Furrow lies in the Sagittal Plane.

In *Strongylocentrotus lividus*, however, the egg has a ring of pigment subequatorial in position (the axis as determined by the point of extrusion of the polar bodies coincides with the axis of segmentation). In ordinary development this pigment passes first into the macromeres and then into the archenteron, and Boveri further states that if the order of segmentation be deranged by pressure or by stretching this pigment ring plays the same part as in normal development. Garbowski has, however, asserted that the ring need not occupy the position assigned to it by Boveri: it may be wholly in the animal hemisphere, or wholly in the vegetative hemisphere, or oblique to the axis. This being so we are compelled to suppose either that the processes of differentiation are indifferent to the previous order of segmentation, or that the pigment is not the organ-forming (archenteric) substance which Boveri imagined it to be.

There are other experiments on Echinoderms, however (due to Driesch), which prove that abnormalities in segmentation do not prejudice the complete normality of differentiation. For Driesch showed that eggs in which the character of segmentation had been altered by heat, pressure, shaking, dilution of the sea-water or the calcium-free sea-water introduced by Herbst, could still give rise to normal embryos. The segmentation of artificially parthenogenetic eggs may also be absolutely irregular (Loeb), but these eggs still produce normal larvae.

There is also the significant fact that the isolated blastomeres of these animals —at least as far as the four-celled stage—(or the vegetative blastomeres after the next division or the macromeres after the fourth division) will give rise to normal larvae, although they segment as though the missing blastomeres were still there, and precisely the same thing is seen in Nemertines (Wilson, Zeleny). The converse of this behaviour is exhibited by the Ctenophore egg, which segments normally when a portion of the vegetative hemisphere is removed, yet gives rise to an embryo devoid of costae and stomodaeum (Driesch and Morgan).

In the eggs of Turbellaria, Annelids and Mollusca (except Cephalopods) it is possible to trace the "lineage" of all the important organs of the larval body back to individual blastomeres or groups of blastomeres, and here, if anywhere, we should expect to find a "mosaic" segmentation. Even in this case, however, the factors which determine the order of division appear to be as distinct from those which determine differentiation as they are elsewhere. An isolated cell, as we know from the researches of Wilson and Crampton, will continue to segment as though its fellows were still present, but provided it contains the indispensable specific material will still produce a whole larva. Thus the CD cell of the two-cell stage or the D cell of the four-cell stage of *Dentalium* segments partially, but yet gives rise to a complete trochophore because it has received the all-essential polar lobe while its companion AB, or A, B and C cells do not. Conversely in *Ilyanassa* the cleavage of separated blastomeres may be made to resemble that of whole ova, by lowering the temperature, yet their developmental capacities are strictly limited. In *Nereis* again the pattern of segmentation may be readily modified by pressure; but the trochophore is still normal; and most recently of all Lillie has shown that in *Chaetopterus* certain endoplasmic granules—associated with gut formation—may be driven by the centrifuge to any point of the egg, at will, but to whatever point they are driven the cleavage is always related to the axis determined by the polar bodies in the same way as in the unaltered egg. Lillie has further succeeded, to some extent, in obtaining differentiation—the formation of the prototroch—though all division was entirely suppressed. We may add that cells which occupy an identical position in similar patterns of cleavage, may have, in different animals, a very diverse fate.

In "typical" development, as Roux would call it, the symmetry of the egg, the symmetry of segmentation, the symmetry of the embryo may all be coincident. This is well seen in Annelids and Molluscs—where the D cell is posterior; in Ascidians—where the First Furrow is in the Plane of Symmetry of the egg and also in the Sagittal Plane of the embryo, the second transverse, the third horizontal; again in the Cephalopoda—where the first is in the Symmetry Plane of the egg and in the Sagittal Plane, the second transverse; in Ctenophora—where the first and second divisions are respectively sagittal and transverse; in the Sea-urchins, and in the undisturbed egg of the Frog. But that coincidence of segmentation with differentiation is not necessary for normal development is made abundantly clear by the experiments we have just considered, experiments which bring out the latent distinction between the two processes.

We can only, it seems to me, conclude that there are present in the egg two sets of factors—those which cause a certain pattern of segmentation and those on which differentiation depends. The former must be sought for in the quantity of yolk in the egg (Balfour's rule), in the relation between the mitotic spindle and the cytoplasm (the rules of Hertwig), in the relation between the dividing centrosomes and the previous spindle axis (Sach's law) and in the capillary properties of the surfaces of the blastomeres. The latter on the other hand will be found—have, indeed, already been found—in certain necessary specific organ-forming substances of the cytoplasm.

A limited part might indeed be assigned to mitosis in the determination of embryonic axes; the asters at the poles of the fertilization spindle might conceivably impress a bilateral symmetry upon the cytoplasm,—and it is as well to bear in mind that there is in the Frog a slight correlation between First Furrow and Sagittal Plane; but speaking generally the two processes are very largely independent of one another. To the developing embryo the order in which its specific materials are cut up in segmentation is a matter of indifference—what is essential apparently is that the material should be subdivided until a definite quantitative relation is established between the dimensions of the nucleus and the dimensions of the cell (Boveri).

Postscript.

It is a very great pleasure to me to find that Professor Brachet, though working by different methods, has arrived at conclusions almost identical with mine.

In his first paper (*Arch. de Biol.* XXI. 1905) he states that the angle between Plane of Symmetry and First Furrow may have any value. One hundred eggs were examined by means of *sections*. Further, according to him, the Plane of Symmetry *always* includes the "penetration" path, and *always* becomes the Sagittal Plane (here I cannot help thinking he goes too far). The relation between the Planes was further investigated by puncturing the egg in the two-celled stage. When the First Furrow coincided with the Plane of Symmetry the result was a hemiembryo lateralis, when at 90° to the Symmetry Plane a hemiembryo anterior (the blastomere not containing the grey crescent having been injured) and when oblique an embryo with one posterior side wanting.

In a second communication (*Arch. Ent. Mech.* XXII. 1906) the effect is described of removing part of the vegetative hemisphere by puncture. When this operation is performed prior to the formation of the grey crescent subsequent development is normal. Up to this time therefore the egg is isotropic about its axis. When the grey crescent has appeared, however, the loss of this substance may entail abnormality or death.

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MISCELLANEA.

I. The Distribution of the Means of Samples which are not drawn at Random.

By STUDENT.

It is one of the advantages of the normal curve that if samples are drawn at random from any population, no matter how distributed, the distributions of the statistical constants of the samples rapidly approach the Gaussian as the samples grow large.

This being so, the result of grouping 2000 in samples of 25 given in Drs Greenwood and White's very interesting paper in *Biometrika* is surprising.

For it is easy to show that if B_1, B_2 be the constants of the distribution of the means of samples of n drawn at random, corresponding to β_1, β_2 in the original frequency distribution, then*

$$B_1 = \frac{\beta_1}{n} \quad \text{and} \quad B_2 - 3 = \frac{\beta_2 - 3}{n}.$$

But in this case $\beta_1 = 1{\cdot}7977$ and $B_1 = {\cdot}4756 : \frac{\beta_1}{n} = {\cdot}0719,$

$$\beta_2 - 3 = 2{\cdot}5790, \quad B_2 - 3 = {\cdot}3185 : \frac{\beta_2 - 3}{n} = {\cdot}1032.$$

Now neither of these can be considered significant with a sample of 80 means but at the same time they are both sufficiently different to suggest that the conditions which led to the theoretical result have not been fulfilled.

The first thing which occurred to me was that as Sheppard's corrections had been used for the means but not for the original distribution it might be well to try applying them to both.

This however makes but little difference, for we get

$$\beta_1 = 1{\cdot}9898 : \frac{\beta_1}{25} = {\cdot}0792,$$

$$\beta_2 - 3 = 2{\cdot}7725 : \frac{\beta_2 - 3}{25} = {\cdot}1109.$$

I next considered the possibility that the samples were not strictly random but that there was some slight correlation between successive observations.

I therefore assumed that the individuals composing the sample were more like each other than to the rest of the population, that in fact there was homotyposis, and working from this hypothesis I found that the slightest correlation produces a very marked retardation in the approach to normality with increase in the size of the sample.

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* Henderson, R.: *Journal Inst. of Actuaries*, Vol. XLI. pp. 429—442.

It will be observed that this is essentially a 'small sample' problem, for with increase in the size of the sample the correlation due to likeness between successive individuals diminishes except in exceptional cases when it becomes manifest as a well-marked heterogeneity.

My results emphasize the necessity of avoiding anything which tends to produce secular variation and as far as possible to neutralise it by repeating observations only after some time has elapsed.

Thus repetitions of analyses in a technical laboratory should never follow one another but an interval of at least a day should occur between them. Otherwise a spurious accuracy will be obtained which greatly reduces the value of the analyses.

In the present case there is not sufficient evidence to show whether correlation was really present, but as in the course of a fairly extended practice I have not yet met with observations in which this tendency was altogether absent, I incline to the belief that it was.

In any case, being ignorant of the technique, I can only suggest as possibilities slight variations from point to point on the slide, differences in light or in the observer as the day went on.

The general problem is as follows:

Let samples of n be drawn from a population with constants μ_2, μ_3, μ_4, β_1, β_2, and let the samples be drawn in such a manner that the individuals composing each sample are correlated with correlation coefficient r, then, assuming linear regression and homoscedastic arrays, the constants of the distribution of their means (M_2, M_3, M_4, B_1, B_2) are as follows:

$$M_2=\frac{\mu_2}{n}\{1+(n-1)r\},$$

$$M_3=\frac{\mu_3}{n^2}\{1+(n-1)r\}\{1+(2n-1)r\},$$

$$M_4=\frac{\{1+(n-1)r\}}{n^3(1+2r)}[\mu_4\{1+(3n-1)r+3n(n-1)r^2\}+3(n-1)(1-r)(1+nr)\mu_2{}^2],$$

$$B_1=\frac{\beta_1}{n}\,\frac{\{1+(2n-1)r\}^2}{(1+r)^2\{1+(n-1)r\}},$$

$$B_2=\frac{\beta_2}{n}\,\frac{\{1+(3n-1)r+3n(n-1)r^2\}}{(1+2r)\{1+(n-1)r\}}+\frac{3(n-1)(1-r)(1+nr)}{n(1+2r)\{1+(n-1)r\}}.$$

As the method of determining the three moment coefficients is the same in each case and it is merely a question of reduction to obtain B_1 and B_2, it will be sufficient for me to give the proof for M_4.

Let $x_1\, x_2 \ldots x_n$ be the values, measured from the mean of the population, of the individuals composing the typical sample, and let there be N such samples.

Then $$M_4=\frac{1}{N}\Sigma\left\{\frac{x_1+x_2+\ldots+x_n}{n}\right\}^4$$

$$=\frac{1}{N}\Sigma\,\frac{S(x_1{}^4)+4S(x_1{}^3x_2)+6S(x_1{}^2x_2{}^2)+12S(x_1{}^2x_2x_3)+24S(x_1x_2x_3x_4)}{n^4}\quad\ldots\ldots\ldots\text{(i)}.$$

Taking each of these six terms in turn we have

$$\frac{\Sigma\{S(x_1{}^4)\}}{Nn^4}=\frac{n\Sigma(n_{x_1}x_1{}^4)}{N\,.\,n^4}=\frac{\mu_4}{n^3}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}.$$

For $S(x_1{}^4)$ has n terms and when they are taken over all the N samples which compose the population there will be $n\,.\,n_{x_1}$ of $x_1{}^4$, n_{x_1} being the number of x_1's in the population and $n_{x_1x_2}$ the number of x_1's associated with x_2's and so on.

Again there are $n(n-1)$ terms in $S(x_1^3x_2)$,

$$\therefore \frac{\Sigma\{4S(x_1^3x_2)\}}{N\,.\,n^4} = \frac{4(n-1)\,\Sigma\,(n_{x_1x_2}\,x_1^3x_2)}{N\,.\,n^3}$$

$$= \frac{4(n-1)\,\Sigma\,(n_{x_1}\,.\,x_1^3\,.\,\text{mean value of } x_2)}{N\,.\,n^3}.$$

But the mean value of x_2 associated in the sample with x_1 will be $\frac{r\sigma_{x_2}}{\sigma_{x_1}}x_1$ or since $\sigma_{x_1}=\sigma_{x_2}$ it is rx_1,

$$\therefore \frac{\Sigma\{4S(x_1^3x_2)\}}{N\,.\,n^4} = \frac{4(n-1)\,\Sigma\,(n_{x_1}\,.\,x_1^4\,.\,r)}{N\,.\,n^3}$$

$$= \frac{4(n-1)\,r}{n^3}\,\mu_4 \quad \text{......................................(iii).}$$

Next
$$\frac{\Sigma\{6S(x_1^2x_2^2)\}}{N\,.\,n^4} = \frac{3(n-1)}{n^3}\,.\,\frac{\Sigma\,(n_{x_1x_2}\,.\,x_1^2x_2^2)}{N}$$

$$= \frac{3(n-1)}{n^3}\,\frac{\Sigma\,(n_{x_1}\,.\,x_1^2\,.\,\text{mean value of } x_2^2)}{N}$$

[Now the mean value of x_2^2 is equal to the square of the S.D. of the x_1 array of x_2's, $\{\mu_2(1-r^2)\}$, added to the square of the mean value of x_2, $(r^2x_1^2)$]

$$= \frac{3(n-1)}{n^3}\,\frac{\Sigma\,n_{x_1}\{r^2x_1^4 + x_1^2\mu_2(1-r^2)\}}{N}$$

$$= \frac{3(n-1)}{n^3}\,\{r^2\mu_4 + (1-r^2)\,\mu_2^2\} \quad \text{......................................(iv).}$$

Again :
$$\frac{\Sigma\{12S(x_1^2x_2x_3)\}}{Nn^4} = \frac{6(n-1)(n-2)}{n^3}\,\frac{\Sigma\,(n_{x_1x_2x_3}\,.\,x_1^2x_2x_3)}{N}$$

$$= \frac{6(n-1)(n-2)}{n^3}\,\frac{\Sigma\,(n_{x_1x_2}\,.\,x_1^2x_2\,.\,\text{mean value of } x_3)}{N}.$$

The mean value of x_3 for values x_1 and x_2 of the other two variables is given by the equation

$$m_{x_3} = -\frac{\sigma_{x_3}}{R_{33}}\left\{\frac{R_{31}x_1}{\sigma_{x_1}} + \frac{R_{32}x_2}{\sigma_{x_2}}\right\}$$

where the R's are the minors of the determinant

$$\begin{vmatrix} 1, & r, & r \\ r, & 1, & r \\ r, & r, & 1 \end{vmatrix} \quad \text{or } m_{x_3} = (x_1+x_2)\,.\,\frac{(r-r^2)}{1-r^2} = (x_1+x_2)\,.\,\frac{r}{1+r}.$$

Substituting we get

$$\frac{\Sigma\{12S(x_1^2x_2x_3)\}}{N\,.\,n^4} = \frac{6(n-1)(n-2)}{n^3}\,.\,\frac{r}{1+r}\,.\,\frac{\Sigma\,(n_{x_1x_2}(x_1^3x_2 + x_1^2x_2^2))}{N}.$$

By (iii) and (iv)
$$= \frac{6(n-1)(n-2)}{n^3}\,.\,\frac{r}{1+r}\,\{r\mu_4 + r^2\mu_4 + (1-r^2)\,\mu_2^2\}$$

$$= \frac{6(n-1)(n-2)}{n^3}\,.\,r\,\{r\mu_4 + (1-r)\,\mu_2^2\} \quad \text{..............................(v).}$$

Lastly :
$$\frac{\Sigma\{24S(x_1x_2x_3x_4)\}}{Nn^4} = \frac{(n-1)(n-2)(n-3)}{n^3}\,\frac{\Sigma\,(n_{x_1x_2x_3x_4}\,x_1x_2x_3x_4)}{N}$$

$$= \frac{(n-1)(n-2)(n-3)}{n^3}\,\frac{\Sigma\,(n_{x_1x_2x_3}\,.\,x_1x_2x_3\,.\,\text{mean value of } x_4)}{N}.$$

As before the mean value of x_4 comes from the multiple regression equation

$$m_{x_4} = -\frac{\sigma_{x_4}}{R_{44}} \left\{ x_1 \frac{R_{41}}{\sigma_{x_1}} + x_2 \frac{R_{42}}{\sigma_{x_2}} + x_3 \frac{R_{43}}{\sigma_{x_3}} \right\},$$

where the R's are minors of

$$\begin{vmatrix} 1, & r, & r, & r \\ r, & 1, & r, & r \\ r, & r, & 1, & r \\ r, & r, & r, & 1 \end{vmatrix}.$$

$$\therefore\ m_{x_4} = (x_1 + x_2 + x_3) \frac{(r(1-r)^2)}{1 - 3r^2 + 2r^3} = (x_1 + x_2 + x_3) \cdot \frac{r}{1+2r}.$$

Substituting we get

$$\frac{\Sigma\{24S(x_1 x_2 x_3 x_4)\}}{Nn^4} = \frac{(n-1)(n-2)(n-3)}{n^3} \cdot \frac{r}{1+2r} \frac{\Sigma\{n_{x_1 x_2 x_3} \cdot x_1 x_2 x_3 (x_1 + x_2 + x_3)\}}{N}$$

$$= \frac{(n-1)(n-2)(n-3)}{n^3} \cdot \frac{r}{1+2r} \cdot \frac{3\Sigma(n_{x_1 x_2 x_3} \cdot x_1^2 x_2 x_3)}{N}.$$

Applying (v)

$$= \frac{3(n-1)(n-2)(n-3)}{n^3} \cdot \frac{r^2}{1+2r} \{r\mu_4 + (1-r)\mu_2^2\} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(vi)}.$$

Substituting (ii) ... (vi) in (i) we get

$$M_4 = \frac{1}{n^3} \Big\{ \mu_4 + 4(n-1) r\mu_4 + 3(n-1)\{r^2\mu_4 + (1-r^2)\mu_2^2\}$$

$$+ 6(n-1)(n-2) . r . \{r\mu_4 + (1-r)\mu_2^2\} + 3(n-1)(n-2)(n-3) \cdot \frac{r^2}{1+2r} \{r\mu_4 + (1-r)\mu_2^2\} \Big\},$$

which reduces to the result given above, viz.

$$M_4 = \frac{\{1 + (n-1) r\}}{n^3 (1+2r)} [\{1 + (3n-1) r + 3n(n-1) r^2\} \mu_4 + 3(n-1)(1-r)(1+nr) \mu_2^2].$$

Using these equations it is possible to find values of r which would satisfy the conditions for the various constants.

Thus (using Sheppard's corrections for both sets of constants) I find that with the given values

of μ_2 and M_2 $r = \cdot 003$,
of β_1 and B_1 $r = \cdot 063$,
of β_2 and B_2 $r = \cdot 033$.

Now clearly if r were fitted by least squares or in any other way from these three values it must clearly come closest to the μ_2 value owing to the lower prob. error of μ_2. As to fit it properly is clearly very complicated owing to the intercorrelations of the constants I have assumed a value $r = \cdot 01$ as a nice round number which gives a value of M_2 higher than that found in the sample before us but not at all impossibly so.

This gives $M_2 = \cdot 1101$, actual $\cdot 1074$,
$B_1 = \cdot 1397$, ,, $\cdot 4756$,
$B_2 = 3 \cdot 2012$, ,, $3 \cdot 3185$.

These constants give a type I. curve

$$y = 97 \cdot 57 \left(1 + \frac{x}{1 \cdot 65}\right)^{24 \cdot 64} \left(1 - \frac{x}{47 \cdot 82}\right)^{714 \cdot 2}.$$

If we assume no correlation I get a curve

$$y = 109 \cdot 0 \left(1 + \frac{x}{1 \cdot 92}\right)^{41 \cdot 7} \left(1 - \frac{x}{58 \cdot 31}\right)^{1266 \cdot 8},$$

whence I get the following 'fits'*.

---

* The figures given are really mid-ordinates, but for such small numbers the difference between the mid-ordinate and the area on the base unit is negligible.

		Below *1·10*	*1·10 1·22*	*1·22 1·34*	*1·34 1·46*	*1·46 1·58*	*1·58 1·70*	*1·70 1·82*	*1·82 1·94*	*1·94 2·06*	*2·06 2·18*	*2·18 2·30*	*2·30 2·42*	*2·42 2·54*	*2·54 2·66*	*2·66 2·78*	Above *2·78*
Actual ...		—	4	8	7	14	12	12	5	7	5	2	1	2	0	1	—
Calculated	No Correlation	1·01	2·42	5·28	8·86	11·69	13·07	12·18	9·97	6·90	4·27	2·36	1·18	—	—	·92	—
	Correlation ·01	1·85	3·27	6·02	8·92	11·01	11·71	10·84	8·95	6·64	4·52	2·82	1·64	·90	—	·85	—

These give $P = \cdot 46$ and $P = \cdot 86$ respectively, the first being a good deal helped by the convention that the tail should not be carried beyond the point at which a single unit may be expected, and the second much less so.

As the empirical curve fitted from the actual moments has a P of ·92, the second curve may be considered fairly good depending as it does on a guess following on calculation. On the other hand a P of ·46 with so few cases as 80 is not particularly good, and as Prof. Pearson has pointed out to me the graph distinctly gives an idea of greater skewness than is represented by the no correlation curve. I do not however wish to contend that the circumstances attending the production of the sample actually conformed to the arbitrary conditions which I found it necessary to assume in order to simplify the analysis. But seeing that the fit is good and that with such a small sample even the divergent B_1 is not altogether impossible, I think it likely that there was some sort of correlation, though probably not that particular kind which has been assumed in this note.

Conclusions:

(1) That the approach to normality of the distribution of means of samples drawn from a non-Gaussian population is delayed by the existence of correlation between the individuals composing the samples.

(2) That on certain arbitrary assumptions the constants of the new distribution can be found given the constants of the old one and r according to formulae given above.

(3) That using the above formulae and choosing a likely looking value of r, a curve can be drawn to represent the sample in Drs Greenwood and White's paper with fair likelihood.

II. A Short Method of Calculating the Coefficient of Correlation in the Case of Integral Variates.

By J. ARTHUR HARRIS, Ph.D., Cold Spring Harbor, Long Island, U.S.A.

For symmetrical correlation tables in which both variates have the same mean and standard deviation, Professor Pearson has suggested* that his difference method "may possibly be of good service," but warns the reader:—

"At the same time too much reliance must not be placed upon the difference method, not only because it assumes normality of distribution but because it involves a somewhat rough method of approximation in the case of the diagonal cell."

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* *Drapers' Company Research Memoirs, Biometric Series*, IV. pp. 4—9, 1907.

In applying this difference method to series of material already worked out by the product moment method I have found considerable variation in the weight to be given to the diagonal cell, but many more series should be examined before suggesting any other value than the one-sixth proposed by Professor Pearson. While engaged in this work some modifications of method which, I think, will lighten considerably the calculation of r in some cases came to my attention.

It is well known that the formula

$$r_{xy}=(\sigma_x^2+\sigma_y^2-\sigma_\nu^2)/(2\sigma_x\sigma_y)\quad\text{(i)},$$

where σ_x and σ_y are the standard deviations of the two characters and σ_ν is the standard deviation of their difference, gives r with the same accuracy as the product moment method irrespective of the nature of the distributions.

In symmetrical tables, in which each individual is used once as a first and once as a second member of a pair, $\sigma_x=\sigma_y$ and the above formula may be written

$$r_{xx}=\frac{2\sigma_x^2-\sigma_\nu^2}{2\sigma_x^2}=1-\frac{1}{2}\frac{\sigma_\nu^2}{\sigma_x^2}\quad\text{(ii)}.$$

From this formula the correlation coefficient may be calculated with great ease and rapidity as follows:

First, determine the positive differences between the first and second member of the pairs from the table, as suggested by Professor Pearson[*] and as illustrated in the example given below.

After the totals of the columns have been found they are multiplied by the squares of the differences as given at the heads of the several columns. Twice the sum of these products divided by N gives σ_ν^2, for the origin is at 0, the plus and minus deviations are equally great and the first moment necessarily 0.

The standard deviation of the character will usually have been obtained for other purposes but if it is not wanted no roots need be extracted to obtain r by this method. The multiplication of the totals of the several columns by the squares of the differences which they represent requires only a little more work than their multiplication by the numbers themselves, and if a Brunsviga or Comptometer be used all of the arithmetical work can be done in a few minutes, even when the tables are rather large. Thus the whole work can be completed almost if not quite as quickly as that for Pearson's formula (vii)[†] and with the same accuracy as the product moment method.

Since all may not have access to Pearson's memoir referred to, I give the following illustration of the arithmetic of the method.

Illustration I. Calculation of correlation from symmetrical table. Dr Fernando De Helguero's symmetrical table for number of flowers per inflorescence in *Cicorium Intybus*[‡] serves as an example. Beginning at the head of each column in the correlation table we copy down the frequencies to and including the diagonal cell—where the difference between the two variates is 0—under appropriate headings in a series of difference columns. The work can be done systematically and rapidly by copying the first number of the first column in the 0 column. In beginning the copying of each succeeding column from the correlation table a new column of differences, one higher than the last, will be begun and be represented by a single entry for that column while the remaining entries will be placed successively one place to the left. The whole process will be clear at once from a comparison of the work of the illustration with the table of data. The numbers in clarendon type show the order in which the entries from the 12th column were written down.

[*] *loc. cit.* p. 7.

[†] *loc. cit.* p. 6.

[‡] *Biometrika*, Vol. v. p. 188.

The same value is obtained by the product moment method, the difference between this and the $r = \cdot613$ obtained by De Helguero being due to the slip in his calculation of the standard deviation. It appears from the above that the calculation of r is a very simple process.

TABLE I.

Inflorescences of Cicorium.

	8	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	Totals
8	—	1	1	1	2	—	—	—	—	—	5
9	1	2	2	15	10	1	2	—	—	—	33
10	1	2	8	19	11	4	1	—	—	—	46
11	1	15	19	134	90	18	1	—	—	—	278
12	2	10	11	90	114	97	9	4	—	—	337
13	—	1	4	18	97	122	53	2	—	—	297
14	—	2	1	1	9	53	96	10	—	—	172
15	—	—	—	—	4	2	10	10	3	—	29
16	—	—	—	—	—	—	—	3	2	1	6
17	—	—	—	—	—	—	—	—	1	—	1
Totals	5	33	46	278	337	297	172	29	6	1	1204

ν = *Difference of x and y.*

0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	
0	1	1	1	**2**	0	0	0	0	0	
2	2	15	**10**	1	2	0	0	0	—	
8	19	**11**	4	1	0	0	0	—	0	
134	**90**	18	1	0	0	0	—	0		
114	97	9	4	0	0	—	0	—		$S(\nu^2)$
122	53	2	0	0	—	0				276
96	10	0	0	—	2					224
10	3	0	—	4						180
2	1	—	20							64
0	——	56								50
——	276									——
488										794

$$N = 1204,$$

$$\sigma_x = 1\cdot3352^*,\ \sigma_x^2 = 1\cdot7828,$$

$$\sigma_\nu^2 = \frac{2 \times 794}{N} = 1\cdot318937,$$

$$r = \left(1 - \frac{1}{2}\frac{\sigma_\nu^2}{\sigma_x^2}\right) = \left(1 - \frac{1}{2}\frac{1\cdot318937}{1\cdot7828}\right) = \cdot630.$$

In working with a symmetrical table we copy down only the diagonal cell and the positive differences. But it would be very easy to continue right down the columns: after passing the diagonal cell the differences would take the negative sign and we could calculate the standard deviation from all the differences instead of from a part of them. In the symmetrical table this would be merely a waste of time, but where the tables are not symmetrical (and the two variates have different means and standard deviations) this process can be used to advantage.

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* The value 1·3536 given by De Helguero was apparently obtained by taking the root of the second rough moment, i.e. without first subtracting the square of the first rough moment.

In fact I believe that in the case of integral variates the calculation with formula (i) will be considerably more rapid than the conventional product moment method.

A further saving of time can be effected by doing away with the necessity for copying the entries from the correlation table into difference columns. This can be made clear by an illustration.

Illustration II. Calculation of correlation from non-symmetrical tables. Pearl's table for his Series V of Ceratophyllum*, showing correlation between the position on the branch and the number of leaves per whorl, is given below with the differences between the two variates indicated above each entry as small indices. These can be entered on a table—preferably in red ink or pencil to avoid confusion—much more quickly than the products of the deviations of the two variables from their assumed origins, and with much less chance of error, especially in the case of large tables. The work can be most rapidly done by taking first the diagonal cell and then the diagonal rows lying on either side of it. All the entries of any diagonal row will have the same index, but it should be put down in every case to avoid confusion. All that now remains to be done is to sum these diagonal columns and calculate the second moment. For this r can be at once calculated from formula (i) above.

TABLE II.

Whorls of Leaves in Ceratophyllum.

Position of Whorl on Branch.

Leaves per Whorl.	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	Totals
4	2^{3}	2^{2}	—^{1}	—^{0}	—$^{-1}$	—$^{-2}$	—	—	—	—	4
5	17^{4}	5^{3}	1^{2}	2^{1}	—^{0}	1^{-1}	1^{-2}	—	—	—	27
6	16^{5}	6^{4}	10^{3}	1^{2}	1^{1}	1^{0}	—$^{-1}$	1^{-2}	—	—	36
7	40^{6}	29^{5}	18^{4}	14^{3}	9^{2}	1^{1}	2^{0}	4^{-1}	1^{-2}	—	118
8	19^{7}	28^{6}	25^{5}	17^{4}	8^{3}	7^{2}	3^{1}	—^{0}	3^{-1}	1^{-2}	111
9	5^{8}	20^{7}	19^{6}	26^{5}	22^{4}	17^{3}	13^{2}	6^{1}	4^{0}	5^{-1}	137
10	7^{9}	5^{8}	6^{7}	4^{6}	6^{5}	10^{4}	8^{3}	14^{2}	4^{1}	3^{0}	67
11	1^{10}	—^{9}	—^{8}	—^{7}	—^{6}	—^{5}	—^{4}	—^{3}	—^{2}	—^{1}	1
Totals	107	95	79	64	46	37	27	25	12	9	501

From our illustrative table we have for the sums of the diagonal rows :—

ν	*−2*	*−1*	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	Total
Frequency ...	4	13	10	17	47	64	90	102	91	45	10	7	1	501

Note that a check for the accuracy of the work is furnished by the identity of the total with the N of the population. In the product moment method there is no convenient check for the accuracy of the multiplications leading to the $S(xy)$. In the present case we find

$$\begin{aligned} &\text{S. D. Position} &&= 2{\cdot}3733, \\ &\text{S. D. Leaves} &&= 1{\cdot}4003, \\ &\text{S. D.}^2\ \text{Difference} &&= 4{\cdot}4380, \\ &r &&= {\cdot}475, \end{aligned}$$

agreeing with the value given by Pearl.

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* Pub. 58, *Carnegie Institution of Washington*, p. 60, Table 31, 1907.

Obviously this method of obtaining the totals of the difference columns can be used for the symmetrical tables as well, where only the diagonal cell and half of the remaining diagonal rows need be considered, thus doing away with the work required by the method of copying the frequencies from the correlation table into difference columns as suggested by Pearson.

Unfortunately these difference methods are limited in their applicability. When integral variates are under consideration, they give the same result as the product moment method when Sheppard's Correction is not applied in the calculation of the standard deviations. But difficulties arise when other than integral variates are considered. The divergence of the coefficients of correlation calculated by different methods is shown by Wright, Lee and Pearson*. But where the limits of their applicability are borne in mind, I think, these difference methods deserve more consideration in practical work than they have hitherto received.

III. Note on Variation in Adoxa.

By J. ARTHUR HARRIS, Ph.D., Cold Spring Harbor, L.I., U.S.A.

In examining Whitehead's paper† on the Moschatel, certain points came to my attention which seem to merit more detailed consideration than he has given them. What is really needed is a more extensive quantitative investigation than is possible on the data hitherto published. Possibly the indication of certain points of interest not discussed by Whitehead may induce some one who has the opportunity of collecting fresh material to treat the problems in the detailed way which they deserve.

Two questions occur to one examining Whitehead's second table:

(*a*) Is there any relationship between the number of flowers which an inflorescence produces and the characteristics of these flowers?

(*b*) What is the similarity of the flowers of an inflorescence?

Both of these, I take it, are of considerable interest to morphologists.

Problem (*a*). The correlation between the number of flowers per inflorescence and the number of the divisions of the corolla.

In our almost complete ignorance of factors influencing the number of parts of the flower, it seems important to investigate every possible interrelationship between the characteristics of the flower and the characteristics of the individual which produces it. Such investigations may not be expected to yield more than a small part of the information which we desire, but before experimental methods are extensively applied, it seems logical to determine whether there is any dependence of the number of floral parts developed, or any dependence of any characteristics of these floral parts, upon the vegetative organs of the individuals which produce them.

In the present case only one characteristic other than those belonging to the flower itself is available for comparison, i.e. number of flowers per inflorescence. This character seems, however, a particularly interesting one for study. In the Moschatel there is one form of inflorescence which is conspicuously the modal one. This type is a very neat, compact, morphological structure and any departure from it appears at once to be "abnormal." The flowers, too, have a normal form, from which a considerable number of deviations are to be found. Are abnormalities in the number of lobes of the corolla associated with abnormalities in the number of flowers borne on the inflorescence? The data for determining the coefficients of correlation are extracted from Whitehead's Table II and presented in the correlation tables

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* *Biometrika*, Vol. v. p. 410.

† *Biometrika*, Vol. II. pp. 108—113, 1902.

I—III. Here the inflorescences are necessarily weighted with the number of flowers which they produce. In all these cases the terminal flower seems to have been present. The number of lateral flowers, then, represents the real variable characteristic of the inflorescence, and has been taken as the first variable in preparing the correlation tables. The constants calculated are given in Table A. The weighting of the inflorescences must be borne in mind by anyone desiring to use these standard deviations or coefficients of variation for other purposes.

The means and variabilities of the three collections show some differences which may be significant, but until further collections from a wider range of habitats are available, little can be profitably said concerning them. Like other kinds of biological work, Biometry cannot make great progress until enough series of data have been reduced to permit of extensive comparisons.

The correlation coefficients are only about five to seven times their probable errors and so cannot be given too much significance*. Furthermore, two of the coefficients are negative while the other is positive. In such cases as these too great significance must not be attached to the probable errors. The actual number of inflorescences is really very small in two of the series, and the tables appear to contain adequate data merely because the inflorescences are necessarily weighted with the number of flowers which they bear. In cases in which deviations from the typical condition are very few the collection of a slightly larger series may materially change the constants for a race†. The Kent series is the only adequately large one of the three, and the correlations derived from it may be considered significantly negative. The other two collections would be sufficiently large to give fairly trustworthy results in characters with more normal variability. But where only 9 of the 163 inflorescences actually involved depart from the normal type—as is the case in the Essex series—it is dangerous to lay much stress upon the results from small samples.

Considering all these difficulties I think we can draw no final conclusion from these data. But it appears from the results of this first examination of the relationship between the number of flowers per inflorescence and the number of divisions of the floral envelopes that the problem is worthy of detailed study by someone living in a region where considerable quantities of *Adoxa* may be obtained‡.

Problem (*b*). The correlation between the number of corolla lobes of flowers of the same inflorescence.

Teratologists have long known that when one organ of an individual is abnormal there is some probability that a second homologous organ of the same individual will be abnormal also. How great is this probability?

For the small deviations from the type of the race which have generally been called variations —or fluctuations by many modern writers—Pearson and others have determined quantitatively the degree of resemblance between the undifferentiated like organs of the individual for a considerable series of species. The interindividual correlations for a form like *Adoxa* will be interesting to compare with the coefficients obtained from other homotypic relationships. *Adoxa* produces ordinarily only a single inflorescence, and the interest of the comparison lies in the fact that all the homotypes are so closely associated. *À priori*, I would have expected such organs to be more highly correlated than those more widely separated on the individual. Even if we suppose that proximity of organs does not indicate that they have any function to perform in common which would tend to render their correlation organic rather than homotypic*, it would seem quite reasonable to think that organs separated in their embryological development by only relatively few cell divisions would be more similar than those separated by an immensely large number. Concretely, the flowers of an inflorescence of *Adoxa* have been built up from a common vegetative point by only a few cell divisions as compared with the inconceivably large number which have intervened between the laying down of the leaves upon the outermost branches of a beech tree. In the laying down of the leaves of a forest tree there is also opportunity for considerable differences in the environments of the individual vegetative points. The actual correlation constants, calculated from the symmetrical homotypic tables IV—VI, are given in Table B.

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* All the probable errors are calculated from the formulae commonly used, but it must be noted that for the number of flowers per inflorescence the percentage of deviations from the normal approaches the limits beyond which these formulae cannot be used with perfect confidence.

† For another illustration see *Report Missouri Botanical Garden*, Vol. 20, 1909.

‡ In this note I have not considered the number of lobes of the terminal flower because the data are too few. This should be taken up when larger series of material are available. I note in passing that there is a slip in Whitehead's Table III, where the frequency of the 5-merous flowers should be given as 2479 instead of 2494.

The conspicuous thing about these constants is that they are not higher, but rather lower, than those usually found for vegetable homotypes. Comparing the results summarized in Pearson's Table XXXII†, we note, however, that Ash, Wild Ivy and one series of Holly give values of about the same general order of magnitude as these. In a study of the correlation between the two leaves of the flowering stalk of *Podophyllum* I have shown‡ that the relationship in two series of material is about ·45, although here too I should have expected higher values. Aside from any bearing which the problem of homotyposis may have on heredity, I think it may throw some light upon a number of morphogenetic problems. One of these is the question of a possible relationship between the homotypic constants for organs which are widely separated ontogenetically and those which are closely associated ontogenetically§.

One further point may be noted. In any critical study of interindividual correlation in the Moschatel the possibility of the individuals being related vegetatively must be taken into account. Pearson found that in *Malva rotundifolia* the homotypic coefficient was considerably lower than that found in most forms, and concluded that this was due to the plants having for the most part spread from one clump by stolons.

TABLE A.

Variation and Organic Correlation Constants for Adoxa.

Material	Means	Standard Deviation	Coefficient of Variation	Coefficient of Correlation
Kent, Flowers ...	4·042 ± ·007	·489 ± ·005	12·09	− ·135 ± ·014
„ Divisions ...	4·819 ± ·006	·440 ± ·005	9·14	
Surrey, Flowers ...	4·190 ± ·024	·604 ± ·017	14·423	− ·214 ± ·038
„ Divisions...	4·803 ± ·020	·500 ± ·014	10·41	
Essex, Flowers ...	3·994 ± ·007	·278 ± ·005	6·95	+ ·163 ± ·026
„ Divisions ...	4·862 ± ·009	·357 ± ·007	7·35	

Flowers = Number of lateral flowers per inflorescence; Divisions = Number of divisions of corolla. Sheppard's Correction not used.

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* See in this connection Pearson's very clear statements of the differences between organic and homotypic correlation in *Biometrika*, Vol. I. p. 341, 1902.

† *Phil. Trans.* 1901, Vol. 197, A, p. 356.

‡ *Bot. Gaz.* Vol. 47, pp. 438—444, 1909.

§ Possibly the statistical difficulties introduced by the "small samples" necessarily used in studies of interindividual correlation in such forms as *Podophyllum* and *Adoxa* may obscure the real biological relationships. See Pearson, *Biometrika*, Vol. VI. p. 403, 1909.

TABLE B.

Interinflorescence Correlations for Number of Petals in Adoxa.

Series of Material	Correlation and Probable Error
Near Chislehurst, Kent ...	·346 ± ·013
Caterham, Surrey ...	·265 ± ·037
Theydon Garnon, Essex ...	·301 ± ·024

TABLE I.

Kent.

Petals.

Lateral Flowers.	*3*	*4*	*5*	*6*	*7*	*8*	Totals
2	—	—	9	—	—	1	10
3	1	15	70	6	1	—	93
4	12	319	1549	12	—	—	1892
5	—	14	36	—	—	—	50
6	8	16	42	—	—	—	66
7	—	1	6	—	—	—	7
Totals	21	365	1712	18	1	1	2118

TABLE II.

Surrey.

Petals.

Lateral Flowers.	*3*	*4*	*5*	*6*	Totals
3	—	1	11	—	12
4	—	41	173	10	224
5	2	15	12	1	30
6	—	6	12	—	18
Totals	2	63	208	11	284

TABLE III.

Essex.

Petals.

Lateral Flowers.	*3*	*4*	*5*	*6*	Totals
3	—	13	8	—	21
4	2	72	541	1	616
5	—	—	5	—	5
6	—	1	5	—	6
Totals	2	86	559	1	648

TABLE IV. *Kent.*

First Flower.

Second Flower.	*3*	*4*	*5*	*6*	*7*	*8*	Totals
3	12	44	22	—	—	—	78
4	44	474	599	12	—	—	1129
5	22	599	4532	30	2	1	5186
6	—	12	30	6	—	—	48
7	—	—	2	—	—	—	2
8	—	—	1	—	—	—	1
Totals	78	1129	5186	48	2	1	6444

TABLE V. *Surrey.*

First Flower.

Second Flower.	*3*	*4*	*5*	*6*	Totals
3	—	4	4	—	8
4	4	94	111	6	215
5	4	111	514	20	649
6	—	6	20	8	34
Totals	8	215	649	34	906

TABLE VI. *Essex.*

First Flower.

Second Flower.	*3*	*4*	*5*	*6*	Totals
3	2	2	2	—	6
4	2	92	152	1	247
5	2	152	1528	2	1684
6	—	1	2	—	3
Totals	6	247	1684	3	1940

IV. On the Association of Drawing with other Capacities in School Children.

By ETHEL M. ELDERTON, Galton Eugenics Laboratory.

An interesting paper* has recently been published by E. Ivanoff discussing the association of power of good drawing in boys and girls with other capacities. The relationships, however, are not expressed in terms of any modern statistical measure of correlation†, and it is accordingly not easy to determine whether drawing capacity is highly or slightly correlated with other characters, or what are the relative degrees of association between drawing and these characters. It seemed therefore worth while considering the data from another standpoint, and expressing it in a form more readily intelligible to the statistician. Each characteristic is divided into three categories, a 'mean' group, and a group above and a group below the mean. The classification of a group which contains 40 to 80°/$_{\circ}$ of the total frequency as 'moyens' is not to be commended. It corresponds, however, to a division into markedly good, markedly poor, and a middle group of 'indeterminates.' Thus for example Ivanoff uses for categories of attention 'Attentif,' 'Distrait' and 'Indéterminé,' and for temper 'Doux,' 'Violent' and 'Indéterminé,' while for most special studies including drawing he uses a 'moyens' group. He does not give, but from his data it is possible to deduce, contingency tables with 9-fold groupings. The coefficients of mean square contingency deduced from such tables will be comparable among themselves, but possibly 30 to 50 per cent. below the true value of the correlation coefficient; eight tables were worked out by the fourfold table method and showed on an average 40 per cent. increase on the contingency values. It is only needful to bear this in mind when we are considering the absolute importance of the contingencies investigated. I divide Ivanoff's characters into the following classes:

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* Recherches expérimentales sur le Dessin des Écoliers de la Suisse Remande. *Archives de Psychologie*, III. VIII. 1908.

† From the standpoint of theory very grave objections indeed might be raised to M. Ivanoff's treatment of correlation, especially to his use of four correlations from a single table which are not independent.

(A) Measures of General Ability, represented by 'Intelligence' and 'General Work.'

(B) Studies which we may *a priori* suppose to be associated with drawing, namely 'Writing,' 'Geography,' on the assumption that it covers map-making, and 'Manual Work.'

(C) Studies less obviously associated with Drawing, namely 'Composition,' 'Language,'* 'History' and 'Arithmetic.'

(D) Psycho-pedagogic Characters, i.e. such characters as are necessarily foremost in the estimate of the teachers. These are 'Attention,' 'Obedience,' 'Industry,' 'Cleanliness' and 'Temper.'

We reach the following values of the association of these characters with Drawing:

		Boys		Girls	
A.	Intelligence ...	·15	·17	·23	·23
	General work ...	·19		·23	
B.	Writing ...	·20	·16	·14	·20
	Geography ...	·14		·26	
	Manual work ...	·14		·21	
C.	Composition ...	·13	·11	·19	·18
	History ...	·12		·16	
	Language ...	·08		·14	
	Arithmetic ...	·09		·21	
D.	Attention ...	·16	·11	·11	·11
	Obedience ...	·13		·08	
	Energy ...	·08		·13	
	Industry ...	·10		·11	
	Cleanliness ...	·16		·10	
	Temper ...	·06		·12	
	Mean of 15 contingencies		·13		·16

We see from this table several results flow at once:

(i) Ability in drawing is more closely associated with other characters in girls than in boys. This holds for every sub-group also, except the pedagogic characters, which in the girl are possibly slightly less associated with drawing capacity than in the case of the boy.

(ii) If we can trust the data, drawing ability has extraordinarily little relationship to any other of the characters observed. If we increase the contingency values by 40 per cent. to reach the true correlation, the highest relationship will be ·36 for geography and drawing in girls; the highest in boys will be ·28 for writing and drawing; and the average value for the boys will be only ·18 and for girls ·22. These are distinctly small relationships, compared with what have been found for the interrelationship of other studies in previous investigations. This may arise possibly from capacity in drawing being a hereditary character having small association with other school measures of fitness.

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* This is equivalent to 'reading' and 'grammar.'

(iii) Measures of general ability head the list for both boys and girls. Drawing is more closely related to general intelligence than even the subjects like writing, geography and manual work, more closely associated with the effective use of the hand.

(iv) In the case of both boys and girls the psycho-pedagogic characters have less relation to proficiency in drawing than the hand studies. For the girls all the correlations are the lowest on the list; efficiency in studies not obviously related to drawing like reading, grammar and arithmetic is far more important. The same result is screened in the boys by the comparatively high values of the attention and cleanliness correlations. Indeed nothing appears to be gained in drawing efficiency by a large capacity for industry or by energy.

(v) There do appear some slight sexual differences. Attention and cleanliness in boys take the place of the geography and manual work of the girls, whose arithmetic and composition are more vital than their handwriting.

The following table gives the exact order of intensity of the correlation of the characters considered with drawing for the two sexes. While Ivanoff's paper thus brings out a number of points of considerable interest and is suggestive of further work, we venture to put forward one or two points for consideration.

Table of contingency coefficients in order of magnitude:

Order	Boys	Girls
1	Writing (·20)	Geography (·26)
2	General work (·19)	Intelligence (·23)
3	Cleanliness (·16)	General work (·23)
4	Attention (·16)	Manual work (·21)
5	Intelligence (·15)	Arithmetic (·21)
6	Geography (·14)	Composition (·19)
7	Manual work (·14)	History (·16)
8	Composition (·13)	Writing (·14)
9	Obedience (·13)	Language (·14)
10	History (·12)	Energy (·13)
11	Industry (·10)	Temper (·12)
12	Arithmetic (·09)	Industry (·11)
13	Energy (·08)	Attention (·11)
14	Language (·08)	Cleanliness (·10)
15	Temper (·06)	Obedience (·08)

In the first place, in actual studies it would seem best to classify the children according to the decile position they take in their respective classes or standards. The ages also should be given, so that the correlations may be corrected for any age influence. In the next place, characters like intelligence, attention, obedience, etc., should, where it is feasible, be divided into five rather than three sub-classes. This would permit of a 25-fold table and the contingency could thus be calculated with considerably closer approach to the true correlation. In making from the statistical side these suggestions for future observations, we do not wish to underrate the value or interest of Ivanoff's data, but merely to press for additional information which will much strengthen the quantitative reasoning on such material.

BOYS.

Intelligence.

Drawing.	Fo	Moy	Fa	Totals
Fo ...	69	207	18	294
Moy ...	117	749	79	945
Fa ...	13	132	21	166
Totals	199	1088	118	1405

General Work.

Drawing.	Fo	Moy	Fa	Totals
Fo ...	135	117	42	294
Moy ...	301	372	272	945
Fa ...	34	66	66	166
Totals	470	555	380	1405

Writing.

Drawing.	Fo	Moy	Fa	Totals
Fo ...	82	182	30	294
Moy ...	180	556	209	945
Fa ...	25	76	65	166
Totals	287	814	304	1405

Geography.

Drawing.	Fo	Moy	Fa	Totals
Fo ...	74	199	21	294
Moy ...	136	706	103	945
Fa ...	17	124	25	166
Totals	227	1029	149	1405

Manual Work.

Drawing.	Fo	Moy	Fa	Totals
Fo ...	44	237	13	294
Moy ...	67	808	70	945
Fa ...	8	139	19	166
Totals	119	1184	102	1405

Composition.

Drawing.	Fo	Moy	Fa	Totals
Fo ...	54	215	25	294
Moy ...	87	745	113	945
Fa ...	13	135	18	166
Totals	154	1095	156	1405

History.

Drawing.	Fo	Moy	Fa	Totals
Fo ...	72	201	21	294
Moy ...	147	698	100	945
Fa ...	17	130	19	166
Totals	236	1029	140	1405

Language.

Drawing.	Fo	Moy	Fa	Totals
Fo ...	100	80	114	294
Moy ...	276	284	385	945
Fa ...	39	63	64	166
Totals	415	427	563	1405

Arithmetic.

Drawing.	Fo	Moy	Fa	Totals
Fo ...	144	71	79	294
Moy ...	366	314	265	945
Fa ...	69	47	50	166
Totals	579	432	394	1405

Attention.

Drawing.	Fo	Moy	Fa	Totals
Fo ...	41	201	52	294
Moy ...	85	649	211	945
Fa ...	20	82	64	166
Totals	146	932	327	1405

Obedience.

Drawing.	Fo	Moy	Fa	Totals
Fo ...	70	214	10	294
Moy ...	172	701	72	945
Fa ...	29	112	25	166
Totals	271	1027	107	1405

Energy.

Drawing.	Fo	Moy	Fa	Totals
Fo ...	76	177	41	294
Moy ...	189	591	165	945
Fa ...	24	112	30	166
Totals	289	880	236	1405

Industry.

Drawing.	Fo	Moy	Fa	Totals
Fo ...	47	227	20	294
Moy ...	83	780	82	945
Fa ...	21	129	16	166
Totals	151	1136	118	1405

Cleanliness.

Drawing.	Fo	Moy	Fa	Totals
Fo ...	41	239	14	294
Moy ...	64	808	73	945
Fa ...	16	121	29	166
Totals	121	1168	116	1405

Temper.

Drawing.	Fo	Moy	Fa	Totals
Fo ...	61	209	24	294
Moy ...	164	685	96	945
Fa ...	24	128	14	166
Totals	249	1022	134	1405

Fo signifies fort; Moy, moyen; Fa, faible.

GIRLS.

Intelligence.

Drawing.

	Fo	Moy	Fa	Totals
Fo ...	26	37	5	68
Moy ...	94	578	65	737
Fa ...	17	174	40	231
Totals	137	789	110	1036

General Work.

	Fo	Moy	Fa	Totals
Fo ...	43	21	4	68
Moy ...	279	270	188	737
Fa ...	45	97	89	231
Totals	367	388	281	1036

Writing.

	Fo	Moy	Fa	Totals
Fo ...	19	37	12	68
Moy ...	113	513	111	737
Fa ...	31	141	59	231
Totals	163	691	182	1036

Geography.

Drawing.

	Fo	Moy	Fa	Totals
Fo ...	24	39	5	68
Moy ...	74	600	63	737
Fa ...	27	152	52	231
Totals	125	791	120	1036

Manual Work.

	Fo	Moy	Fa	Totals
Fo ...	26	33	9	68
Moy ...	90	575	72	737
Fa ...	31	158	42	231
Totals	147	766	123	1036

Composition.

	Fo	Moy	Fa	Totals
Fo ...	22	39	7	68
Moy ...	92	571	74	737
Fa ...	17	173	41	231
Totals	131	783	122	1036

History.

Drawing.

	Fo	Moy	Fa	Totals
Fo ...	21	43	4	68
Moy ...	102	558	77	737
Fa ...	20	172	39	231
Totals	143	773	120	1036

Language.

	Fo	Moy	Fa	Totals
Fo ...	43	13	12	68
Moy ...	285	236	216	737
Fa ...	91	57	83	231
Totals	419	306	311	1036

Arithmetic.

	Fo	Moy	Fa	Totals
Fo ...	37	15	16	68
Moy ...	182	300	255	737
Fa ...	43	75	113	231
Totals	262	390	384	1036

Attention.

Drawing.

	Fo	Moy	Fa	Totals
Fo ...	18	42	8	68
Moy ...	96	523	118	737
Fa ...	26	158	47	231
Totals	140	723	173	1036

Obedience.

	Fo	Moy	Fa	Totals
Fo ...	15	49	4	68
Moy ...	139	561	37	737
Fa ...	31	192	8	231
Totals	185	802	49	1036

Energy.

	Fo	Moy	Fa	Totals
Fo ...	19	39	10	68
Moy ...	155	457	125	737
Fa ...	25	152	54	231
Totals	199	648	189	1036

Industry.

Drawing.

	Fo	Moy	Fa	Totals
Fo ...	16	50	2	68
Moy ...	121	576	40	737
Fa ...	21	196	14	231
Totals	158	822	56	1036

Cleanliness.

	Fo	Moy	Fa	Totals
Fo ...	7	56	5	68
Moy ...	93	596	48	737
Fa ...	12	199	20	231
Totals	112	851	73	1036

Temper.

	Fo	Moy	Fa	Totals
Fo ...	22	44	2	68
Moy ...	123	583	31	737
Fa ...	30	194	7	231
Totals	175	821	40	1036

Fo signifies fort; Moy, moyen; Fa, faible.

V. Variability in Shirley Poppies from Pretoria.

Dr Maynard sends me the following data for the variability of the stigmatic bands on three Shirley Poppies, self-sown from a single poppy. The first poppy had no less than 468 capsules, the second 425, the third had probably hardly fewer, but when about one-third of the capsules (123) had been counted, the garden boy removed the plant.

The results obtained were as follows:

	Capsules	Mean	S. D.
First Poppy	468	11·079	1·233
Second Poppy	425	12·595	1·519
Third Poppy	(123+)	13·085	—
Together	1016	11·955	1·576

I found in 1899* that my two most prolific Shirley Poppies had 126 and 85 capsules respectively, giving S.D.'s of 1·329 and 1·589. There thus appears no striking increase in the variability of the individual plant after transfer to Pretoria. The mean values are possibly rather low, but plenty of individual poppies show as low a result. Only two out of 15 crops† gave a mean value less than 12 on English soil. On the whole the chief difference appears to be the prolific character of the Shirley Poppy in the Transvaal.

Dr Maynard has fitted his data to frequency curves, reproduced in the accompanying diagrams. It will be seen that the individual variation closely accords with Type II, or is symmetrical but hardly Gaussian. The total material exhibits a very slight skewness, caused probably by the heterogeneity resulting from three large groups. That this is not more marked is almost certainly due to the fact that the three poppies have all the same mother-plant, and were possibly the result of self-fertilisation.

K. P.

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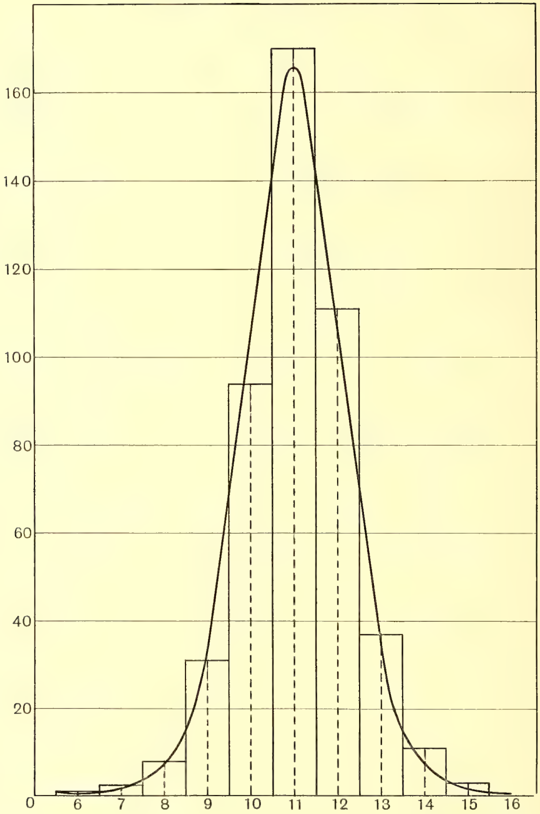
* *Phil. Trans.* Vol. 195, A, p. 316.

† *Biometrika*, Vol. IV. p. 401.

SHIRLEY POPPY. No. 1.



Bands on Capsules

$N=468$

$\sigma = 1{\cdot}233 \pm {\cdot}027$

$\mu_3 = -\ {\cdot}199 \pm {\cdot}143$

$\sqrt{\beta_1} = {\cdot}106 \pm {\cdot}076$

$\beta_2 = 4{\cdot}051 \pm {\cdot}153$

$\kappa = {\cdot}0042$

Mean $= 11{\cdot}079 \pm {\cdot}038$

Type II.

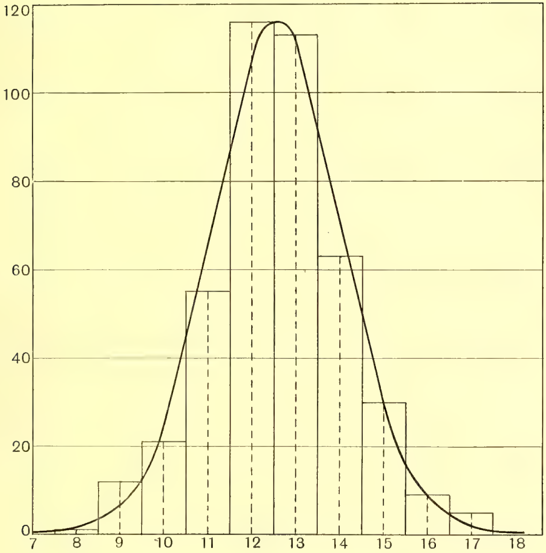
$m = -\ 5{\cdot}354$

$a^2 = -11{\cdot}711$

No.	Observed	Calculated
6	1	·37
7	2	1·65
8	8	7·83
9	31	34·36
10	94	106·84
11	170	165·70
12	111	106·84
13	37	34·36
14	11	7·83
15	3	1·65
16	0	·37
	468	467·8

$P = {\cdot}79$

SHIRLEY POPPY. No. 2.



$N = 425$

$\sigma = 1{\cdot}519 \pm {\cdot}035$

$\mu_3 = 0{\cdot}264 \pm {\cdot}286$

$\sqrt{\beta_1} = {\cdot}075 \pm {\cdot}081$

$\beta_2 = 3{\cdot}366 \pm {\cdot}160$

$\kappa = {\cdot}006$

Mean $= 12{\cdot}595 \pm {\cdot}050$

Type II.

$m = -10{\cdot}700$

$a^2 = -42{\cdot}470$

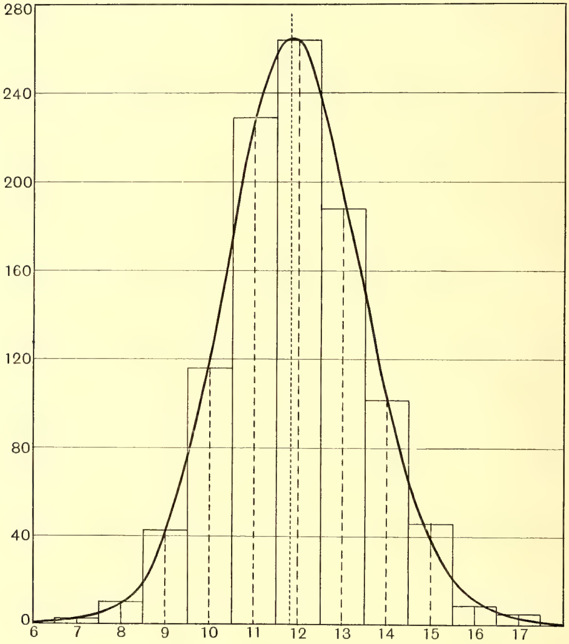
Bands on Capsules

No.	Observed	Calculated
7	0	·32
8	1	1·54
9	12	6·77
10	21	24·06
11	55	62·27
12	116	106·30
13	113	111·60
14	63	71·49
15	30	29·67
16	9	8·78
17	5	2·07
18	0	·43
	425	425·3

$P = {\cdot}34$

BANDS ON POPPY CAPSULES.

Three Plants. Nos. 1 *and* 2 + *part of a third.*



$N = 1016$
$\sigma = 1{\cdot}578 \pm {\cdot}024$
$\mu_3 = {\cdot}551 \pm {\cdot}203$
$\sqrt{\beta_1} = {\cdot}140 \pm {\cdot}052$
Mean $= 11{\cdot}956 \pm {\cdot}033$
Mode $= 11{\cdot}862$
$P = {\cdot}71$

$\kappa = {\cdot}027$
$\beta_2 = 3{\cdot}308 \pm {\cdot}104$

$r = 24{\cdot}694$
$m = 13{\cdot}347$
$\nu = -4{\cdot}098$
$a = 7{\cdot}575$

TYPE IV.

No.	Observed	Calculated
6	1	·25
7	2	1·75
8	9	9·66
9	43	40·74
10	116	118·70
11	229	223·5
12	264	264·2
13	189	200·0
14	102	102·7
15	46	38·71
16	9	11·61
17	6	3·0
	1016	1014·82

VI. Some Recent Criminological Works.

By CHARLES GORING, M.D.

In a bulky volume, entitled *Les femmes homicides**, Dr Pauline Tarnowsky records the results of an anthropometrical survey she has carried out upon 160 Russian women imprisoned for murder. The preface states, very modestly, the essential purpose of the author's undertaking. "I regard this book," she says, "as nothing more than a collection of material, and shall be happy if future workers may find some use for it in constructing the edifice of criminal anthropology which still awaits the great architect." Throughout the work, there is abundant evidence that Dr Tarnowsky has collected her material with a devotion and conscientious industry which insures its reliability; and these qualities, combined with the excellent arrangement and lucid exposition of her facts, cannot fail to make the book of value to all in need of criminal statistics.

The outline of the author's plan is, briefly, as follows. Each of the individuals studied was, firstly, submitted to physical measurement: all the principal measurements recognised by anthropologists were taken of head, face and body. Secondly, the majority of these criminals were examined with reference to the functional condition of their sense organs and, at the same time, the condition of their reflexes and muscular power and the presence of any deviation from the normal in physical structure, including the so-called stigmata of degeneration, were noted. Lastly, so far as they could be ascertained, the more important particulars of the family antecedents were noted for each individual:—such as the character, inclinations, habits, occupation and standard of living of the parents; special emphasis being laid upon the occurrence of heritable diseases and upon the more or less marked inclination towards alcoholic excess. Apart from some preliminary chapters dealing historically with general principles, the great bulk of the volume is made up of the records of these measurements and observations. The record of each individual is presented seriatim and upon a definite plan, and is supplemented in each case by a very readable and psychologically interesting narrative of the personal history of the murderess, the circumstances leading up to her crime, its probable motives, and the methods of its execution. Finally, the salient features contained in the records are brought together and classified in the form of tables. Accompanying tables of similar records relating to different classes of criminal and non-criminal Russian women are presented for purpose of comparison. The work concludes with an account of some general deductions which, in the opinion of the author, follow from the analytical and comparative study of her data. *Les femmes homicides* is a valuable contribution of conscientious spade-work, and as such will find general acceptance.

Having said this we are bound to add that, in the treatment of her evidence, the author has been animated too much by a spirit of partisanship for the old school of criminologists; we mean that school associated with the name of César Lombroso—to whom, by the way, the author dedicates her work. We think that the evidence from her data is altogether too slight to justify so emphatic and far-reaching a statement as the following, which the author affirms to be her fixed conviction—"that there exists an *enormous* physical and psychical difference between the female criminal and the normal woman of the same race and class and that this conclusion follows directly (1) from the measurements, (2) from the facts of heredity, and (3) from the evidence of the degeneration stigmata contained in the records." First, with regard to the evidence from the measurements. For comparison with the table of average measurements of "les femmes homicides," three corresponding tables are provided relating to (1) "femmes instruites,"

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* Paris, Félix Alcan.

(2) "femmes villageoises illettrées," (3) "femmes voleuses et prostituées." As the author admits, between the murderesses and thieves the difference in the average measurements is insignificant and, we would add, is in every case within the limits of the probable error due to random sampling. But, says the author, "ce qu'il vous saute aux yeux" is the difference in length, breadth, and circumference of head between non-criminal peasant women and women of the criminal class. As a matter of fact what strikes the attention particularly is the precisely similar difference, in an opposite direction, between non-criminal peasant women and women of the educated class: and, the question arises, is not this difference due to the same cause in both cases? We note, from the records, that of the 160 individuals measured, five were of illegitimate parentage, and 13 were orphans from birth; and these, combined with the fact that 70°/₀ of the parents of the remainder were debauched with drink, suggest the provisional proposition that the descending scale of head measurements, from "les femmes instruites" at one extreme to "les femmes homicides" at the other, may be related to an increasing scale of poverty, and is more likely an expression of defective nutrition, during the early years of life and growth, than of inherited criminal diathesis.

The facts of heredity are interesting and suggestive, although we look in vain for the evidence in support of the alleged inheritance of a criminal type which the author contends is contained therein. We note from the records of these facts that 134 criminal children had 12 criminal parents, that seven criminal lunatics had two parents who were insane and two who were epileptic, and that 141 criminal children had 12 parents who died of phthisis. Now, while these figures suggest the existence of a positive correlation in the direct parental inheritance of criminality, insanity, and phthisis, their proportions are not in conformity with the plausible hypothesis in the mind of the author, that insanity and phthisis in one generation may influence the appearance of criminality in the next. In fact, from the evidence of the records, the only condition that might seem likely to give rise to physical degenerative changes associated with a criminal diathesis in the descendants, is parental alcoholism. The records give for the parents of "homicides," 71°/₀ alcoholic; for the parents of non-criminal peasant women, 16°/₀ alcoholic. However, in view of the fact, that the majority of the homicidal women were also themselves alcoholic, the significance of this alcoholism in the parents is somewhat modified. What we want to know is, not the relative incidence of alcoholism in the parents of criminal and non-criminal children, but its relative incidence amongst the parents of alcoholic children who are, or are not, criminal as well.

Of the value of the stigmata of degeneration, quoted as proof of the existence of a criminal type, we have only space here to say that the objection to this kind of observation, so popular with criminologists of the old school, is that no one can precisely define what it is that is being observed; and that the error due to any unconscious bias in the observer may be so large as to render the value of such evidence entirely nugatory. That the criminal women were found to have ten times as many stigmata as the women who were not criminal may seem to be conclusive evidence that ought to tell. Scientifically, however, such evidence is unfruitful and leaves the mind cold and unconvinced.

We wish we had more space in which to discuss some other points in this interesting book. But we have already strayed from our chief purpose which is to congratulate Dr Pauline Tarnowsky on her work and to recommend it to the attention of biometricians.

---

*Britain's Blot** is the title of a book dealing with "Recidivism"—a general term including all kinds of Habitual Criminality and Habitual Petty Delinquency. The meaning of this figurative title is given in the introduction of the book. "The face of Society," writes the author, "is mottled all over by the actions of its law breakers and transgressors, and on this mottled area

* G. F. Sutherland, M.D., F.R.S.E.: W. Green and Sons, Edinburgh.

there stands out in bold relief the dark and discreditable blot of recidivism, perpetually hanging like a pall"..., &c. &c. The author has evidently had a wide experience of the evil he describes; but we are not convinced that he sees very clearly either the inherent nature or the intent or causation of this evil. For instance, in one part of the book the recidivist is described as "an irrepressible Frankenstein, rearing its hydra-head, and stalking through the land with limbs of brass." From this we take it that the recidivist looms in the imagination of the author as a monster, uncontrolled, and at large, a constant menace to Society, which is responsible for its creation. In another place, however, the writer states in plain, dogmatic English his confident belief that not only is every kind of social, economic, and environmental condition a principal cause of recidivism, but that another principal cause is to be found in the spiritual constitution of the delinquent. This second opinion, besides negativing the Frankenstein theory, leaves us in wonderment as to what other possible agency, apart from the supernatural, could be further quoted as a cause of recidivism. It is evident the book has been written to influence social reformers, and others of the general public interested in social questions, rather than to be a guide to serious students of sociology.

The subject of this book is treated in the main discursively, and with a general sprinkling of statistics that are more pretentious than helpful. In Chapter I, dealing with general statistics of crime, some figures and diagrams are presented of the number of the general population who are apprehended, imprisoned and convicted every year. It is stated that from these figures "Great Britain's recidivism can be seen at a glance"—a statement based upon the inference that the number of recidivists existing in prison corresponds to the total number of recidivists in existence; a most fallacious inference. To estimate the true dimensions of *Britain's Blot*, it is obviously necessary to know not only the number of imprisoned recidivists, relatively to the population at large, but also how many of the population at large are recidivists out of prison. That is to say, how many recidivists now free have already been in prison, and how many will find their way to prison before they die. No attempt has been made to answer this interesting, subtle and elusive statistical question. Chapters II and III deal with the age, education, occupation and geographical distribution of delinquents. The part played by illiteracy in the making of criminals, is asserted and deplored; as is also deplored the fact that a large majority of criminals are drawn from the labouring classes. In view of the fact that out of Britain's population of 38 millions, over 30 millions belong to the class of general labourers, and are presumably illiterate, it is quite idle, without the aid of a correlation calculus, to attempt to form any estimate of the causal association between these conditions and crime. In Chapter V, which consists of eight pages, a general survey is undertaken of no fewer matters than criminal anthropology, criminal anthropometry, criminal physiognomy, biometrics, and national eugenics. Some measurements are also given of the mean head length, head breadth, and stature of 370 criminals. The next chapter grants five pages to an exposition of criminal lunacy and eight pages to a discussion of legal and metaphysical conceptions of free will, criminal responsibility, and punishment. In Chapter VII, which deals with the causation of recidivism, degeneracy and heredity, answers to all of these problems are successfully begged in twelve pages. The conclusion that physical and mental characters are inherited in the same way, and at the same rate, seems to the author "a natural," as well as "a feasible" conclusion: but he asserts it to be "the climax of absurdity" to suppose that the criminal breeds the criminal. The remaining chapters deal with criminal jurisprudence and penology.

To those who prefer a wide approximate view to an accurate perception of detail, who prefer opinions to facts and rhetoric to figures the book should appeal sympathetically. To biometricians it will probably not be of very great interest.

VII. Note on Partial Leucosis in a Hen.

We owe the following account and photographs to the kindness of Mr W. Coles Finch of Luton, Chatham. Inquiry seems to show that the experience is uncommon among poultry-breeders, and its publication may lead to further observation of like phenomena, which would possibly throw light on whether plumage without moult can change by loss of pigment and thus otherwise than by abrasion.

The father of the hen in question was a pure Indian Game and the mother a true Houdan. Seven broods of this cross came under notice. In five the birds, watched to the third year, remained black. In the other two cases the broods of chickens were all true black. In one brood, however, one bird, a hen, turned from black to spotted in her first year, she has been spotted, not white, ever since, but the spots have diminished each year. Her present condition is given in Fig. 1. She has been crossed with an Indian Game and had two spotted black and white chickens. In the second brood one hen on the approach of her first winter gradually got paler and paler, turned into gray and finally pure white before the end of the year. No photograph unfortunately was taken. She remained white all next summer but after moult renewed her plumage to spotted black and white. She is shown in Fig. 2 with a normal sister for comparison. She remained spotted all next summer, but when her sisters moulted, she shed her tail-feathers only ; on wing and body feathers the black gradually changed to white. This commenced in beginning of October and by November 1 she was nearly quite white, as shown in Fig. 3*. By the middle of November she was practically pure white (the mark on wing occurring on one feather being the last vestige of black) as is shown in Fig. 4, a sister being given for comparison (the light colour is only reflection of sheen). On Fig. 5 black and white feathers plucked from time to time from October to November are given to indicate that the change appears to be one of pigment and scarcely of feather or due to abrasion. The bird has thus been once black, once white, then once spotted and again white. It was mated this year with an Indian Game but unfortunately has died without laying.

* The plumage is somewhat disarranged after a struggle in catching her.

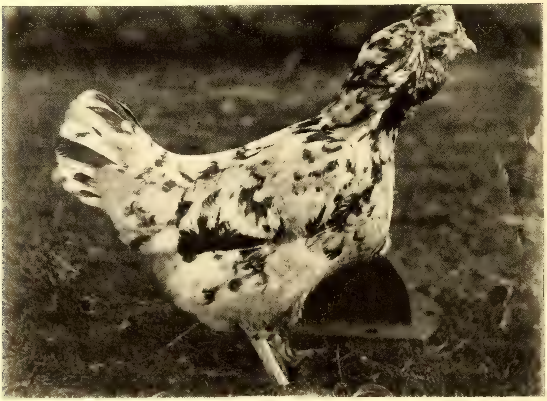


Fig. 1.

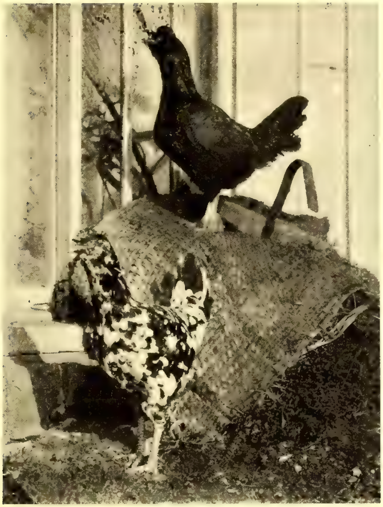
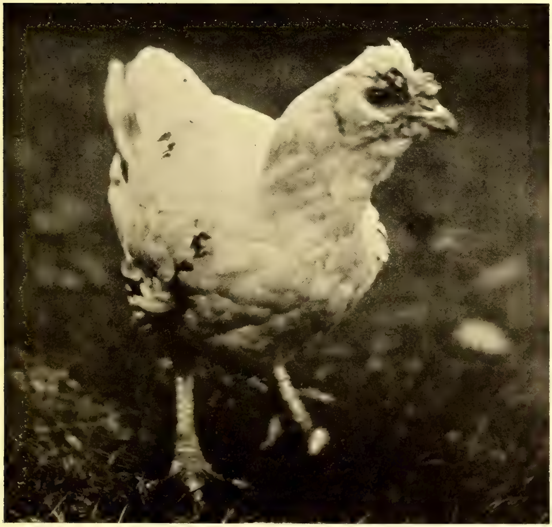


FIG. 2.



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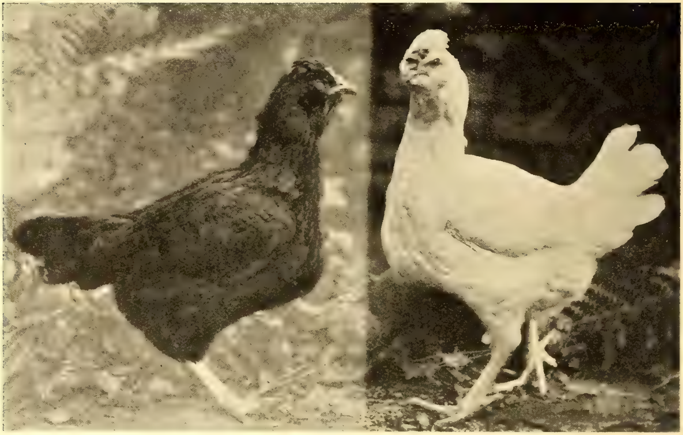

Fig. 4.

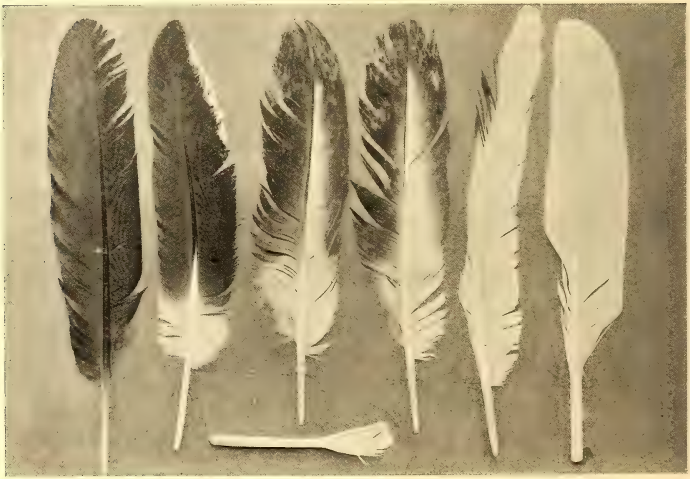

Fig. 5.

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HISTOLOGICAL EXAMINATION OF A CASE OF ALBINISM.

BY J. E. ADLER, F.R.C.S. AND J. M^c^INTOSH, M.D.

(From the Bacteriological Laboratory, London Hospital, E.)

ALBINISM has now long been a subject of interest to medical science, and many papers have appeared from time to time descriptive of its clinical peculiarities. The literature on the histological findings of the affected parts is, however, most scanty, and is usually confined to descriptions of pieces of the iris removed at operations for traumatic and other affections.

In looking through the literature only one record of a complete post-mortem examination on an albino could be found. This was made by Buzzi in 1783 (2). The case from his description appears to have been a complete albino, but his descriptions of the minute anatomy are far from satisfactory. Carron du Villards (1) and Maury (8) record dissections of human albino eyes; the former with absence of all pigment, the latter with traces of pigment.

It is a pity that a more uniform system of classification has not been adopted by the various observers in the description of their cases, so that it might have been possible to get a correct idea of the condition observed.

All the cases on which histological observations have been made, with the exception of Buzzi's case, would come under the groups of incomplete albinism, according to the classification proposed by Geoffroy Saint Hilaire (3).

In the case of partial albinism recorded by Mueller (9), there was a complete absence of all pigment in certain areas of the iris. In Nettleship's case there was a complete absence of pigment in the portion of the iris examined which was probably a fair sample. On the other hand, in Usher's cases of incomplete albinism recorded by Nettleship (10), there was diminished pigment in both choroid and retina. Clinically Usher's cases might pass for complete albinos, and this would mean that we could not distinguish by clinical methods between cases of incomplete and complete albinism.

The case with which we are about to deal, and which we had the opportunity of examining, was that of a female child of 10 weeks. The child was the second albino in the family, and the mother had taken the matter so much to heart that she committed suicide. She left the child exposed, and from the effects of this it died.

Family History. The family history of the child was specially investigated in order to ascertain whether there was a hereditary history of albinism; but no evidence of albinism could be found in any of the ancestors or relations in the four previous generations. The description of the family tree (see p. 243) connected with the case is as follows (unless marked as albino, all members are normal):—

Generation IV. 12. B. C., aet. 10, albino—pink eyes, nystagmus, white hair, big healthy boy, very bright. 13. L. C., aet. 8, ♂ healthy. 14. —. C., aet. 6, ♀ healthy. 15. N., aet. 4, ♂. 16. O. C., aet. 3, ♀ healthy. 17. C. C., aet. 6 weeks, ♀ albino, the case described. IV. 1. S. W., aet. 25, ♂ healthy. 2. L. W., aet. 24, ♂ healthy. 3. E. W., aet. 22, ♀ healthy. 4. B. W., aet. 22, ♀. 5. P. K., aet. ?, ♂. 6. A. K., aet. ?, ♀. 7. G. D., aet. 13, ♂. 8. D. E., aet. 8, ♂. 9. J. E., aet. 9, ♂. 10. S. E., aet. 10, ♀. 11. M. E., aet. 11, ♀. 18. B. S., aet. 9, ♂. 19. D. S., aet. 7½, ♀.

Generation III. 1. R. W., father of family IV. 1—4. 2. M. A. W., wife of III. 1, healthy (eldest sister). 3. A. K., sister of III., 2. 4. P. K., her husband, father of family IV. 5—6. 5. L. C., mother of IV. 7, and wife of III. 6. 6. G. D., husband. 7. L. C., ♀, died. 8. E. C., ♀, died. 9. D. C., ♂, married to III. 10. 10. S. E., ♀, became wife of III. 9, mother of IV. 8—11. 11. A. C., ♂, died. 12. A. C., ♂, youngest child, father of case, married III. 27 (L. R.), alive. 13—17, Children of W. C., II. 3—4, all unaffected, but all details unknown. 18, 19, 20, and 21, Children of A. C., II. 5—6, all unaffected, details unknown. 22. A. R., ♀, spinster, pale, but not a true albino. 23. F. R., ♀. 24. G. R., ♂, in asylum. 25. H. R., ♂, married D. S., ♀, III. 26. ♀, wife of IV. 18, 19. 27. ♀, L. R., married A. C., III. 11, died examined, found unaffected, mother of the two albinos in the family. 28. ♂, S. R., just married to III. 29. ♀. 30. ♀, P. R., just married to III. 31. ♂, B. 32. If any, and how many, unknown. 33—34. Offspring of II. 11 and 12, age and sex unknown, known however to be normal. 35. If any, no details known. 36—45. Offspring of II. 15—16, all unaffected, but no details as to order and sex.

Generation II. 1. D. C., alive, ♂, aet. 74, unaffected. 2. M. A. S., ♀, wife of above, unaffected, parents of III. 1—12. 3. W. C., ♂, died, age unknown, unaffected. 4. ♀, his wife, unaffected. 5. A. C., ♂, died, age unknown, unaffected. 6. ♀, his wife, unaffected. 7. A. C., ♀, married G. R. (8), ♂, parents of family III. 22—30. 9. S. C., ♀, married 10. ♂, age and initials unknown. 11. L. C., ♀, married 12, ♂, age and initials unknown. 13. A. C., ♀, married 14, ♂, age and initials unknown. 15. M. C., ♀, married 16, ♂, age and initials unknown; II. 15—16 parents of III. 36—45. 17. ♀, sister of G. R., II. 8, unaffected. 18. ♀, sister of G. R., II. 8, unaffected. 19. ♀, sister of G. R., II. 8, unaffected.

Generation I. 1. D. C., ♂, died, aet. 72, unaffected. 2. ♀, wife of above, unaffected, all other details unknown. 3. R., ♂, unaffected, all details unknown. 4. ♀, unaffected.

Pathological Anatomy—Post-mortem Notes.

External Appearances. The body was that of a well nourished female child of about six weeks. Rigor mortis was absent.

There were small papular erythematous patches on the extensor aspects of both wrists, forearms and legs; symmetrically arranged ecchymoses were seen on the upper and inner aspects of both thighs.

The back, shoulders, and sides were covered with fine white downy hairs. The nipple, umbilicus, perineum, and anus were unpigmented. The scalp was covered with a large quantity of fine silky hair about an inch long. No hairs were pigmented. The eyebrows consisted of white hairs, while the eyelashes were regular, even, and white.

The eyes were pink; the pupils equal, and moderately small. The irides were entirely devoid of all pigment, and were pinkish in colour, with fine greyish white radially arranged striations.

Weight, 7 lbs. 14 ozs. Circumference of head, 14½ ins.

On opening the thorax, the thymus gland was found to be enlarged, and reached down as far as the level of the third costal cartilage; weight, ½ oz. There was no excess of fluid in either pleural cavity, and there were no pleuritic adhesions. The lungs were collapsed, definite capillary bronchitis being present. The mucous membrane of the bronchi was congested, and the larger bronchi contained some yellowish pus. There was no consolidation. Weight, right lung, 1¼ ozs.; left lung, 1¾ ozs.

The Heart. There was no excess of fluid in the pericardium. The foramen ovale was not completely closed, and admitted a fine probe. The valves and aorta were normal in appearance. Weight, ¾ oz.

The Stomach. The stomach was almost empty, and very pale in colour; two or three ecchymotic spots were noticed on its anterior surface near the greater curvature. The stomach contained about two or three drachms of mucous fluid, which was odourless. The mucous membrane appeared healthy.

The Liver protruded one finger's breadth below the costal margin, and, on being cut into, appeared to be normal. No thickening of the capsule was noticed. The gall bladder was filled with the usual yellow bile, and the ducts were found patent.

The Spleen (¼ oz. in weight) was firm in texture. On section, the malpighian bodies were not distinctly marked.

The Kidneys (¼ oz. each in weight) were lobulated. Their capsule stripped readily, and the perirenal fat was plentiful and normal in colour. The ureters were patent, and were followed into the bladder. The latter was empty, and was healthy in appearance. Both suprarenal glands were present, and appeared normal.

The small intestine was, for the most part, thin and empty, and the mucous membrane, though pale in colour throughout, appeared to be healthy; while the Peyer's patches presented a normal appearance. The large intestine, appendix, and rectum were examined and found to be healthy. The pancreas appeared to be normal, both in regard to structure and weight; the duct of Wirsung was patent. The mesenteric, bronchial, cervical, and axillary glands were normal in size, and presented the usual appearance on section. The tongue, tonsils, and oesophagus were normal. The uterus and cervix were infantile in shape, but the ovaries were both enlarged and cystic. Pieces were removed from each organ, and placed in a solution of formalin (1 in 10) for histological examination.

The Brain was removed intact (weight, 1 lb. ½ oz.), and was very soft. The dura mater, as is usual in children, was adherent at the suture lines. The cerebral sinuses were found to be normal. The pituitary body with the sella turcica appeared to be normal, and were removed en masse.

Histopathology. All the organs and tissues were examined microscopically, but, apart from a complete absence of any pigment in certain parts, no histological peculiarities were observed. With regard to the liver, spleen, lungs, kidneys, pancreas, thymus, thyroid, heart, and muscles, no abnormality was made out; while in the other structures, namely, brain, eyes, internal ear, suprarenal gland, skin, and hair, the only noticeable feature was a complete absence of pigment.

Brain. Sections of the cerebrum, cerebellum, crura cerebri, pons Varolii, and medulla oblongata were examined, both in the stained and unstained condition. Search was made for pigment in those parts in which pigmentation normally exists in the brain cells in the adult. The various nuclei of the pons, medulla, and cerebellum, the substantia nigra and the grey matter were examined, but contained no pigment granules.

Sections of these areas were also stained with fluorescein, but no granules were found.

The pituitary body was apparently normal in structure.

The Eyes. Both eyes were examined completely, one was fixed in formalin (10 °/₀) and the other in spirit, and then they were frozen solid and cut into right and left halves. The internal aspect of the eyeballs was like yellowish white parchment. The vessels were clearly outlined on their surface, while the fovea centralis appeared as a distinct spot of a slightly deeper tint than the surrounding retina.

Paraffin and celloidin sections were made and examined under the microscope by day and artificial light, without even a trace of pigment being found in any of the structures of the eye, whether of an ectodermic or mesodermic origin, retina, epithelium and stroma of iris or choroid : see Plate II.

Owing to the entire absence of pigment in the eyes, the anatomical structure of the irides and retina was shown to great advantage, while the arrangement of the delicate connective tissue of the iris and its continuity with the choroid and ciliary processes were clearly seen. The arrangement of the two sets of muscle fibres could be followed. The iris also showed a well-marked crypt. Unfortunately the retina had not been fixed soon enough after death to preserve the rods and cones layer intact, their situation however can be easily recognised.

The choroid and retina were examined in the stained and unstained condition.

Internal Ear. The petrous portion of the temporal bone containing the middle and internal auditory apparatus was removed entire and decalcified. The internal ear contained no pigment in the mesoblastic cells of the perilymphatic spaces.

The Suprarenal Gland. The medulla and the cortex appeared to be of the usual proportions; no brown granules were to be observed in those cells of the cortex which are usually found to contain pigment in adult life.

The Skin. Except for the absence of pigment, the only noticeable feature seen in sections of the skin was the non-granular appearance of the rete mucosum.

The Hair contained no pigment; after the hairs were cleared in xylol they became quite transparent, and could be seen only with considerable difficulty.

In conclusion, one must say that the case described above is one of complete albinism, on account of the complete absence of any melanin pigment in those structures which normally contain it.

This child, though it might also be described in Professor Karl Pearson's terminology as an internal albino, does not solve the question as to whether internal albinism exists in adult albinos, because practically no pigment exists in any part of the brain of a child before the sixth month of life (Marinesco (7)). The same is also true for those glands, such as the suprarenal, which contain pigment in adult life (Koelliker (5)).

There was no evidence of any maldevelopment or mental feebleness in any member of the family, such as has been reported in certain albinotic stocks. But though there is undoubted proof that in certain instances deficiency of pigment does go along with such conditions, we have no reason to suppose that there is a constant relation between them.

In all probability this child, had it lived, would have remained a complete albino even in spite of the fact that the colour of the iris in all infants darkens considerably during the first year of life, because it is very unlikely that pigmentation would occur after so long a period of complete absence, namely, over 10 months, as melanin pigment, according to Koelliker (4), makes its appearance in the retina towards the fourth week of intra-uterine life. The specimens kindly lent by Dr Low (6) showed distinct pigment in the retina at this period: see Plate I.

Where the albinism entered this family is not at all clear; if the condition is not a sport it is most likely that the condition was handed down from the (paternal) grand-parents, as the parents of the child were cousins. From a Mendelian view both parents might be described as carrying recessive albinism, but we have really no evidence from this single case to support this view.

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DESCRIPTION OF PLATES.

Plate I. Fig. 1. Section of Eye in 4th week embryo.
Fig. 2. Section of Eye in 10th week foetus.
We owe these sections to the kindness of Dr A. Low.

Plate II. Fig. 3. Section of retina of albino case.
Fig. 4. Section of retina of normal infant.
Fig. 5. Section of iris of albino case.
Fig. 6. Section of iris from normal infant.

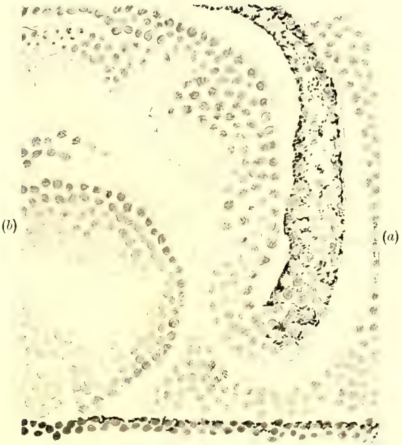


Fig. 1. Section of eye in 4th week embryo (10 mm. long).
(a) Pigment granules in pigment layer of retina.
(b) Lens.
[Dr Low's section]

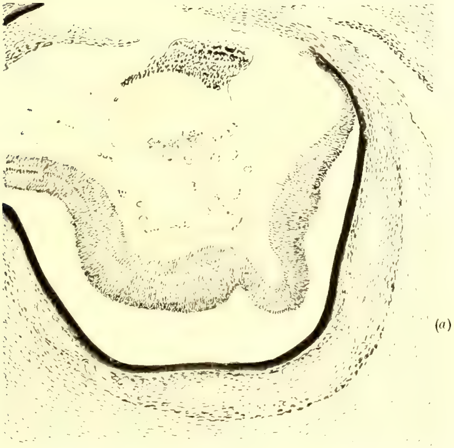


Fig. 2. Section of eye, 10th week foetus (28 mm. long).
(a) Pigment layer of retina.
[Dr Low's section]

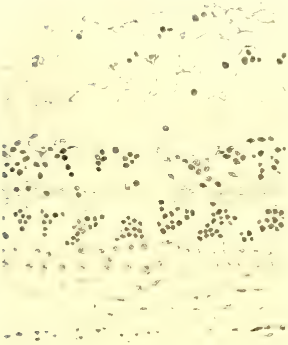


FIG. 3. Section of retina of albino child, with absence of all pigment.

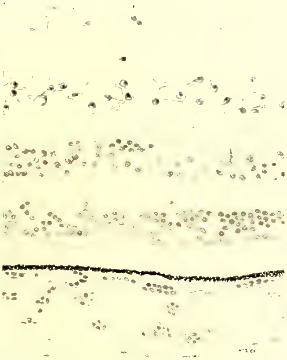


FIG. 4. Section of retina of normal child of same age, showing pigment layer.

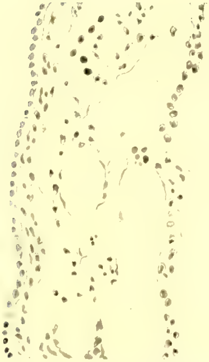


FIG. 5. Section of iris of albino child, with absence of all pigment.

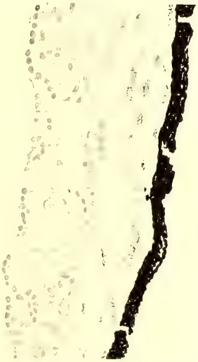
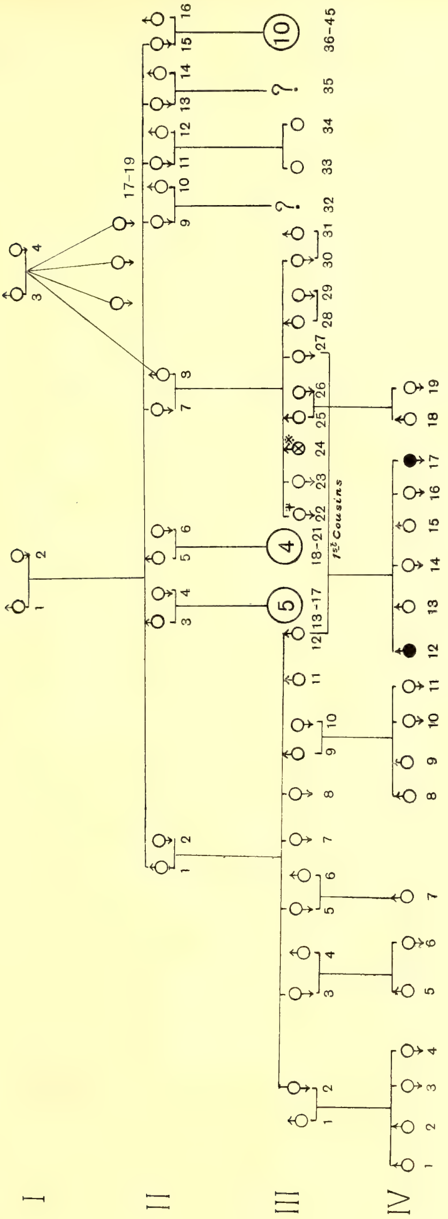


FIG. 6. Section of iris from normal child of same age.


I
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* In an asylum.

NOTE ON INTERNAL ALBINISM.

By KARL PEARSON, F.R.S.

THE authors of the previous paper having referred to my use of the term *internal albinism*, it may not, perhaps, be out of place to say a few words on that subject. This is not done because any conclusions on this point have been so far worked out, but because the investigations upon which they must turn will perforce have to be made by others with not only the adequate training but the requisite material,—a material which is rarely available, and which without such suggestions, as I wish to bring forward, may possibly not be used to the maximum advantage when it does occur. This must be my excuse for putting forward some rather vague suggestions instead of demonstrated facts; I hope that, among many other questions of interest, they may be partially answered when the opportunity arises.

With the exception of an occasional eye or part of an eye no post-mortem account of a human albino has appeared in print as far as I am aware between 1783 and 1910. Buzzi in 1783 had not the histological knowledge which would have enabled him to throw light on our modern problems. In Adler and M^cIntosh's case the age of the child was too slight for the case to give information as to the internal pigment of a fully developed human albino. In the case of the post-mortem examination recorded below the answers to several of my problems could have been given, but the problems were not in the mind of the medical man who made the examination. It is because such examinations must be very infrequent that I venture to throw out my queries and suggestions in the hope that they may reach physiologists and pathologists; and that although originating in a layman they may consider them worthy of investigation.

In the case of albinism in man there is no doubt of the largely relative character of the absence of superficial pigment*. Almost every grade of albinism, whether we consider the eyes, hair, or skin, occurs. But there is something more than graduated intensity of pigmentation; there may be local complete absences of pigment; besides incomplete albinism, we have partial albinism and this in a great variety of stages. Intensity and distribution of the superficial pigment varies immensely in man. Now besides the superficial pigment, there occurs in man internal pigment, notably in certain brain centres, internal ear, liver, kidneys, suprarenals, lungs, etc. It can scarcely be doubted that this internal pigmentation plays some part in the case of several of the organs mentioned, possibly a most important part in the case of the brain centres, the suprarenal glands, and the internal ear. Now is there any reason for supposing that this pigmentation varies like the superficial pigmentation in intensity and locality? If so, what are the signs, outward and visible, of incomplete or partial internal albinism? Where would it be most reasonable to look for the occurrence of such cases? It is noteworthy that pigment superficially injected into an albino does not remain on the surface, but is fixed by the suprarenals, liver, spleen and lungs; it is eliminated by the kidneys and intestinal epithelium. In Addison's disease, not only bronze areas but *leucotic* areas occasionally appear. It would seem reasonable therefore to suppose that there is not absolute independence between internal and external pigmentation; other cases could be cited, but the suggestion made is merely that it is not wholly unreasonable to look first for defects of internal pigment in those individuals or stocks where we find defects of external pigment.

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* The terms "superficial pigment" and "external albinism" are used to cover the visible pigment, or pigment exposed to the action of light. They include the retinal pigment although its morphological origin brings it into closer relation with the internal pigment of the brain centres.

Of course it is logical in the first place to demonstrate that internal pigment occurs in albinos at all. Adler and McIntosh's case throws no light from the human albino on this point; no more did Buzzi's. But Mr O'Donoghue* kindly investigated some albino rabbits, rats and mice for me and he assures me that internal pigment does occur in these as in normal rats and mice. We may assume that this is true for human albinos, but this is just one of the points which need elucidation as the occasion arises. However, the suggestion made is that it might be reasonable to look for defects, partial or incomplete of internal pigment, in the same stocks as we find albinism occurring in—either in the case of albinotic, or in the case of non-albinotic members.

I now pass to some special points. The partially albinotic cat, the cat with white hair and blue eyes, is invariably deaf. Recent histological examination of such cats has shewn that among other important deviations from the normal in the internal ear, the walls of the perilymph chamber lack pigmentation. We are not justified as yet in saying that this is related to the deafness, but the possibility that it is must be borne in mind. I am not aware of any careful histological examination of the pigment in the internal ear of congenital deaf-mutes. That some forms of congenital deaf-mutism arise from partial internal albinism is conceivable and worth inquiring into as opportunity arises.

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* "I have found that the absence of pigment so noticeable in the external appearance of albinos does not seem to affect the internal organs. In examining the viscera of albino rabbits I found it impossible to discover any difference in colour that would enable them to be distinguished from those of normal animals. The spleen, liver, bile, kidney, adrenal body, Peyer's patches and salivary glands are to all appearances quite similarly coloured. Two rats and two mice, one of each pair being an albino and the other normal, gave similar results on examination and in addition the mesenteries of both normal and albino were alike in containing pigment cells whose presence was demonstrated by silver nitrate." Chas. H. O'Donoghue.

The disappearance of superficial pigment is one of the marks of senility, it affects the eyes and hair alike. Is it possible that this loss of superficial pigment can ever be accompanied by a loss of internal pigment? For example, possibly in the case of the brain centres? Senile imbecility and in its milder form mere senility might possibly be associated with a weakening of the intensity of pigmentation in certain of the brain centres. The bigger question now arises: Is it conceivable that any forms of imbecility, not all, but *any*, are associated with defective brain pigmentation? This opens up an exceedingly interesting inquiry for those who are able to examine the brains of imbeciles and the senile insane —what, if any, differentiation is there in the centres of brain pigmentation*? Ophthalmoscopic investigation shows a high percentage of incomplete albinism of the eye among the insane.

The Japanese mouse is usually a piebald with completely albinotic eyes, it possesses the singular habit of dancing, or better of spinning. This has been attributed to various causes. Is it wholly and absolutely certain that the spinning characteristic of this mouse may not also be associated with its partial albinism? A special type of human imbecile is the spinning idiot, who rotates like a top.

Fawn or red pigmentation occurs in close association with many types of albinism. Some types of red hair are, we have just found, wholly lacking in pigment granules. I have some reason for believing that a larger proportion of red hair will be found among congenital deaf-mutes than in the general population; it will in a forthcoming memoir on albinism by Messrs Nettleship, Usher and myself be shown to be much in excess in albinotic stocks. Examining albinotic stocks there appears, partly in the albinotic and partly in the non-albinotic members to be an excessive proportion of imbecility, idiocy and deaf-mutism†. It may well be that these associated defects are only stigmata of general degeneracy, but the possibility of their having some relation to absence of internal pigment is, I venture to think, worth bearing in mind. So much for the suggestions I wish to make; they at least suggest observations which can be made by those so fortunate as to be able to examine histologically cases of albinism, imbecility and deaf-mutism. They may be quite illusory but even an erroneous suggestion is often a useful guide in directing observation to unregarded points, and the authors of the above-mentioned memoir on albinism have sought in vain for any report on possible differentiation in the internal pigment of albinos—it does not appear to have been dealt with.

In the account which I give below of a third autopsy on an albino, the subject was a boy, who combined deaf-mutism and spinning idiocy with albinism. Any two of these characters would on the theory of probability be an extraordinarily rare chance combination, but the existence of all three seems almost to demonstrate some interrelation in pathological origin. This deaf-mute spinning albino was the son of a deaf-mute mother. Dr Alfred Miller, the Medical Superintendent of the

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* Dr G. E. L. Keyes, I am glad to say, has this investigation in hand.

† Actual statistics will be given in the memoir referred to above.

Warwick County Lunatic Asylum, to whom I owe a copy of the Asylum notes on this case, has made full inquiries as to the family history of the patient, but without success, this being no doubt due to the fact that the patient was illegitimate. I have cordially to thank Sir James Crichton-Browne, for mentioning that he had many years ago seen this boy, and suggesting inquiry at the Asylum.

Particulars as to a Deaf-mute Albino Idiot.

Admitted, July, 1852. "W. L. aet. 9. A congenital idiot. Is the natural son of a deaf and dumb mother. For some time after going to the Birmingham Asylum where he has been about 16 months he had frequent bleeding from the nose, and used to scream a good deal in the night. He is deaf and dumb; but not destitute of intelligence. He is very active, running and jumping about, climbing chairs, tables, etc., and has a habit of spinning like a top, though not nearly so much as formerly. He is of an affectionate disposition." There is nothing interesting related in the notes after this date. He is stated to have had frequent attacks of severe epistaxis—and generally to have been in poor health.

He died on Sept. 18, 1865.

"Post-mortem examination 23 hours after death. Body extremely emaciated, rigor mortis well marked. Skull thick and dense. Dura mater thickened and firmly adherent. Arachnoid slightly thickened. Convolutions of cerebrum adherent.

Cerebrum.—Middle lobes very prominent and the corresponding fossae at the base of the skull proportionately deep. Grey matter of convolutions firmer than usual.

Punctae sanguineae very numerous. About three drachms of fluid in each lateral ventricle. Choroid plexuses granular. Right optic thalamus eroded superficially along the tract of the taenia semicircularis and its substance soft and easily broken down. Anterior part of the floor of the 3rd ventricle of a gelatinous appearance and a buff colour. Pineal body soft and friable. Corpora quadrigemina soft and almost diffluent. Cerebellum.—A small patch of superficial softening on each side of the anterior part of the under surface. Olivary bodies small and ill-defined. Pleurae adherent throughout. Pericardium universally adherent externally. Lungs full of tubercular matter.

I certify that the apparent cause of death of the above-named W. L. was Phthisis (chronic)."

W. H. PARSEY, *Medical Officer.*

(Copied and forwarded to K. P. by Alfred Miller, Medical Superintendent, in letter of Jan. 25, 1909.)

ON A NEW METHOD OF DETERMINING CORRELATION, WHEN ONE VARIABLE IS GIVEN BY ALTERNATIVE AND THE OTHER BY MULTIPLE CATEGORIES.

By KARL PEARSON, F.R.S.

(1) In a recent paper* I have dealt with the case when one variable is given by alternative categories and the other proceeds by quantitative groupings, for example when a population whose ages are recorded is classed into anaemic and non-anaemic; or a population whose cephalic indices have been measured is classed into conscientious and unconscientious sections. The object of the present paper is to carry the idea involved in such double row correlation tables a stage further by supposing the variable classified into multiple categories to be purely qualitative. Such variables I have elsewhere spoken of as categoric variables, to distinguish them from graduated variables. As illustrations we may take the eye colour of an individual and the presence or absence of pulmonary tuberculosis. Here the eyes may be grouped into seven or eight classes, but we can only record the presence or absence of the disease; it is true that the immunity of the individual is almost certainly a graded variable, but we are not able at present to measure it, and must be contented with an alternative classification. Again, the presence of malaria and the skin tint is another illustration from the same field; for while skin tint is undoubtedly a graduated variable, no medical inquirer has probably the energy or time to do more than group into multiple classes—perhaps seven or eight—separated by certain skin tint mosaics.

Hitherto such double row contingency tables could only be reduced by using the fourfold process, originally published by me in the *Phil. Trans.* Vol. 195 A, pp. 1—4. Such a process has two disadvantages; it assumes that a graduated variate which follows the Gaussian law is at the bottom of both classifications; and further it requires us to make *one* fourfold grouping, where many are possible. The choice left to the operator is not unique, and different selections may modify somewhat the result. Even if the mean result of taking several divisions be adopted, we do not get rid of an arbitrary element in the process, and the labour may occasionally be excessive.

The fourfold process, absolutely necessary as it often is, should on the whole be reserved for those cases in which a fourfold division has arisen from the very nature of the data and not be applied to double row tables, where by using it we lose all the advantage of having one variable in multiple categories. It is further to be noted that in many cases it is impossible to suppose any adequately graduated variable behind the multiple categoric variate. For example, if we wish to find the relationship between crime and occupation, we may easily select 10 or 12 groups of kindred occupations, and discuss the number of convictions in each such group of individuals, but it would be unreasonable to suppose any continuous variable behind this grouping, while we might reasonably suppose a continuous variable behind the tendency to criminality. The present method has the advantage of giving a unique solution for double row contingency tables; it makes less appeal to hypothesis than the fourfold division method applied to such cases, and it is far more rapid in execution.

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* *Biometrika*, Vol. VII. pp. 96—105.

Finally, the necessity for some such method has been forced upon my notice by the great frequency with which such double row contingency tables have recently occurred in the work of those dealing with medical, sociological and criminological statistics in my laboratory.

(2) The theory of the method is very simple. Let y be the categoric variate in multiple classes, and x the alternative variate. We suppose x to be ultimately continuous, but by using y we do not suggest continuity. For such a system the correlation ratio, η, has a perfectly definite meaning, it is the ratio of the standard deviation of the weighted means of the y-arrays of x's to the standard deviation of the whole population; in symbols:

$$\eta^2 = \frac{S\{n_y(x_y - \bar{x})^2\}}{N\sigma_x^2} \quad \text{..................................(i)},$$

where n_y is the number of individuals in any y-category; x_y is the mean x for this category and $\bar{x}$ and σ_x are the mean and standard deviation of the x's of the whole population N. This value of η, it is well-known*, must lie between 0 and 1; it becomes equal to r, the correlation coefficient, when the regression is linear. Further the mean standard deviation of the weighted arrays is known to be $\sigma_x\sqrt{1-\eta^2}$, a value which becomes small as η approaches unity, or x becomes absolutely defined by the y-category in which it lies.

Now let ${}_y\sigma_x$ be the standard deviation for the x's which fall into a special category y, then we may take:

$$\eta^2 = \frac{1}{N} S\left(n_y \frac{x_y^2}{{}_y\sigma_x^2} \cdot \frac{{}_y\sigma_x^2}{\sigma_x^2}\right) - \left(\frac{\bar{x}}{\sigma_x}\right)^2 \quad \text{........................(ii)},$$

since $$S(n_y x_y) = N\bar{x}.$$

Now we have just seen that the mean value of ${}_y\sigma_x^2 = \sigma_x\sqrt{1-\eta^2}$, we shall therefore assume that the distribution is sufficiently homoscedastic for us to replace ${}_y\sigma_x^2/\sigma_x^2$ by its mean value $(1-\eta^2)$. Hence:

$$\eta^2 = (1-\eta^2)\, S\left(\frac{n_y}{N} \frac{x_y^2}{{}_y\sigma_x^2}\right) - \left(\frac{\bar{x}}{\sigma_x}\right)^2,$$

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* *Roy. Soc. Proc.* Vol. 71, p. 303. "On the General Theory of Skew Correlation and Non-linear Regression." *Drapers' Research Memoirs*, 1905. Dulau & Co.

and from this it follows that

$$\eta = \left[\frac{\dfrac{1}{N} S\left\{n_y\left(\dfrac{x_y}{{}_y\sigma_x}\right)^2\right\} - \left(\dfrac{\bar{x}}{\sigma_x}\right)^2}{1+\dfrac{1}{N}S\left\{n_y\left(\dfrac{x_y}{{}_y\sigma_x}\right)^2\right\}}\right]^{\frac{1}{2}} \text{..........................(iii).}$$

This is the value of the correlation ratio, based, so far, solely on the assumption that the arrays are sufficiently homoscedastic to permit of our replacing the ratio ${}_y\sigma_x/\sigma_x$ by its mean value. There has been no hypothesis of linear regression, still less of a Gaussian distribution.

It now remains to consider on what hypothesis we shall determine $x_y/{}_y\sigma_x$ and $\bar{x}/\sigma_x$, which are not directly determinable from the data. We shall assume that the mean of each array of x's as well as the mean of the whole population may be found in terms of their respective standard deviations by tables of the probability integral. In other words we assume the distribution of the x-variable to be approximately Gaussian. Thus we find uniquely the correlation ratio of a double row contingency table by the following assumptions:

(i) The alternative variate is sufficiently Gaussian to admit of the means being found by tables of the probability integral.

(ii) The arrays are approximately homoscedastic. This latter condition is unnecessary, if the alternative variable be replaced by one grouped in *three* classes, for then the actual value of ${}_y\sigma_x/\sigma_x$ can be found for each array and we need not give it the mean value $\sqrt{1-\eta^2}$. With regard to the multiple categoric variable no assumption as to the nature of its distribution has been, or need be, made.

Illustrations.

(3) I will take first some illustrations from a forthcoming paper dealing with the relation of pigmentation to psychical characters.

(A) *Conscientiousness in Boys and Hair Colour.* The conscientiousness was classified as "keen" and "dull" with an intermediate class, which was found to be relatively very small and distributed accordingly between the two main groups. We have the following table:

Hair Colour.

Conscientiousness.	Red	Fair	Brown	Dark	Jet Black	Totals
Keen ...	59·5	441·5	433	354·5	20·5	1309
Dull ...	24·5	194·5	188	162·5	16·5	586
Totals	84	636	621	517	37	1895

We have, using Sheppard's Tables:

$$\bar{x}/\sigma_x = -\cdot 4980,$$

and for the arrays in order from red to jet black:

$$\bar{x}_1/\sigma_{x_1} = -\cdot 5485, \quad \bar{x}_2/\sigma_{x_2} = -\cdot 5077, \quad \bar{x}_3/\sigma_{x_3} = -\cdot 5166,$$
$$\bar{x}_4/\sigma_{x_4} = -\cdot 4837, \quad \bar{x}_5/\sigma_{x_5} = -\cdot 1359.$$

There is thus an almost continuous decrease in conscientiousness as we pass from red to jet black hair*. Or, there is a very slight but sensible relationship of hair pigmentation and conscientiousness. Using formula (iii) we find

$$\frac{1}{N} S \left\{ n_y \left(\frac{x_y}{\sigma_y} \right)^2 \right\} = \cdot 2515.$$

Hence
$$\eta^2 = \frac{\cdot 2515 - \cdot 2480}{1\cdot 2515} = \cdot 00276,$$

and
$$\eta = \cdot 053.$$

The assumption that conscientiousness is a continuous variate approximately following the normal distribution appears a reasonable one.

(B) *Self-consciousness and Hair Colour.* The data for 1914 schoolboys are given in the following table:

Hair Colour.

	Red	Fair	Brown	Dark	Jet Black	Totals
Selfconscious ...	42·5	344·25	296·5	238	14·25	935·5
Unselfconscious ...	36·5	322·75	351·5	251·5	16·25	978·5
Totals	79	667	648	489·5†	30·5†	1914

Here $\bar{x}/\sigma_x = +\cdot 02816$, and for the arrays:

$$\bar{x}_1/\sigma_1 = -\cdot 0953, \quad \bar{x}_2/\sigma_2 = -\cdot 0404, \quad \bar{x}_3/\sigma_3 = +\cdot 1066,$$
$$\bar{x}_4/\sigma_4 = +\cdot 0346, \quad \bar{x}_5/\sigma_5 = +\cdot 0823.$$

These give
$$\frac{1}{N} S \left\{ n_y \left(\frac{x_y}{\sigma_y} \right)^2 \right\} = \cdot 0052,$$

whence we find
$$\eta = \cdot 066.$$

It will be seen that hair colour is more closely related to self-consciousness than to conscientiousness, the red and fair boys being more self-conscious than the dark haired boys—an experience which may I think be confirmed on adults. The means of the arrays are, however, not so smooth* as in the former case and there is some sign of the regression curve being heteroclinal.

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* The values of x reduced to a common unit σ_x are: $\bar{x}_1/\sigma_x = -\cdot 5477$, $\bar{x}_2/\sigma_x = -\cdot 5070$, $\bar{x}_3/\sigma_x = -\cdot 5159$, $\bar{x}_4/\sigma_x = -\cdot 4830$, $\bar{x}_5/\sigma_x = -\cdot 1357$.

† One boy was placed intermediate in hair colour between dark brown and jet black.

(C) *Handwriting and Eye Colour.* I have chosen this illustration because it admits of using formula (ii) directly. The material is represented in the following table:

Handwriting.

		Very Good	Good	Moderate	Poor	Bad	Totals
Eye Colour.	Light ...	45	249	312·75	112·5	28·75	748
	Medium ...	60	252	302·75	99	27·25	741
	Dark ...	41·5	137·5	182	63	8	432
	Totals	146·5	638·5	797·5	274·5	64	1921

Let us call the range of medium eye colour h and measure the means first from the boundary of light and medium (x) and then from the boundary of medium and dark (x'). We have†

$$
\begin{array}{llll}
\bar{x}/\sigma_x = {\cdot}2809, & \bar{x}'/\sigma_x = {\cdot}7558, & h/\sigma_x = 1{\cdot}0368, & \\
\bar{x}_1/\sigma_1 = {\cdot}5039, & \bar{x}_1'/\sigma_1 = {\cdot}5731, & h/\sigma_1 = 1{\cdot}0770, & \sigma_1/\sigma_x = {\cdot}9626, \\
\bar{x}_2/\sigma_2 = {\cdot}2794, & \bar{x}_2'/\sigma_2 = {\cdot}7880, & h/\sigma_2 = 1{\cdot}0674, & \sigma_2/\sigma_x = {\cdot}9713, \\
\bar{x}_3/\sigma_3 = {\cdot}2737, & \bar{x}_3'/\sigma_3 = {\cdot}7448, & h/\sigma_3 = 1{\cdot}0184, & \sigma_3/\sigma_x = 1{\cdot}0180, \\
\bar{x}_4/\sigma_4 = {\cdot}2280, & \bar{x}_4'/\sigma_4 = {\cdot}7405, & h/\sigma_4 = {\cdot}9684, & \sigma_4/\sigma_x = 1{\cdot}0705, \\
\bar{x}_5/\sigma_5 = {\cdot}1276, & \bar{x}_5'/\sigma_5 = 1{\cdot}1504, & h/\sigma_5 = 1{\cdot}2780, & \sigma_5/\sigma_x = {\cdot}8112.
\end{array}
$$

We see that the last column gives us the means of finding the ${}_y\sigma_x/\sigma_x$ of the formula (ii). Also we have alternative methods of determining η according as to whether we use the first or second column, i.e. x or x'.

Using the first column we find:

$$\eta^2 = {\cdot}0835 - {\cdot}0789 = {\cdot}0046,$$

or:

$$\eta = {\cdot}068.$$

Using the second column we find:

$$\eta'^2 = {\cdot}5754 - {\cdot}5713 = {\cdot}0041,$$

or:

$$\eta' = {\cdot}064.$$

The close accordance of these results speaks well for the application of the method to the present material. We may test it again, assuming that medium and dark eyes are classed together, and using formula (iii), we find in this case

$$\eta'' = {\cdot}070,$$

---

* Reduced to a common unit σ_x, they are $\bar{x}_1/\sigma_x = -{\cdot}0951$, $\bar{x}_2/\sigma_x = -{\cdot}0403$, $\bar{x}_3/\sigma_x = +{\cdot}1064$, $\bar{x}_4/\sigma_x = +{\cdot}0345$, $\bar{x}_5/\sigma_x = +{\cdot}0821$.

† Reduced to σ_x we have $\bar{x}_1/\sigma_x = {\cdot}4851$, $\bar{x}_2/\sigma_x = {\cdot}2714$, $\bar{x}_3/\sigma_x = {\cdot}2780$, $\bar{x}_4/\sigma_x = {\cdot}2441$, $\bar{x}_5/\sigma_x = {\cdot}1035$.

agreeing for all practical purposes with the value found for η above, and indicating that although the arrays as shown in the fourth column are heteroscedastic, it is practically sufficient to give ${}_y\sigma_x/\sigma_x$ its mean value. Examining Column 1, we see that the mean eye colour of the arrays shifts continuously towards the light as we pass from good to bad handwriters. The association is small, but can hardly be without significance.

(4) Alcoholism is a character which may be reasonably treated as one of graduated intensity, and one accordingly to which our method may be reasonably applied. I take as an illustration some statistics of crime most kindly provided by Dr Charles Goring.

(D) *Alcoholism and Type of Crime.* In the following table the crime categories have been arranged in the order of relative intelligence as ascertained from Dr Goring's correlation investigations on crime and intelligence.

Class of Crime.

	Arson	Rape	Violence	Stealing	Coining	Fraud	Totals
Alcoholic ...	50	88	155	379	18	63	753
Non-alcoholic ...	43	62	110	300	14	144	673
Totals	93	150	265	679	32	207	1426

We find: $\bar{x}/\sigma_x = +\cdot 0704$, and for the arrays*:

$$\bar{x}_1/\sigma_1 = +\cdot 0946,\quad \bar{x}_2/\sigma_2 = +\cdot 2190,\quad \bar{x}_3/\sigma_3 = +\cdot 2145,$$

$$\bar{x}_4/\sigma_4 = +\cdot 1463,\quad \bar{x}_5/\sigma_5 = +\cdot 1573,\quad \bar{x}_6/\sigma_6 = -\cdot 5119.$$

Using formula (iii), we determine, $\eta = \cdot 234$. There is thus quite a sensible correlation between the extent of alcoholism in the criminal and the type of crime he will commit†. Starting with fraud, there is increased alcoholism as we pass to stealing, coining, violence and rape. An exception, however, occurs in the case of arson, the class of crime whose perpetrators are least intelligent among criminals. Those who commit arson appear to be less alcoholic than coiners and thieves—criminals who require considerable sobriety for success in their callings. Probably the coiners would stand above thieves in the scale, did not the returns include the offenders who pass false coin as well as the producers; further, highly skilled but alcoholic workmen, who cannot retain on that account their berths, may in the case of certain trades tend to gravitate into this class of criminal. While rape and crimes of violence arise from a low grade of intelligence combined with alcoholism, it would seem that arson has a low grade of intelligence for its source, but probably some character such as revenge rather than alcoholism ultimately at its source.

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* Reduced to a common unit: $\bar{x}_1/\sigma_x = +\cdot 0920$, $\bar{x}_2/\sigma_x = +\cdot 2129$, $\bar{x}_3/\sigma_x = +\cdot 2085$, $\bar{x}_4/\sigma_x = +\cdot 1422$, $\bar{x}_5/\sigma_x = +\cdot 1529$, $\bar{x}_6/\sigma_x = -\cdot 4977$.

† The reader must remember that this has no bearing on the problem of whether alcoholism leads to crime.

(E) *Alcoholism in Parents and Intelligence in the Offspring.* I owe the data for this illustration to Miss Elderton, who is treating alcoholism in parents and its effect on children at length in a forthcoming memoir. Some of her data are given in the accompanying table:

Intelligence of Sons (Edinburgh Data).

		Excellent	Good	Medium	Dull	Defective	Totals
Parents.	Alcoholic ...	19	125	116	46	7	313
	Non-alcoholic ...	23	97	99	45	9	273
	Totals	42	222	215	91	16	586

We have: $\bar{x}/\sigma_x = -\cdot 0857$, and for the arrays*:

$$\bar{x}_1/\sigma_1 = +\cdot 1197,\quad \bar{x}_2/\sigma_2 = -\cdot 1587,\quad \bar{x}_3/\sigma_3 = -\cdot 0993,$$
$$\bar{x}_4/\sigma_4 = -\cdot 0138,\quad \bar{x}_5/\sigma_5 = +\cdot 1573,$$

this last value being of very small weight. The results are most remarkable; if we omit the excellent intelligences, we obtain an almost uniformly descending intensity of alcoholism in the parent, as the intelligence of the child diminishes; the least alcoholic parents have the less intelligent offspring. If we treat the table by a fourfold division, between "good" and "medium" intelligences we find for the correlation of parental sobriety and intelligence in offspring the value $r = -\cdot 03$. It is clear that in the "excellent" group some special cause is at work, which gives the regression its non-linear character, and leads to a correlation ratio η of ·086. The same want of regular increase of intelligence of offspring with sobriety of parent is found in the case of parents and daughters, where the correlation coefficient is zero and the correlation ratio $\eta = \cdot 102$, the "excellent" as well as the "dull" and "defective" groups of daughters having in this case the more alcoholic parents. It is clear that the relation of alcohol in parents to intelligence in children is not of a close kind and the association, such as it is, is not of a continuous character and apparently different in the two sexes.

(F) *Health of Daughters and Alcoholism of Fathers.* Miss Elderton has provided me with the following table:

Health of Daughters (Manchester Data).

		Healthy	Fair	Delicate	Phthisical and Epileptic	Died Young	Totals
Fathers.	Alcoholic ...	104	10	24	8	38	184
	Non-alcoholic ...	254	16	59	41	83	453
	Totals	358	26	83	49	121	637

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* Reduced to a common standard: $\bar{x}_1/\sigma_x = +\cdot 1193$, $\bar{x}_2/\sigma_x = -\cdot 1581$, $\bar{x}_3/\sigma_x = -\cdot 0989$, $\bar{x}_4/\sigma_x = -\cdot 0989$, $\bar{x}_5/\sigma_x = -\cdot 0137$, $\bar{x}_6/\sigma_x = +\cdot 1567$.

$$\bar{x}/\sigma_x = +{\cdot}5567.$$

Here[*] $\bar{x}_1/\sigma_1 = +{\cdot}5519$, $\bar{x}_2/\sigma_2 = +{\cdot}2934$, $\bar{x}_3/\sigma_3 = +{\cdot}5559$,
$\bar{x}_4/\sigma_4 = +{\cdot}9811$, $\bar{x}_5/\sigma_5 = +{\cdot}4844$.

The correlation ratio for this table is $\eta = {\cdot}133$, but a little examination shows that there is no definite association of poor health in child and alcoholism in father. Of the healthy children and of the delicate children the fathers have equal intensity of alcoholism; the phthisical and epileptic children come in an overwhelming proportion from the sober parents. The parents of those who die young are somewhat more alcoholic than those who survive. The small difference may well be due to more accidents and not to worse health in the children of alcoholic parents.

(F) *bis. Health of Sons and Alcoholism of Fathers.* Miss Elderton's table is as follows:

Health of Sons (Manchester Data).

		Healthy	Fair	Delicate	Phthisical and Epileptic	Died Young	Totals
Fathers.	Alcoholic ...	122	9	24	8	42	205
	Non-alcoholic...	328	37	71	37	133	606
	Totals	450	46	95	45	175	811

Here[†] $\bar{x}/\sigma_x = +{\cdot}6658$, and for the arrays:

$\bar{x}_1/\sigma_1 = +{\cdot}6095$, $\bar{x}_2/\sigma_2 = +{\cdot}8573$, $\bar{x}_3/\sigma_3 = {\cdot}6662$,
$\bar{x}_4/\sigma_4 = +{\cdot}9239$, $\bar{x}_5/\sigma_5 = +{\cdot}7063$.

The correlation ratio is here $\eta = {\cdot}089$, but the correlation coefficient obtained from a fourfold table of healthy against the remainder is $-{\cdot}06$, or the children of alcoholic fathers are the more healthy. It is quite easy to see from the above results how little evidence there is for alcohol in fathers affecting the health of sons. As in the case of girls, but more sensibly so, the healthy children have fathers somewhat more alcoholic than the average; the delicate children have fathers not less sober than the average; the phthisical and epileptic children have fathers markedly more sober than the average population, and the early deaths are slightly more frequent among the children of sober parents. It is difficult to see in either of these tables evidence for alcohol as a chief source of bad health or poor intelligence in the offspring.

(5) As a last illustration I take certain statistics of the severity of attack in cases of smallpox and the strength of immunity provided by vaccination. I owe the following data to Dr John Brownlee of the City of Glasgow Hospital.

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[*] Reduced to a common standard: $\bar{x}_1/\sigma_x = +{\cdot}5470$, $\bar{x}_2/\sigma_x = +{\cdot}2908$, $\bar{x}_3/\sigma_x = +{\cdot}5510$, $\bar{x}_4/\sigma_x = +{\cdot}9725$, $\bar{x}_5/\sigma_x = +{\cdot}4801$.

[†] Reduced to a common unit: $\bar{x}_1/\sigma_x = +{\cdot}6071$, $\bar{x}_2/\sigma_x = +{\cdot}8539$, $\bar{x}_3/\sigma_x = +{\cdot}6635$, $\bar{x}_4/\sigma_x = +{\cdot}9202$, $\bar{x}_5/\sigma_x = +{\cdot}7035$.

(G) *Severity of Attack and Strength of Immunity due to Vaccination.* I look first at the data arranged as a two row correlation table.

Severity of Attack.

Years since Vaccination.	Haemorrhagic	Confluent	Abundant	Sparse	Very Sparse	Totals
Since vaccination *0* to *25* years	5	38	120	176	148	487
Over *25* years or unvaccinated	44	251	388	308	211	1202
Totals	49	289	508	484	359	1689

$$\bar{x}/\sigma_x = +\cdot5583.$$

Here*
$$\bar{x}_1/\sigma_1 = +1\cdot2700,\quad \bar{x}_2/\sigma_2 = +1\cdot1194,\quad \bar{x}_3/\sigma_3 = +\cdot7185,$$
$$\bar{x}_4/\sigma_4 = +\ \cdot3488,\quad \bar{x}_5/\sigma_5 = +\ \cdot2218.$$

From these results we find $\eta = \cdot321$, a very sensible relationship, and we see how the mean vaccination immunity increases as we pass from haemorrhagic to very sparse cases. There is very little doubt that we are dealing here with a perfectly graduated and continuous cause and effect, remarkably different in its character from the alcoholic influences we have just discussed.

To illustrate this point, I proceed to investigate the same data arranged as a three rowed table, using formula (ii). We have:

Severity of Attack.

Years since Vaccination.	Haemorrhagic	Confluent	Abundant	Sparse	Very Sparse	Totals
0—25 years ...	5	38	120	176	148	487
25—45 years ...	29	155	299	268	181	932
Over *45* or unvaccinated	15	96	89	40	30	270
Totals	49	289	508	484	359	1689

We have†:
$$\bar{x}/\sigma_x = \cdot9951,\quad \bar{x}'/\sigma_x = \cdot5583,\quad h/\sigma_x = 1\cdot5533;$$
$$\bar{x}_1/\sigma_1 = \cdot5069,\quad \bar{x}_1'/\sigma_1 = 1\cdot2700,\quad h/\sigma_1 = 1\cdot7769,\quad \sigma_1/\sigma_x = \cdot8742;$$
$$\bar{x}_2/\sigma_2 = \cdot4329,\quad \bar{x}_2'/\sigma_2 = 1\cdot1194,\quad h/\sigma_2 = 1\cdot5523,\quad \sigma_2/\sigma_x = 1\cdot0007;$$
$$\bar{x}_3/\sigma_3 = \cdot9338,\quad \bar{x}_3'/\sigma_3 = \cdot7185,\quad h/\sigma_3 = 1\cdot6524,\quad \sigma_3/\sigma_x = \cdot9401;$$
$$\bar{x}_4/\sigma_4 = 1\cdot3875,\quad \bar{x}_4'/\sigma_4 = \cdot3488,\quad h/\sigma_4 = 1\cdot7363,\quad \sigma_4/\sigma_x = \cdot8946;$$
$$\bar{x}_5/\sigma_5 = 1\cdot3815,\quad \bar{x}_5'/\sigma_5 = \cdot2218,\quad h/\sigma_5 = 1\cdot6032,\quad \sigma_5/\sigma_x = \cdot9689;$$

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[*] Reduced to a common unit: $\bar{x}_1/\sigma_x = +1\cdot2028$, $\bar{x}_2/\sigma_x = +1\cdot0602$, $\bar{x}_3/\sigma_x = +\cdot6805$, $\bar{x}_4/\sigma_x = +\cdot3303$, $\bar{x}_5/\sigma_x = +\cdot2101$.

[†] Reduced to a common unit: $\bar{x}_1/\sigma_x = \cdot4431$, $\bar{x}_2/\sigma_x = \cdot4332$, $\bar{x}_3/\sigma_x = \cdot8779$, $\bar{x}_4/\sigma_x = 1\cdot2413$, $\bar{x}_5/\sigma_x = 1\cdot3385$.

leading, according to our horizontal division, to $\eta = \cdot319$ and $\eta' = \cdot337$ with a mean value ·328, in excellent agreement with the two row table value ·321.

Lastly the whole data has been taken as a contingency table:

Severity of Attack.

Years since Vaccination.	Haemorrhagic	Confluent	Abundant	Sparse	Very Sparse	Totals
0—10	—	1	6	11	12	30
10—25	5	37	114	165	136	457
25—45	29	155	299	268	181	932
Over *45*	11	35	48	33	28	155
Unvaccinated	4	61	41	7	2	115
Totals	49	289	508	484	359	1689

Worked out as a 25-fold contingency table by mean square contingency we have

$$C_2 = \cdot335,$$

which lies between the two values of η found from the three rowed table by the new method, and is within the probable error of either. This agreement is very satisfactory evidence that it was legitimate to apply the new method to a case of this kind, and that in this particular instance the material was closely normal.

(6) The illustrations given will, I think, show that a wide range of problems can be dealt with by the new method, and that results found by it are closely comparable with results obtained by other processes. It does not assume linearity of regression, and in fact as in the examples on alcoholism it brings out in a very effective way the deviations from linearity. The hypothesis used is: that in the case of the variable with alternate categories, we can suppose it continuous and sufficiently normal in character to have the mean values found from tables of the probability integral. If the arrays are not sufficiently homoscedastic for us to replace their standard deviations by their mean value, then we must use if we can a three and not a two rowed table. It is always desirable to have if possible such a table, because it enables us to test the nature of the variability in the arrays. Our last illustration, however, shows that very considerable variations in the standard deviations of the arrays do not sensibly modify the result obtained by using for those standard deviations their mean value. On the whole this new method, which replaces the fourfold table method for cases in which the latter does not give a unique answer, is, I think, likely to prove of some service.

ON THE EFFECT OF A DIFFERENTIAL FERTILITY ON DEGENERACY.

By KARL PEARSON, F.R.S.

A NEW YEAR'S GREETING TO FRANCIS GALTON, 1910.

(1) I HAVE indicated in several earlier papers* the very marked changes which are produced when there is correlation between fertility and any character in a species. I have termed the changes which result in the mean and variability of the character *genetic or reproductive selection*, and have shown that if such correlation exists reproductive selection may wholly defeat or largely neutralise the results of natural selection. In actual wild life, it is very difficult to find any character sensibly correlated with fertility, and it would appear that the low value of correlations of this character is an essential condition for rapid progress by natural selection. Nature has in some way—at present not clear to us—prevented this reversal of the survival of the fitter by suppressing all *marked* correlation between the physical characters and the fertility of a species. Half a century ago, I think, this absence of correlation between fertility and physique would have been found to hold for man in this country. It is practically certain that it does not hold to-day. Artificially a differential fertility has been created; the better mental as well as physical characters can be shown to be associated with a lessened fertility and a reproductive selection has been called into play, which not only impedes, but possibly reverses natural selection. The object of the present paper is to obtain—at any rate to a first approximation—some measure of the secular changes in a race which must flow from a correlation of fertility with any character of an organism. The problem in the case of bisexual reproduction is not a wholly straightforward one. It is influenced in the first place by the intensity of assortative mating in the species. In the next place we have to ask: Does the fertility depend on the intensity of the character in one or in both parents? And lastly we must find a reasonable form for the relation between fertility and the intensity of the character. These points will be considered in the following sections.

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* See *Phil. Trans.* Vol. 187, p. 258; Vol. 192, pp. 259, 314; *R. S. Proc.* Vol. 59, p. 303. *The Chances of Death*, Vol. I. p. 63 et seq.

(2) We will suppose that some character—e.g. grade of intelligence—influences the fertility. We have then to consider whether this character in the male and female influences equally or unequally the fertility of the pair. It has been asserted by some that academic training in the woman lessens her power of child-bearing. In this case the more intelligent women would have fewer offspring, but owing to assortative mating such women marry the more intellectual men, and whether intelligence was or was not associated with a lesser grade of fertility in the male, assortative mating would handicap the fertility of the intelligent male, and the more intelligent males would be practically less fertile.

Let ξ_1 and ξ_2 be the deviations from their respective sex means of male and female for any character, σ_1 and σ_2 the standard deviations with respect to this character in the two sexes, represented by the subscripts 1 and 2. Let r_{12} be the correlation of ξ_1 and ξ_2, i.e. the intensity of assortative mating.

Then it is reasonable to assume that the fertility of the pair is some function of x, where x is a linear function of ξ_1 and ξ_2, or:

$$x = c_1 \frac{\xi_1}{\sigma_1} + c_2 \frac{\xi_2}{\sigma_2} \qquad \text{(i).}$$

The mean of x is clearly zero, and we can free ourselves from the influence of either parent by putting c_1 or c_2 zero. If fertility were related to x by a simple linear correlation then we should obtain, it is well known, the highest correlation of fertility (y) and x by taking (the subscript 3 denoting fertility):

$$c_1 = \frac{r_{13} - r_{12} r_{23}}{1 - r_{12}^2}, \qquad c_2 = \frac{r_{23} - r_{12} r_{13}}{1 - r_{12}^2} \qquad \text{(ii),}$$

or proportional to these quantities. We need not at present however assume any special values for c_1 and c_2. Further, it is desirable to suppose that the curve of mean fertility for each value of x is not necessarily linear, but of a more general type, allowing us to make fertility a maximum at other grades than the extreme values of the character.

(3) This leads us to the next point. What law of fertility seems reasonable? We want a law of fertility which will allow closely of the fertility (a) increasing nearly uniformly and at any given rate with the character, (b) decreasing nearly uniformly and at any given rate with the character, and (c) being concentrated with any degree of intensity round any grade of the character, and falling away on both sides of this maximum. All these conditions are fulfilled if we assume the mean fertility y for any grade of the character x, not to be given by a line but by a normal curve, e.g.

$$y = y_0 e^{-\frac{1}{2}(x-k)^2/\sigma_0^2} \qquad \text{(iii).}$$

Here by a proper choice of σ_0 and k, we can make the distribution of fertility fall or rise with the character (i.e. k positive or negative and both k and σ_0 very large,—k so taken that the centre of fertility lies outside the region of the range of values of x), or concentrate the fertility on any grade of the character (i.e. make

k equal that value, and take σ_0 small). It appears therefore that the distribution indicated, if not ideal, will still serve as a first approximation to the type of cases likely to occur.

There is, however, an argument of still greater weight for the suggestion that a fertility curve of this kind cannot be widely removed from the actual state of the case; namely, when we measure almost any human character (i) in parents and (ii) in their offspring we find both distributions very approximately normal. This could not possibly be the case unless the fertility curve were itself approximately normal. We include of course under this the case in which σ_0 is indefinitely great, or the fertility is not associated with the character in question, which is probably what we may describe as the *organic* condition of affairs. Of course in actual working we have to regard many special features. For example, in the matter of intelligence, it is fairly obvious that the abler men and women have fewer children; but is it true that the fertility falls again towards the extremely dull? This is a point which needs careful consideration; I think it probable, especially when we deal with net and not with gross fertility. Extreme mental defect leads to neglect of offspring, to life in institutions and limited marriage. For example, pauper fertility is higher than criminal fertility. I shall illustrate later this point.

(4) Let us now turn to the direct problem. A parentage has the deviation x of the character and σ_x is the variability of this deviation; the deviation in the offspring is z with a variability σ_z and a correlation ρ. Then, if there were no weighting with a differential fertility, the frequency surface would be

$$u = u_0 e^{-\frac{1}{2}\frac{1}{1-\rho^2}\left(\frac{x^2}{\sigma_x^2} - \frac{2\rho xz}{\sigma_x\sigma_z} + \frac{z^2}{\sigma_z^2}\right)} \quad \text{........................(iv)}$$

in the usual way. But we must multiply this by the average fertility of each grade to obtain the frequency distribution. This will be

$$v = v_0 e^{-\frac{1}{2}\frac{(x-k)^2}{\sigma_0^2}} \times e^{-\frac{1}{2}\frac{1}{1-\rho^2}\left(\frac{x^2}{\sigma_x^2} - \frac{2\rho xz}{\sigma_x\sigma_z} + \frac{z^2}{\sigma_z^2}\right)} \quad \text{........ ....(v)}$$

$$= v_0 e^{-\frac{1}{2}\frac{k^2}{\sigma_0^2}} \times e^{-\frac{1}{2}\frac{1}{1-\rho^2}\left\{x^2\left(\frac{1}{\sigma_x^2} + \frac{1-\rho^2}{\sigma_0^2}\right) - \frac{2xk(1-\rho^2)}{\sigma_0^2} - \frac{2\rho xz}{\sigma_x\sigma_z} + \frac{z^2}{\sigma_z^2}\right\}} \quad \text{...(v) } bis$$

$$= v_0' \times e^{-\frac{1}{2}\frac{1}{1-R^2}\left\{\frac{(x-m_1)^2}{\Sigma_1^2} - \frac{2(x-m_1)(z-m_2)R}{\Sigma_1\Sigma_2} + \frac{(z-m_2)^2}{\Sigma_2^2}\right\}} \quad \text{...............(vi),}$$

where v_0' is a fresh constant, i.e. independent of x and z, and m_1 and m_2 are the means of effective parentages (i.e. weighted parentages) and of actual sons, Σ_1 and Σ_2 their standard deviations and R their correlation. We have by finding the centre of the elliptic contour of (v) *bis* in the usual way:

$$m_1 = k\sigma_x^2/(\sigma_0^2 + \sigma_x^2) \quad \text{..............................(vii),}$$

$$m_2 = m_1\rho\sigma_z/\sigma_x \quad \text{....................................(viii).}$$

The first of these equations shows the shifting of the parental mean, the change being due to differential fertility; the second equation shows that the shift of the offspring mean exactly follows the shift of the parental mean, or the mean of the offspring is the offspring of the mean parent. This gives at once the secular change due to weighting the parentages with their fertility.

Further we must have

$$\frac{1}{(1-R^2)\Sigma_1^2} = \frac{1}{1-\rho^2}\left(\frac{1}{\sigma_x^2} + \frac{1-\rho^2}{\sigma_0^2}\right),$$

$$\frac{1}{(1-R^2)\Sigma_2^2} = \frac{1}{1-\rho^2}\left(\frac{1}{\sigma_z^2}\right),$$

$$\frac{R}{(1-R^2)\Sigma_1\Sigma_2} = \frac{\rho}{1-\rho^2}\frac{1}{\sigma_x\sigma_z}.$$

From these flow readily

$$R = \frac{\rho}{\sqrt{1+(1-\rho^2)\dfrac{\sigma_x^2}{\sigma_0^2}}} \quad\text{.............................(ix)},$$

$$\Sigma_1 = \frac{\sigma_x}{\sqrt{1+\dfrac{\sigma_x^2}{\sigma_0^2}}} \quad\text{..(x)},$$

$$\Sigma_2 = \sigma_z\sqrt{1-\frac{\rho^2}{1+\dfrac{\sigma_0^2}{\sigma_x^2}}} \quad\text{..............................(xi)}.$$

These results show us at once that differential fertility: (i) reduces the variability (Σ_2) of the offspring, (ii) reduces the variability of the effective parentages, (iii) reduces the apparent correlation between parentage and offspring. It should be noted that the non-differential fertility is obtained by putting $\sigma_0 = \infty$; thus any differential fertility up to $\sigma_0 = 0$, which denotes breeding from a single grade, will give lowered values for R, Σ_1 and Σ_2.

While the variabilities and correlation are thus changed, it is worth noting that the regression of offspring on parents, i.e. $R\Sigma_2/\Sigma_1 = \rho\sigma_z/\sigma_x$ and is therefore unchanged. Further the equation to the regression line is

$$z - m_2 = \rho\frac{\sigma_z}{\sigma_x}(x - m_1),$$

or since $m_2 = m_1\rho\sigma_z/\sigma_x$, it follows that

$$z = \rho\frac{\sigma_z}{\sigma_x}x$$

or is identical with the regression line for non-differential fertility. This is, of course, only a case of the general principle found long ago* that selection of a character A does not alter the regression of a second character B on A, but it is interesting to note that the effect of differential fertility is as it were a pushing of the population down its regression line.

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* *Phil. Trans.* Vol. 197, pp. 20, 21.

(5) We may next inquire into the mean fertility and variability of fertility in the population.

The fertility of the parentage x is

$$y = y_0 e^{-\frac{1}{2}(x-k)^2/\sigma_0^2} \text{ ....(iii)},$$

and the frequency of such parentages is given by

$$w = \frac{N}{\sqrt{2\pi}\,\sigma_x} e^{-\frac{1}{2}x^2/\sigma_x^2}$$

where N is the total number of such parentages.

Hence, if $\bar{f}$ = the mean fertility:

$$N\bar{f} = \int_{-\infty}^{+\infty} \frac{Ny_0}{\sqrt{2\pi}\,\sigma_x} e^{-\frac{1}{2}\left\{\frac{x^2}{\sigma_x^2}+\frac{(x-k)^2}{\sigma_0^2}\right\}} dx,$$

and accordingly:

$$\bar{f} = \frac{y_0\sigma_0}{\sqrt{\sigma_0^2+\sigma_x^2}} e^{-\frac{1}{2}\frac{k^2}{\sigma_0^2+\sigma_x^2}} \text{ ....(xii)}.$$

This is one equation linking up the indeterminate quantities y_0, σ_0, k with observable quantities like $\bar{f}$ the mean size of family in the population. We can now find the standard deviation Σ_0 of the size of families. Clearly:

$$\Sigma_0^2 = \text{Sum } (y^2)/N - \bar{f}^2,$$

$$N(\Sigma_0^2 + \bar{f}^2) = \int_{-\infty}^{+\infty} wy^2 dx$$

$$= \frac{Ny_0^2}{\sqrt{2\pi}\sigma_x} \int_{-\infty}^{+\infty} e^{-\frac{1}{2}\left(\frac{x^2}{\sigma_x^2}+\frac{(x-k)^2}{\frac{1}{2}\sigma_0^2}\right)} dx$$

$$= \frac{Ny_0^2\frac{1}{2}\sigma_0}{\sqrt{\frac{1}{2}\sigma_0^2+\sigma_x^2}} e^{-\frac{1}{2}\frac{k^2}{\frac{1}{2}\sigma_0^2+\sigma_x^2}}$$

as before. Hence:

$$\Sigma_0^2 = y_0^2\left\{\frac{\sigma_0}{\sqrt{\sigma_0^2+2\sigma_x^2}} e^{\frac{-k^2}{\sigma_0^2+2\sigma_x^2}} - \frac{\sigma_0^2}{\sigma_0^2+\sigma_x^2} e^{-\frac{k^2}{\sigma_0^2+\sigma_x^2}}\right\},$$

$$V_f^2 = \left(\frac{\Sigma_0}{\bar{f}}\right)^2 = \frac{\sigma_0^2+\sigma_x^2}{\sigma_0\sqrt{\sigma_0^2+2\sigma_x^2}} e^{\frac{k^2\sigma_x^2}{(\sigma_0^2+\sigma_x^2)(\sigma_0^2+2\sigma_x^2)}} - 1 \text{ ....(xiii)}.$$

V_f is an observable quantity and this equation may be used to find k if σ_0 has been otherwise determined. The previous equation will then give y_0. We should have to go to the third moment coefficient to determine y_0, σ_0 and k from the fertility distribution, and it may be doubted whether the result would be of much value; it seems better to endeavour to determine σ_0 and k from observing the average fertilities of those exceeding certain grades of the character.

(6) Determination of the constants of the fertility distribution from the average fertilities of those possessing more than a certain grade of the character.

Let n_h be the number of the population, N, of pairs of parents who exceed the value h of the character x; then:

$$n_h = \frac{N}{\sqrt{2\pi}\,\sigma_x} \int_h^\infty e^{-\frac{1}{2}x^2/\sigma_x^2}\,dx,$$

$$n_h/N = \frac{1}{\sqrt{2\pi}} \int_{h/\sigma_x}^\infty e^{-\frac{1}{2}x'^2}\,dx'.$$

This equation determines h/σ_x from Sheppard's Tables, since n_h/N is known.

Further if $\bar{f}_h$ be the mean fertility of those over grade h:

$$n_h \bar{f}_h = \int_h^\infty \frac{N}{\sqrt{2\pi}\sigma_x} e^{-\frac{1}{2}x^2/\sigma_x^2}\, y\,dx,$$

$$\frac{n_h}{N}\bar{f}_h = \int_h^\infty \frac{y_0}{\sqrt{2\pi}\sigma_x} e^{-\frac{1}{2}\left(\frac{x^2}{\sigma_x^2} + \frac{(x-k)^2}{\sigma_0^2}\right)} dx$$

$$= \frac{y_0 e^{-\frac{1}{2}\frac{k^2}{\sigma_0^2+\sigma_x^2}}}{\sqrt{2\pi}\,\sigma_x} \int_h^\infty e^{-\frac{1}{2}\frac{\sigma_0^2+\sigma_x^2}{\sigma_x^2\sigma_0^2}\left(x - \frac{\sigma_x^2 k}{\sigma_0^2+\sigma_x^2}\right)^2} dx$$

$$= \bar{f}\,\frac{1}{\sqrt{2\pi}} \int_{H'}^\infty e^{-\frac{1}{2}x'^2}\,dx' \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\text{xiv}),$$

where
$$H' = \frac{\sqrt{\sigma_0^2+\sigma_x^2}}{\sigma_x\sigma_0}\left(h - \frac{\sigma_x^2 k}{\sigma_0^2+\sigma_x^2}\right).$$

Now $n_h\bar{f}_h/N\bar{f}$ is clearly known, therefore H' can be found from the tables.

We will suppose two grades of the character to be h_1 and h_2 and the corresponding average fertilities $\bar{f}_{h_1}$ and $\bar{f}_{h_2}$, then H_1' and H_2' will be known; and there results

$$H_1' \frac{\sigma_x\sigma_0}{\sqrt{\sigma_0^2+\sigma_x^2}} = h_1 - \frac{k\sigma_x^2}{\sigma_0^2+\sigma_x^2},$$

$$H_2' \frac{\sigma_x\sigma_0}{\sqrt{\sigma_0^2+\sigma_x^2}} = h_2 - \frac{k\sigma_x^2}{\sigma_0^2+\sigma_x^2},$$

$$\frac{H_1' - H_2'}{h_1 - h_2} = \frac{\sqrt{\sigma_0^2+\sigma_x^2}}{\sigma_x\sigma_0}.$$

Thus:
$$\sigma_0 = \frac{\sigma_x}{\sqrt{\sigma_x^2\left(\frac{H_1'-H_2'}{h_1-h_2}\right)^2 - 1}} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\text{xv}).$$

Further:
$$\frac{H_1'(h_1-h_2)}{H_1'-H_2'} - h_1 = -k\,\frac{\sigma_x^2}{\sigma_0^2+\sigma_x^2}$$

$$= -k\,\frac{\sigma_x^2(H_1'-H_2')^2 - (h_1-h_2)^2}{\sigma_x^2(H_1'-H_2')^2}.$$

Or, finally:

$$k = \frac{H_1'h_2 - H_2'h_1}{H_1' - H_2'} \frac{1}{1 - \dfrac{(h_1 - h_2)^2}{\sigma_x^2 (H_1' - H_2')^2}} \qquad \text{..................(xvi).}$$

Thus σ_0 and k are determined, and $\bar{f}$ being known, we can find y_0. In fact

$$y_0 = \bar{f} \frac{H_1' - H_2'}{(h_1 - h_2)/\sigma_x} e^{\frac{\frac{1}{2}\{(H_1'h_2 - H_2'h_1)/\sigma_x\}^2}{(H_1' - H_2')^2 - \{(h_1 - h_2)/\sigma_x\}^2}} \qquad \text{...........(xvii).}$$

Thus the constants of the fertility distribution are fully determined.

(7) Determination of the constants of the fertility distribution, from a knowledge of the fertility of certain grades of the population. For example, when $x = -m_1\sigma_x$, let the fertility be y_1, and when $x = m_2\sigma_x$, let the fertility be y_2. Then

$$y_1 = y_0 e^{-\frac{1}{2}\left(\frac{m_1\sigma_x + k}{\sigma_0}\right)^2},$$

$$y_2 = y_0 e^{-\frac{1}{2}\left(\frac{m_2\sigma_x - k}{\sigma_0}\right)^2}.$$

Let $\sigma_x/\sigma_0 = \lambda$, $k/\sigma_0 = \mu$; we have:

$$\sqrt{\frac{2(\log y_0 - \log y_1)}{\log e}} = m_1\lambda + \mu,$$

$$\sqrt{\frac{2(\log y_0 - \log y_2)}{\log e}} = m_2\lambda - \mu,$$

$$\left.\begin{aligned} \frac{1}{m_1 + m_2}\left\{\sqrt{\frac{2(\log y_0 - \log y_1)}{\log e}} + \sqrt{\frac{2(\log y_0 - \log y_2)}{\log e}}\right\} &= \lambda \\ \frac{1}{m_1 + m_2}\left\{m_2\sqrt{\frac{2(\log y_0 - \log y_1)}{\log e}} - m_1\sqrt{\frac{2(\log y_0 - \log y_2)}{\log e}}\right\} &= \mu \end{aligned}\right\} \text{....(xviii).}$$

The signs of the roots are arbitrary and must be selected so as to accord with the needs of the problem. For example, if $m_1 = m_2 = m$, either λm or μ may be equal to

$$\sqrt{\frac{2(\log y_0 - \log y_1)}{\log e}} \pm \sqrt{\frac{2(\log y_0 - \log y_2)}{\log e}},$$

and which we choose depends on whether the maximum fertility lies between the two grades or outside and beyond the grade of larger fertility.

The above equations assume that y_0 is known, it is the maximum average fertility of any grade in the community. A third equation to determine y_0 from the average fertility of the whole community is provided on p. 262, Equ. (xii). But the equations then become troublesome and can only be solved by approximation. It will be found also that small changes in the fertility curve are not very influential in modifying the main results to be drawn from the equations set forth in this paper.

(8) Numerical Illustrations. I propose in this section to illustrate the main results of the above investigation by selecting possible values for some of the constants. I am far from asserting that these are actually existing values, but they appear not improbable values—say, for the case of ability. We are only at present acquiring knowledge of the relationship between ability and fertility in the case of man, and the examples I give are merely illustrative and intended to indicate how the algebraical formulae can be dealt with. At the same time, I think, they throw some light on rather urgent national problems. It is desirable to bring home to the minds of the thinking classes what it really does mean, if the fittest in any character have not a third of the fertility of the least fit.

Illustration I. The upper decile of a population has an average fertility rate of 2, the 50 °/₀ showing the lower values of the character an average fertility rate of 6 The fertility of the whole population has a mean value of 5. Find the changes in the population during one generation.

Probability integral tables give at once $h_1 = 1{\cdot}2816\sigma_x$.

To determine H_1', we have

$$n_{h_1} \times \bar{f}_{h_1}/N \times \bar{f} = {\cdot}1 \times 2/5 = {\cdot}04,$$

therefore

$$H_1' = 1{\cdot}7500.$$

Clearly $h_2' = 0$, $n_{h_2}\bar{f}_{h_2}/N \times \bar{f} = {\cdot}5 \times 6/5 = {\cdot}6$, from which the tables give us $H_2' = {\cdot}2533$.

A rough diagram indicates at once the sign of the quantities and shows that we must have

$$H_1' = h_1 + \sigma_x^2 k/(\sigma_x^2 + \sigma_0^2)$$
$$H_2' = \sigma_x^2 k/(\sigma_x^2 + \sigma_0^2) - h_2$$

or in this case

$$1{\cdot}7500\sigma_x\sigma_0/\sqrt{\sigma_0^2 + \sigma_x^2} = 1{\cdot}2816\sigma_x + \sigma_x^2 k/(\sigma_0^2 + \sigma_x^2),$$
$${\cdot}2533\sigma_x\sigma_0/\sqrt{\sigma_0^2 + \sigma_x^2} = \sigma_x^2 k/(\sigma_0^2 + \sigma_x^2);$$

or, subtracting:

$$1{\cdot}4967\sigma_x\sigma_0/\sqrt{\sigma_0^2 + \sigma_x^2} = 1{\cdot}2816\sigma_x,$$

whence

$$\sigma_0 = 1{\cdot}6579\sigma_x,$$

and on substituting

$$k = {\cdot}8131\sigma_x.$$

The equation

$$\bar{f} = \frac{y_0\sigma_0}{\sqrt{\sigma_0^2 + \sigma_x^2}} e^{-\frac{1}{2}\frac{k^2}{\sigma_0^2 + \sigma_x^2}}$$

will now give us y_0. On substituting the values of σ_0 and k and 5 for $\bar{f}$ we find

$$y_0 = 6{\cdot}439.$$

The distribution of fertility is accordingly given by

$$y = 6{\cdot}439 e^{-\frac{1}{2}\left(\frac{x - {\cdot}8131\sigma_x}{1{\cdot}6579\sigma_x}\right)^2}.$$

It is clear therefore that the fertility rises to a maximum at $\cdot 8131\sigma_x$ and then falls again, so that the lowest values of the character have a lowered fertility, but are still in excess of the corresponding high values. Thus m_1, the mean of the effective parentages, is given by

$$m_1 = k\sigma_x^2/(\sigma_0^2 + \sigma_x^2) = \cdot 2169\sigma_x,$$

measured *below* the old mean, and

$$\Sigma_1 = \text{standard deviation of effective parentages}$$
$$= \sigma_0\sigma_x/\sqrt{\sigma_x^2 + \sigma_0^2} = \cdot 8563\sigma_x,$$

or, the effective parentages have 16 per cent. less variability than individual parentages, i.e. parentages unweighted with their offspring. We can now find the average fertility of the lowest decile of the population:

$$\frac{n_{-h_1}\bar{f}_{-h_1}}{N\bar{f}} = \frac{1}{\sqrt{2\pi}}\int_q^\infty \frac{1}{\Sigma_1} e^{-\frac{1}{2}\left(\frac{x''}{\Sigma_1}\right)^2} dx''$$
$$= \frac{1}{\sqrt{2\pi}}\int_{q'}^\infty e^{-\frac{1}{2}x'^2} dx',$$

where

$$q' = q/\Sigma_1 = \frac{h_1 - m_1}{\Sigma_1}$$
$$= \frac{1\cdot 2816 - \cdot 2169}{\cdot 8563} = 1\cdot 2434.$$

Hence

$$\frac{n_{-h_1}\bar{f}_{-h_1}}{N\bar{f}} = \frac{1}{10}\frac{\bar{f}_{-h_1}}{5} = \cdot 10686,$$

or,

$$\bar{f}_{-h_1} = 5\cdot 34;$$

that is to say, that while the average value of the uppermost decile's fertility is only 2, that of the lowermost decile is 5·34, i.e. is still in excess of the average fertility, 5, of the population. Indeed for only about 6 per cent. of the population with the very lowest values of the character does the fertility fall below some 4 per cent.

If we take N as before for the number of pairs of parents, then the distribution of effective parentages is given by

$$w = \frac{5N}{\sqrt{2\pi}}\frac{1}{\cdot 8563\sigma_x} e^{-\frac{1}{2}\left(\frac{x - \cdot 2169\sigma_x}{\cdot 8563\sigma_x}\right)^2}.$$

We have now to consider the distribution of offspring from these effective parentages. We have to consider ρ of p. 260 in relation to σ_x and σ_z. Four hypotheses are possible:

(i) We may suppose that the character is equally influential in the case of the fertility of both sexes; thus brain capacity and resulting intellectual energy might mark equal reduction of fertility in both sexes. In this case the proper value for x is*

$$x = \sigma_1\left(\frac{\xi_1}{\sigma_1} + \frac{\xi_2}{\sigma_2}\right).$$

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* Here as later it must be borne in mind that a constant factor is of no importance.

This gives at once

$$\sigma_x = \sigma_1 \sqrt{2}\sqrt{1 + r_{12}},$$

$$\rho_{xz} = \frac{1}{\sqrt{2}}(r_{1z} + r_{2z})/\sqrt{1 + r_{12}}.$$

Here if the population were stable we should have $\sigma_1 = \sigma_z$, 1 and 2 representing as before mother and father. According to our present knowledge $r_{1z} = r_{2z} = {\cdot}4$ to ·5, = say, ·45, and r_{12} lies between ·15 and ·25, say, ·2. We have then

$$\sigma_x = \sigma_z \times 1{\cdot}5492$$

and ρ_{xz}, $= \rho$ of our formulae on pp. 260—1, = ·5809.

(ii) We may suppose that the character is equally influential with the fertility of both sexes, but that there is no assortative mating. In this case $r_{12} = 0$, the coefficients of $\frac{\xi_1}{\sigma_1}$ and $\frac{\xi_2}{\sigma_2}$ must still be equal and we have at once

$$\sigma_x = \sigma_z \times 1{\cdot}4142, \quad \rho = {\cdot}6364.$$

(iii) It is conceivable that the character affects the fertility in one sex only and that there is no assortative mating. In this case we may take $x = \xi_1$, and if the population is stable $\sigma_x = \sigma_z$, while $\rho = {\cdot}45$.

(iv) The character may be supposed to affect one sex only, but there may be assortative mating. In this case, returning to our formula on p. 259, $r_{23} = 0$ and x may be taken proportional to

$$\frac{r_{13}}{1 - r_{12}^2}\frac{\xi_1}{\sigma_1} - \frac{r_{12}r_{13}}{1 - r_{12}^2}\frac{\xi_2}{\sigma_2},$$

or we may write

$$x = \sigma_1\left(\frac{\xi_1}{\sigma_1} - r_{12}\frac{\xi_2}{\sigma_2}\right),$$

which leads to

$$\sigma_x = \sigma_1\sqrt{1 - r_{12}^2}, \quad \text{and} \quad \rho_{xz} = \frac{r_{1z} - r_{12}r_{2z}}{\sqrt{1 - r_{12}^2}},$$

or on the same assumptions as before:

$$\sigma_x = \sigma_z \times {\cdot}9798, \quad \text{and} \quad \rho = {\cdot}3674.$$

In all these cases σ_z and ρ are of course the variability of offspring for the given character and their correlation with the parentage character on the assumption of one child to the parentage. We now proceed to discuss the constants of the offspring distribution as resulting from effective parentages, i.e. the quantities m_2, Σ_2 and R of our p. 261. Here m_2 is the reduced mean value of the character,

Constants of Offspring Distribution. Illustration I.

Hypothesis ...	(i)	(ii)	(iii)	(iv)
m_2/σ_z ...	·1260	·1380	·0976	·0797
Σ_2/σ_z ...	·9539	·9445	·9747	·9818
R/ρ ...	·8976	·9065	·8805	·8721

i.e. the regression owing to differential fertility, Σ_2 is the variability of the offspring generation and should be compared with σ_z or σ_1, while R is the correlation between offspring and effective parentages.

An examination of this table shows the following general results: (*a*) the mean value of the character in the population is reduced 8 to 13 per cent. of its standard deviation according to the hypothesis we make with regard to the influence of the character on fertility. That is to say, the mean character of the population stands one-third to a half-decile lower with each new generation. (*b*) In the next place the variability is lowered two to five per cent. The probability therefore of extreme deviations is sensibly reduced; but this reduction tells only on excess of the character because the mean of the distribution has been markedly shifted towards defect of the character; in other words, if extremes of the character in excess correspond to noteworthy men, they are much less likely to occur in a population where this differentiation of fertility has set in. Suppose we term one man in a thousand "noteworthy," then such a man corresponds to the grade $3{\cdot}090\sigma_1$ in our original population. His position in the offspring generation will be

$$m_2 + 3{\cdot}090\sigma_1 = 3{\cdot}216\sigma_1 = \frac{3{\cdot}216}{{\cdot}9539}\Sigma_2 = 3{\cdot}37\Sigma_2.$$

But this corresponds to four such individuals in 10,000, or 1 in 2500; or there will be less than half as many noteworthy men. This is on the first hypothesis; the second hypothesis shows only 1 in 3300, and the fourth or less influential hypothesis 1 in 1700. It will therefore be obvious that if mental powers are hereditary, any differentiation of fertility will not only alter rapidly the average intelligence of the community, but will produce that dearth of noteworthy ability, or of genius, which is fatal to the continued existence under modern competitive conditions of a great national life. The whole system of curves corresponding to this first Illustration is given in Diagram I.

Illustration II. In the previous illustration we considered a case where the maximum fertility did not coincide with the minimum of the character, although the lowest values of the character had markedly higher fertility than the highest. It will be of interest to determine what, if any, differences arise in our general conclusions, if we fix on the fertility of individuals high and low in the scale, instead of on average fertility values. We will suppose that the individual parentage which stands sixth in the 1000 for the given character has only a fertility of 1·5, and that which stands last but six has a fertility of 6·5; the maximum fertility of any grade shall be 7, and this shall occur *below the grade* of the individual parentage with a fertility of 6·5. Using the notation of p. 264, we may take $m_1 = m_2 = 2{\cdot}5$, say. Hence

$$\pm\sqrt{\frac{2(\log 7 - \log 1{\cdot}5)}{{\cdot}434{,}2945}} = 2{\cdot}5\lambda + \mu = \pm 1{\cdot}75525,$$

$$\pm\sqrt{\frac{2(\log 7 - \log 6{\cdot}5)}{{\cdot}434{,}2945}} = 2{\cdot}5\lambda - \mu = \pm {\cdot}38503.$$

Since λ must be positive, these give us

$$2{\cdot}5\lambda = {\cdot}6851, \quad \mu = 1{\cdot}0701, \quad \text{or} \quad 2{\cdot}5\lambda = 1{\cdot}0701, \quad \mu = {\cdot}6851.$$

In order that the maximum fertility may be outside the range $-2{\cdot}5\sigma_x$ to $2{\cdot}5\sigma_x$, we must take the first pair of roots. We easily find

$$\sigma_0 = 3{\cdot}6490\sigma_x, \quad k = 3{\cdot}9049\sigma_x,$$

and from equation (xii) $\bar{f} = 4{\cdot}2613.$

Hence the fertility distribution is

$$y = 7e^{-\frac{1}{2}\left(\frac{x - 3{\cdot}9049\sigma_x}{3{\cdot}6490\sigma_x}\right)^2}.$$

The maximum fertility 7 occurs so far from the mean value of the character, that there are practically no parentages with a low value of the character and a low fertility, i.e. the number of *individual* parentages with an extremely low value of the character and a fertility under 7 is only 1 in the 20,000. We may therefore look upon our curve as one which sensibly describes a fertility constantly decreasing as the value of the character increases.

Turning to the distribution of effective parentages we find:

$$m_1 = {\cdot}2728\sigma_x, \quad \Sigma_1 = {\cdot}9644\sigma_x.$$

Thus there is rather a greater shift than in Illustration I. of the average effective parentage, but the parentages are more variable. The distribution of effective parentages is:

$$w = \frac{4{\cdot}261N}{\sqrt{2\pi} \times {\cdot}9644\sigma_x} e^{-\frac{1}{2}\left(\frac{x - {\cdot}2728\sigma_x}{{\cdot}9644\sigma_x}\right)^2}.$$

The number of *effective* parentages with low values of the character and fertility less than the maximum will be found from determining the area of this w curve for values of x greater than $x = 3{\cdot}9049\sigma_x - {\cdot}2728\sigma_x = 3{\cdot}6321\sigma_x$, or by looking up in the Tables $x' = 3{\cdot}6321/{\cdot}9644 = 3{\cdot}77$; this gives 1 in 12,500, a negligible frequency.

In view of our previous results for Illustration I, it may be of interest to note the average fertilities of the top and bottom deciles of our individual parentages, they are to be found from:

$$\frac{n_h \bar{f}_h}{N\bar{f}} = \frac{1}{\sqrt{2\pi}} \int_{q'}^{\infty} e^{-\frac{1}{2}x'^2 dx'},$$

where

$$q' = \frac{h - m_1}{\Sigma_1} = \frac{\pm 1{\cdot}2816 - {\cdot}2728}{{\cdot}9644},$$

whence by aid of the tables we find 2·28 for the average fertility of the top decile of the individual parentages, and 6·30 for the bottom decile. These values may be compared with the corresponding 2 and 5·34 of Illustration I.

We now turn to the distribution of offspring. We have:

$$m_2 = \cdot 2728\rho\sigma_z, \quad \Sigma_2 = \sigma_z \sqrt{1 - \cdot 06986\rho^2},$$

$$R/\rho = 3{\cdot}6490/\sqrt{14{\cdot}3152 - \rho^2}.$$

We can now determine these in precisely the same manner as they were tabled for the four hypotheses in Illustration I.

Constants of Offspring Distribution. Illustration II.

Hypothesis ...	(i)	(ii)	(iii)	(iv)
m_2/σ_z ...	·1585	·1736	·1228	·1002
Σ_2/σ_z ...	·9881	·9858	·9929	·9953
R/ρ ...	·9759	·9783	·9713	·9689

We see at once from this table that whichever hypothesis we take the variability of the new generation is only lessened about 1 per cent. and the correlation about 3 per cent., values markedly less than in the case of Illustration I. This is obviously due to the fact that the fertility distribution is markedly less concentrated. But the shift of the population average is now 10 to 17 per cent. of the standard deviation, or roughly the average has receded through $\frac{1}{2}$ to $\frac{2}{3}$ of a decile in the character. We may measure the extreme effect of this by inquiring how far "noteworthy" individuals, the individual one in a thousand of the old generation, have been reduced in number. What we have to find is the value of $(m_2 + 3{\cdot}090\sigma_1)/\Sigma_2$; for the first hypothesis this is 3·288, which corresponds to 1 in 2000, or the halving of these noteworthy individuals in the second generation, and we have practically the same rate on the second hypothesis. We thus see that on the numbers of this second Illustration, the average population will degenerate considerably faster, 25 per cent. faster approximately, but the dearth of noteworthy individuals will be slightly less. The reader must not, however, conclude from these numbers that our second Illustration represents a less undesirable type of degeneracy. Beside the dearth of the noteworthy, the multiplication of the extremely unfit has to be measured. Let us suppose that the individual whose character has the least value in the 1000 of the original population is by virtue of this deviation a non-useful member of the community—physically or mentally unfit. We must clearly then find out in the second generation to what number of the population this standard applies, i.e. we want $(3{\cdot}090\sigma_1 - m_2)/\Sigma_2$. This is equal to 2·967 on the first and 2·958 on the second hypothesis, corresponding in both cases to 1 in 667 instead of 1 in 1000. On the other hand it will be found that in the first illustration the corresponding numbers are 3·107 and 3·125 on hypotheses (i) and (ii) respectively, giving practically 1 in 1000, and the same standard of unfitness repeating itself in the second generation. Thus Illustration II, with a slightly reduced rate of decrease of noteworthy individuals, has an increased rate of unfit individuals, when compared with

Illustration I, and a general or average decadence greater by 25 per cent. It would be idle to assert that one or other corresponds to a greater racial degeneracy until we have settled whether it is more important to a nation to have in the case of a desirable character (i) a high average value, (ii) a larger percentage of noteworthy individuals, or (iii) a smaller percentage of unfit individuals. We do not know at present what relative weight is to be given to these categories; but we can see that in any quantitative theory of the socially desirable characters in a community, they naturally arise from the analysis of a differential fertility. Such factors as dearth of leaders, lowered general intelligence of the community, multiplication of the unfit, are seen, even by a preliminary analysis of this kind, not to be unrelated causes, by aid of which the historian accounts in a vague manner for racial and political changes. They are associated factors, explicable and measurable when we know the outlines of a theory of differential fertility.

No special stress must be laid on the particular numbers chosen above to illustrate this discussion. They are round numbers not wholly inappropriate, perhaps, to what we know of intelligence and fertility in man. They are used to suggest the main outlines of a discussion of such problems. Considering the immense masses of statistical, especially demographic data now being accumulated in all civilised countries, it seems to me that the historian of the future will have the means of testing in a way, never yet feasible, the changing fitness or unfitness of nations. In that future absolute and differential fertility, the abundance or dearth of leaders, the average intelligence of a nation, the reduction or multiplication of its unfit, will be recognised as the basal factors in racial progress, the biological sources of evolution in political history. Then history becoming biological will for the first time be admitted as a branch of scientific inquiry, and will also for the first time provide the requisite training for the statesman.

If we turn to examine the possibilities of immediate work with regard to differential fertility, there are several directions in which investigations can at present be carried out with reasonable hope of success. If we confine ourselves to the artizan class, wages are a very reasonable measure of capacity, and the correlation between these and fertility or fecundity* might fairly easily be ascertained. Intelligence might be measured by craft classification extending from engineer to general labourer. Much may be done, if we pass from the artizan class alone, by comparing the size of families of members of various professions, of those following mercantile pursuits, of retail traders and of artizans. Further within a narrow class like the graduates of the Universities, it ought to be feasible actually to tabulate fertility against a scale of intelligence as measured by academic distinction. Fertility of sane and insane stocks, of mentally defective and normal stocks, of alcoholic and sober parentages can be ascertained, although the task, of course, needs careful social inquirers. Lastly grades of physical fitness might well be investigated; the family of the athletic might be compared with that of the non-athletic; and those of the smith and the navvy with those of the cobbler, tailor, and street hawker. It would, I think, be quite possible even without a reformed census, for which every sociologist should work, to obtain many results of value bearing on differential fertility by inquiries of the above nature.

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* Fertility might be measured by nett or gross family when complete, or for marriages of 15 or 20 years' duration. It would, I think, be possible to measure fecundity by some function of the intervals between the births of the first three or four children.

Diagrams.

Each diagram consists of five normal curves. They are worked out on Hypothesis (i), i.e. that the fertility of the parentage is equally affected by the character in both parents, and that there is an assortative mating in the community. The coefficient of parental heredity is taken as ·45 and that of assortative mating as ·20. x is measured from the mean of the character in the parentage of the first generation. Expressing everything in terms of σ_1 the variability, say, of males in the first generation we have the following curves:

Diagram I. Representing Illustration I.

Curve I. Distribution of Males in first generation:

$$y = \frac{n}{\sqrt{2\pi}\,\sigma_1} e^{-\frac{1}{2}\frac{x^2}{\sigma_1^2}}.$$

Curve II. Distribution of Individual Parentages in first generation:

$$y = \frac{n}{\sqrt{2\pi}\,1{\cdot}55\sigma_1} e^{-\frac{1}{2}\frac{x^2}{(1{\cdot}55\sigma_1)^2}}.$$

Curve III. Curve of Fertility of Individual Parentages of first generation:

$$y = 6{\cdot}439\, e^{-\frac{1}{2}\left(\frac{x - 1{\cdot}2603\sigma_1}{2{\cdot}5697\sigma_1}\right)^2}.$$

(N.B. Only part of this curve is drawn.)

Curve IV. Distribution of Effective Parentages in first generation:

$$y = \frac{5n}{\sqrt{2\pi}\,1{\cdot}3273\sigma_1} e^{-\frac{1}{2}\left(\frac{x - {\cdot}3362\sigma_1}{1{\cdot}3273\sigma_1}\right)^2}.$$

Curve V. Distribution of Males in second generation:

$$y = \frac{5n}{\sqrt{2\pi}\,{\cdot}9539\sigma_1} e^{-\frac{1}{2}\left(\frac{x - {\cdot}1260\sigma_1}{{\cdot}9539\sigma_1}\right)^2}.$$

Diagram II. Representing Illustration II.

Curve I. Distribution of Males in first generation:

$$y = \frac{n}{\sqrt{2\pi}\,\sigma_1} e^{-\frac{1}{2}\frac{x^2}{\sigma_1^2}}.$$

Curve II. Distribution of Individual Parentages in first generation:

$$y = \frac{n}{\sqrt{2\pi}\,1{\cdot}55\sigma_1} e^{-\frac{1}{2}\frac{x^2}{(1{\cdot}55\sigma_1)^2}}.$$

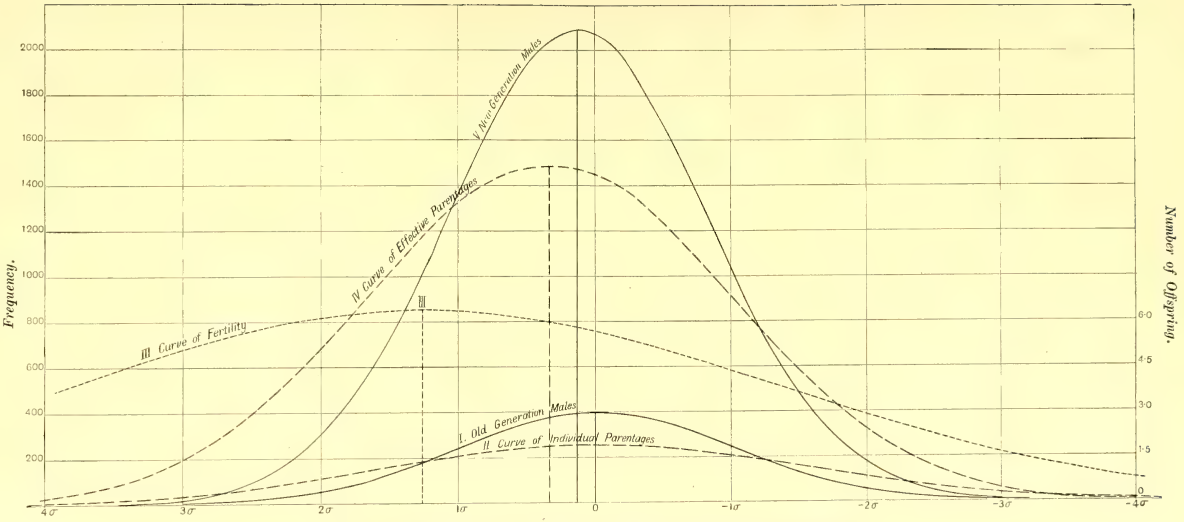

DIAGRAM I. Fertility has its maximum well within range of parental variation.

In this diagram the value of the character is supposed to *increase* from left to right, but x is positive when it is a *defect* from the mean value.

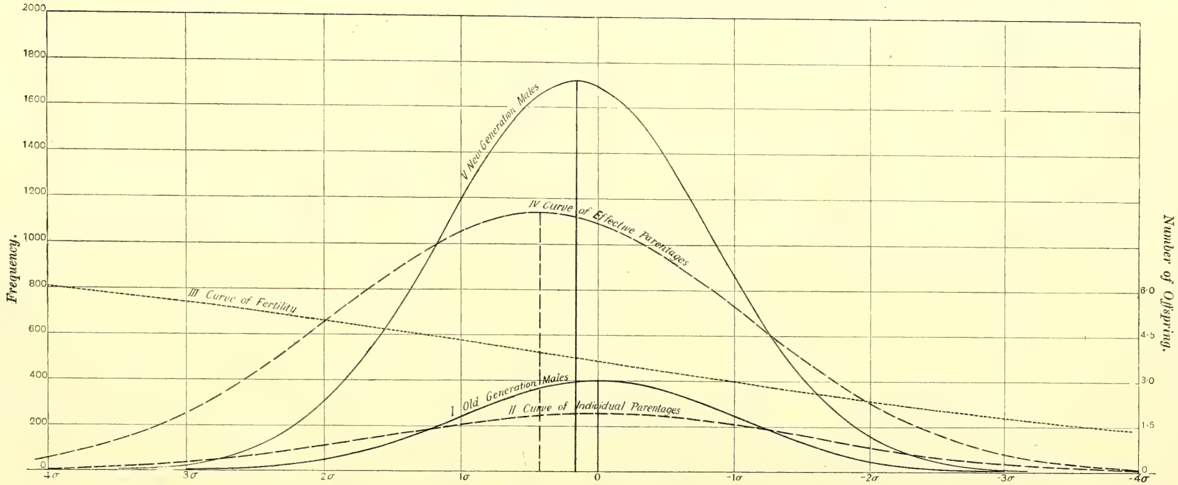

DIAGRAM II. Fertility has an almost uniform gradient within the range of parental variation, its maximum lying practically outside that range.

In this diagram the value of the character is supposed to *increase* from left to right, but x is positive when it is a *defect* from the mean value.

Curve III. Curve of Fertility of Individual Parentages of first generation:

$$y = 7e^{-\frac{1}{2}\left(\frac{x - 6\cdot0526\sigma_1}{5\cdot6560\sigma_1}\right)^2}.$$

(N.B. Only one half of this curve is drawn.)

Curve IV. Distribution of Effective Parentages in first generation:

$$y = \frac{4\cdot261n}{\sqrt{2\pi}\,1\cdot4948\sigma_1} e^{-\frac{1}{2}\left(\frac{x - \cdot4228\sigma_1}{1\cdot4948\sigma_1}\right)^2}.$$

Curve V. Distribution of Males in second generation:

$$y = \frac{4\cdot261n}{\sqrt{2\pi}\,\cdot9881\sigma_1} e^{-\frac{1}{2}\left(\frac{x - \cdot1585\sigma_1}{\cdot9881\sigma_1}\right)^2}.$$

A STATISTICAL STUDY IN CANCER DEATH-RATES.

By G. D. MAYNARD, F.R.C.S.E., Pretoria.

As there is still much that remains obscure in the aetiology of Cancer, any new light that can be thrown on this important subject seems worthy of careful consideration.

It is with a sense of great diffidence that I bring these figures forward, knowing full well how liable to error death-rates may be, and how easy it is to draw false conclusions from the study of figures. But when all is said and done it is to figures that the final appeal must be made, and if the death-returns are so unreliable as to be worthless, then it is certain that there are no more reliable data to be obtained. The comparatively few cases that come within any individual experience are, from the point of view of exact knowledge, of little or no value.

If therefore we are to learn anything from the history of the past, it is to figures that we must go, guardedly and with great caution, bringing all the collateral evidence that we can obtain, to check and control our results, and above all using accurate methods of analysis before drawing any conclusions.

Theories are frequently being advanced, in medical literature, in conflict with the records, these either being ignored, or set aside with the remark that they are so unreliable that no attention should be paid to them. Marked divergencies in the rates are dismissed as being due to errors in registration, the increasing cancer death-rate is regarded as being due to better diagnosis, and so on.

Before entering on any discussion as to the causes of the varying cancer death-rates as observed in different countries—and even in the cities and towns of the same country—it will be advisable to make a few general remarks on how the data in this paper have been obtained and dealt with, and to answer a few of the objections that may be raised as to the value of the returns themselves.

At the outset one must admit that many imperfections are inherent in all statistics of death which are not based on post-mortem examinations. But because the figures are not absolutely correct are we to neglect them, or regard them as absolutely useless? One must remember that where large masses of figures are being dealt with the errors in excess tend to balance those in defect, and wrong inclusions in a group must be set against the accidental exclusions. For instance, is it reasonable to consider that in two cities of the same country, during the same period, and under similar Registration Laws, rates of 1184 and 446 per million are merely fortuitous or due to errors of registration? Such a suggestion means that either more than half the cases are overlooked in the one town or more than 100 °/。 too many are recorded in the other, unless the error is to be divided between them. Before dismissing all differences in rates as being due to errors in diagnosis or imperfections of record it is well to realise exactly what this suggestion entails.

Not only is it often said that better registration accounts largely for the varying rates in different places, but that much of the increase in the cancer death-rate, as also that occurring in the diabetes death-rate, is due to better diagnosis. The statistics show that both these diseases are very markedly on the increase. As a rule before death occurs both diseases give a very definite clinical picture, and run a more or less protracted course. When dealing with death returns, difficulties in early diagnosis are of no consequence. These factors are in favour of a correct diagnosis being made, and probably fewer mistakes are made in the diagnosis of these groups than in almost any others. In the last 30 years there has been very little added to our knowledge, that would aid the general practitioner to a more accurate diagnosis, in the final stages of these diseases. Doubtful tumours are more often submitted to the microscope for confirmation of diagnosis than formerly, but the effect of this on the returns would rather be to exclude from the cancer group cases that would otherwise have been found there; for a fatal tumour of doubtful origin would most probably have been classed as cancer in the absence of definite knowledge to the contrary. There does not seem therefore, sufficient ground for believing that the increase in the rates is in any great part due to improved medical knowledge, or that this will account for the growth in the cancer and diabetes death-rates.

If we refer to the cancer death-rates for "occupation groups" we find that those consisting of men who by their social status can best afford highly skilled medical care, or those who have State medical attendance provided, are the groups showing the lowest death-rates, while the labouring and agricultural classes show distinctly higher rates. This is the reverse of what one would be led to expect if the explanation that better diagnosis accounts for the increased rate—is the correct one.

There is still a further reason for thinking that the group returned as cancer is fairly homogeneous in character. It will be seen from Fig. 1, how good a fit a Pearson Curve gives for the male death-rates, tabulated in five-yearly periods. The dotted polygons at ages 92·5 and 97·5 are the original observations and the constants of this curve are given below. Sheppard's adjustments were not used as I did not expect to find high contact at both ends. Finding however that there was high contact I recalculated the curve, using Sheppard's adjustments and altering the two groups referred to above, as indicated in the diagram. As these groups are based on very small numbers, this step seemed justifiable. The general fit is slightly improved thereby*, although the form of the curve is not altered. In Fig. 2, the female curve is given. Here the observations are more irregular, which may possibly be due to the greater difficulty in obtaining correct ages in the case of women. Sheppard's adjustments were used, although the result shows that we are not justified in supposing high contact. The total number of cases employed in finding the rates for the various age periods was male deaths 11,436, female deaths 18,039. The areas of the frequency polygons represent the number of cancer deaths per million of population living at each age.

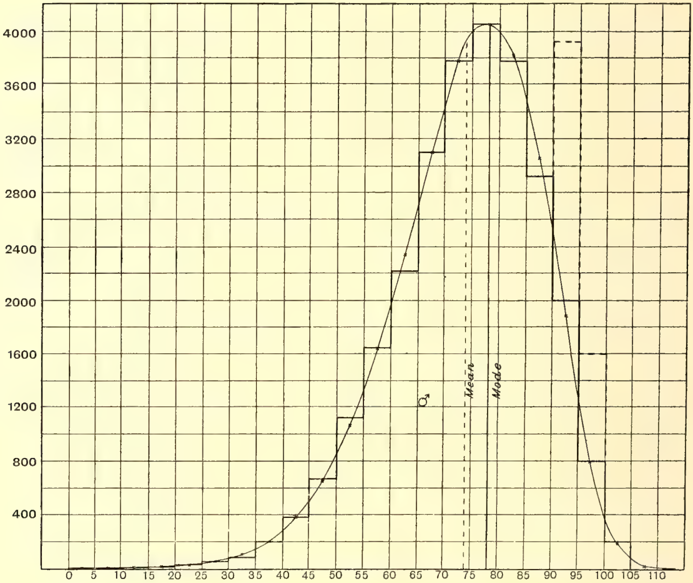

Fig. 1. Cancer Death-Rate, Males, U.S.A., 1900.

$$y = y_0 \left(1 + \frac{x}{52 \cdot 3713}\right)^{45 \cdot 6} \left(1 - \frac{x}{7 \cdot 0852}\right)^{6 \cdot 17}.$$

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* That the theoretical start of the modified curve falls so long before birth is, perhaps, due to the fact that there is really not high contact at the old age end after modification.

The constants of the curves are :—

	Male, Fig. 1		Female, Fig. 2
	Unaltered	Altered	
Mean ...	75·527375	73·906005	72·220620
Mode ...	78·764200	78·116220	78·681705
S. D. ...	2·969692	2·730771	3·048332
μ_3 ...	−7·422077	−11·657840	−14·515937
$\sqrt{\beta_1}$ ...	·283393	·572483	·512457
β_2 ...	2·628036	3·377277	2·818529
κ_1 ...	− ·984864	− ·228657	− 1·150781
Start ...	14·972165	−83·740300	2·406680
End ...	112·200885	113·542075	102·542040

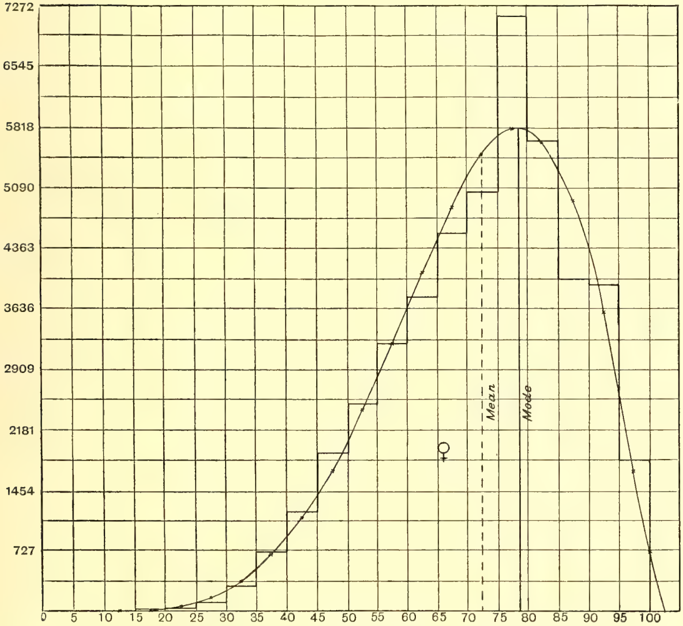

FIG. 2. Cancer Death-Rate, Females, U.S.A., 1900.

$$y = y_0 \left(1 + \frac{x}{15\cdot2550}\right)^{4\cdot656} \left(1 - \frac{x}{4\cdot7721}\right)^{1\cdot456}.$$

Readers who have fitted curves to "disease" distributions grouped in this manner will I think agree, that a fit as good as that given in Fig. 1 indicates the homogeneous nature of this group. I have frequently been struck with the excellent fit given by diseases, like puerperal septicaemia, where the chance of a wrong diagnosis is unlikely, and greatly disappointed with the bad fit obtained by a disease that presents difficulty in diagnosis, such for instance as enteric fever in hot climates, where the heterogeneous character of the group is undoubted. The excellence of fit in the case of the male cancer curve is I think evidence of some value as to the homogeneity of these returns.

The goodness of fit as measured by the usual χ^2 formula[*] for the modified male curve is $P = \cdot 4$ and excluding the irregular groups at 2·5 and 107·5, $P = \cdot 73$.

The curve in the case of the diabetes distribution is not satisfactory. No single curve can be fitted to the figures which will even approximately represent the frequency polygon. It seemed advisable to see if two or more curves could be found that would serve to express the recorded rates. It is fairly easy to see that these curves cannot all be normal. The old age component of diabetes is clearly skew, and it was not hard to obtain a good representation of this part of the mortality. The earlier mortality appears to consist of two components; a diabetes of youth with a mean age of 15 to 20, and a diabetes of middle age with a mean age of about 40. These components would agree to some extent with those found by Pearson in his resolution of the general mortality curve[†]. The discussion of this point has been reserved for another paper. Further figures are needful before it can be definitely stated that the disease now known as *Diabetes melitus* can be certainly resolved into two or more age groups. The English death returns for this disease show the same general features as those obtained from the United States[‡].

Before any comparisons of the cancer death-rates can be profitably made it is necessary that they should be corrected for the age constitution of the population. Cancer is mainly a disease of advanced life, and as this is the portion of the population most unevenly distributed between town and country, the difference in crude death-rates due to this cause must be borne in mind when rates are being compared. Similarly no useful comparisons can be made between the rates as found for the different occupations until corrected for age. The group of bankers for instance has a very different age composition from that of clerks, as the following correction factors show: bankers = 0·3516: clerks = 1·3157. Thus the crude rates of 414 and 280 become with correction 150 and 368 respectively.

The usual method[§] of obtaining the correction factor has been adopted, and the method by which the standard death-rate used was obtained is given in the Supplement to the 65*th Annual Report* of the Registrar-General, for the years 1891—1900. The standard population is that of England and Wales for the same period. It is immaterial what standard is adopted so long as all crude rates are reduced to the same base, as they can then be compared *inter se*, although they do not represent the actual rate obtaining in the district considered.

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[*] *Biometrika*, Vol. I. p. 155.

[†] *Phil. Trans.* Vol. 186, p. 407.

[‡] The female diabetes frequency polygon exhibits the same features as the male distribution, the rates being similar. The early age portions of both cancer and diabetes frequencies require special consideration, which I hope to give on another occasion.

[§] R = death-rate in standard population, $r_1, r_2 \ldots r_n$ = death-rates at age periods, $x_1, x_2 \ldots x_n$ = numbers of population in similar age groups, for district to be corrected, where X is total population. Then if $R' = S(xr)/X$, R/R' is the correction factor.

The only source at my disposal from which it was possible to obtain the necessary data to correct the rates for towns and districts, was the United States Census of 1900. Fortunately this country covers a large area, and is subject to many varying influences of climate and social conditions. It is thus a particularly suitable country in which to study the conditions which may predispose to the development of this disease. Unfortunately, however, the whole country is not under a uniform registration law; the field is therefore restricted to those states and cities which are. The statistics from these Registration States and Cities are probably fairly accurate; and I think the marked differences in rates that are observed, cannot be due to faults of registration, and point to real differences in predisposing causes. As a check on the accuracy of the death returns the Census Department made an enumeration of the deaths occurring in the year ending May 31st, 1900. This was carried out in all districts where there was any doubt as to the state of the registration. The results show that so far as the cities dealt with in this paper are concerned, the error in no case exceeds 9·5 °/o. In only one city was the error greater than 6·5 °/o, while the mean error is 3·5 °/o with a coefficient of variation of 2·225 °/o.

In addition to the correction for age, a sex correction might also have been introduced. In towns of over 100,000 inhabitants—and none with a smaller population than this have been used in this paper—the variation in sex distribution is not, from our present point of view, of very great importance, while the extra arithmetical labour involved in calculating over 150 further correction factors is very considerable. In a few cases where the sex distribution seemed to vary considerably from the usual proportions, I calculated the sex correction factor, but in no case did the alteration in the corrected rate amount to 0·5 °/o. It should be remembered that the cancer death-rate is highest for females, while the diabetes death-rate is highest for males, thus any error resulting from this omission would tend to lower the correlation that exists between these diseases.

In dealing with large areas of mixed urban and rural populations another source of disturbance must be borne in mind, viz. the different age distributions in the two districts, with its effect on the death-rates in each. The correction factor for the rural population of the ten Registration States in 1900 is for cancer ·7585, while for the urban areas of the same States it is 1·0617. Now as the proportion of the population in the urban and rural districts differs in each State, this in itself will cause an appreciable difference in the observed rates. The following table shows the change in rate that would be made from this cause alone, were the age distribution of rural and urban districts uniform for each State.

Cancer Death-Rates.

State	Crude	Corrected
Maine	837	692
New Hampshire ...	668	568
Vermont	818	647
Massachusetts ...	664	648
Rhode Island ...	623	590
New York	612	587
Connecticut	611	563
New Jersey	504	472
Michigan	580	498

It will be noted that this correction although lowering all the rates does not materially disturb their order.

The correction factor as used for the various States is based only on the age distribution of the whole State without reference to the proportions in the urban and rural areas. The necessary figures to make this latter correction were unobtainable.

Throughout the paper the correction factors are based on the census of 1900 while the death-rates of the cities are those of the five-year group 1900—1904; it being assumed that the age distribution has kept fairly uniform, or at least has not altered sufficiently to affect materially the conclusions arrived at. In the case of the States the 1906 death-rate is compared with the insanity figures for 1904. This was unavoidable as no others were obtainable. I do not suppose however that the rates changed so greatly in two years that all value from these comparisons is lost.

In calculating the coefficients of correlation and their probable errors a doubt arose as to whether it was right to give every city the same weight. Should a city with three million inhabitants be given greater weight than one of only 100,000? If it were proved that any factor, such for instance as a meteorological condition, determined the incidence of cancer, it is clear that weighting for size would be wrong. Again if overcrowding was a marked factor it would be unjustifiable to weight for size. To take 100,000 inhabitants as a unit was quite arbitrary; I have therefore regarded each city or State as a unit. This reduces to a minimum the value of the denominator in the probable error, which is therefore of maximum size.

When first approaching this problem I was of opinion that in spite of the trend of modern research it might yet be demonstrated that cancer was of infectious origin, either bacterial or protozoic in nature. Many infectious diseases have a very marked seasonal incidence, and are in certain cases closely correlated with various meteorological conditions. That this is so in respect to enteric fever in the Transvaal I have shown elsewhere, and it is I think reasonable to suppose that if an organism has an existence apart from its human host it is quite likely that this should be so. Cancer being a disease of fairly long duration, and of uncertain date of onset in relation to the time of death, there was no way of finding out whether the disease was contracted more frequently at one time of the year than another. If therefore meteorological conditions have an influence on causation, it must be sought in place rather than in time. For this purpose I have obtained from the U.S.A. Government publication *Climatology of the United States,* average meteorological conditions for 27 Registration cities with populations of over 100,000 inhabitants.

Corrected Cancer Death-Rate, 27 *Cities,* 1900—1904.

Sunshine in hours	$r = -{\cdot}0142 \pm {\cdot}1323$
Mean Temperature	$r = -{\cdot}1478 \pm {\cdot}1294$
Mean Maximum Temperature	$r = -{\cdot}2236 \pm {\cdot}1257$
Mean Minimum Temperature	$r = -{\cdot}0148 \pm {\cdot}1323$
Rainfall	$r = -{\cdot}2517 \pm {\cdot}1239$
Days of Rain	$r = -{\cdot}1621 \pm {\cdot}1288$
Absolute Humidity	$r = -{\cdot}2014 \pm {\cdot}1269$

Cancer Death-Rate, for the 17 *Cities.*

Sunshine	$r = {\cdot}1268 \pm {\cdot}1659$
Rainfall	$r = {\cdot}3163 \pm {\cdot}1518$
Mean Temperature	$r = -{\cdot}0732 \pm {\cdot}1685$
Mean Maximum Temperature	$r = -{\cdot}1509 \pm {\cdot}1600$
Mean Minimum Temperature	$r = {\cdot}1254 \pm {\cdot}1659$
Rel. Humidity 8 a.m. ...	$r = {\cdot}3101 \pm {\cdot}1519$
Rel. Humidity 8 p.m. ...	$r = {\cdot}1603 \pm {\cdot}1600$
Range of Humidity	$r = {\cdot}0724 \pm {\cdot}1685$

It will be observed that out of the seven correlations appearing in the table none of the values are really significant when compared with their probable errors. I had previously calculated the correlations for 17 of the larger cities and values of the same numerical order were obtained, but in many cases the sign was different. They are appended for the purpose of comparison. These values must therefore be considered as zero.

It will be seen from these figures that so far as 27 cities of the U.S.A. are concerned there is no correlation between the average climatic differences obtaining in the city and the prevalence of cancer, as measured by its average death-rate over the five years 1900—1904.

It has frequently been stated that cancer is less prevalent in hot climates, and there are facts to be obtained in support of this general statement. It is not however necessary to assume that this is due directly to climate, and another explanation is, I think, probably the right one. Marked differences of race, temperament, and pressure of life are to be found in tropical and sub-tropical countries, as compared with temperate and cold ones; the exception to this being the case now under discussion, the United States of America.

So far then as these figures go we have no evidence to connect meteorological factors with the variable incidence of cancer. This, although a negative result, is not, I think, without its importance.

Finding that the theory of the infectious origin of this disease gained no support from the values thus discovered, it occurred to me to correlate the cancer rates with those of some other disease. For this purpose I chose diabetes, and for the following reasons. (1) Both diseases have very much the same age distribution. (2) They stand almost alone as being on the increase, while other causes of death show declining rates. (3) The aetiology of both diseases is obscure. (4) Both being diseases of old age, the heredity factor cannot have been increased by the results of modern medical skill, for the prolongation of life that might thus have been procured would have been mainly at a period when the procreative power was passed. In fact the tendency of modern times is rather to postpone marriage to a later period of life, and this might be expected to reduce any heredity factor there may be. (5) If there were a common factor in the causation of the dual increase a correlation between these diseases might be discovered.

There seemed then sufficient justification for undertaking the labour of calculating correction factors for the diabetes rates. As was to be expected both correction factors are similar in value.

Professor Pearson has shown that when correlation tables are formed between rates, in such a way that a common factor occurs in both the variables, a "spurious correlation" is obtained. In this case therefore the usual formula $r = S(xy)/N\sigma_x\sigma_y$ can no longer be employed. On his advice* I have therefore used the following formula when dealing with the correlation as existing between rates:

$$\rho_{xy} = \frac{r_{xy} - r_{zx}r_{zy}}{\sqrt{(1 - r^2_{zx})(1 - r^2_{zy})}}.$$

Here x = the number of deaths from one disease, y = the number from a second, and z = the number of individuals in the district. Throughout this paper where the symbol ρ occurs it refers to values found by this partial correlation formula.

In dealing with diseases like cancer and diabetes, a significant correlation will probably occur when uncorrected deaths are used, due to their similar age incidence. This will be most marked in the value found for the races, and occupation groups, and least so for the cities. This will be seen by comparing the "Coefficient of Variability" for the cancer correction factors for the different groups:

Cities	$V_c = 11{\cdot}46\ {}^\circ/_\circ$.
States	$V_s = 14{\cdot}57\ {}^\circ/_\circ$.
Races	$V_r = 38{\cdot}39\ {}^\circ/_\circ$.
28 Occupation Groups	$V_o = 43{\cdot}86\ {}^\circ/_\circ$.

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* [This formula gets rid of the "spurious correlation" due to forming "rates," i.e. the common population totals, but it does not of course get over the high spurious correlation of the so-called "age correction factors." Ed.]

As, therefore, using corrected deaths introduces "spurious correlation" into the value of ρ, and as using uncorrected deaths will introduce a correlation due to the age factor, the values of ρ calculated, both with and without the use of a correction factor, are given*. The symbol ρ' will be used to denote values found from corrected deaths. The necessity for correcting for age is not of such importance when comparing cancer with suicides or insanity.

The partial correlation coefficients found for cancer and diabetes are as follows:

40 Cities, 1900—1904, $\rho = \cdot 6896 \pm \cdot 0559$, $\rho' = \cdot 7325 \pm \cdot 0494$.
15 States, 1906, $\rho = \cdot 9088 \pm \cdot 0303$, $\rho' = \cdot 8258 \pm \cdot 0554$.

These values were sufficiently striking to lead one to further consider the matter.

From figures obtained from *Vital Statistics* (Vol. I.) U.S.A. Census, 1900, I calculated correction factors for the cancer and diabetes death-rates of the various races residing in the Registration States, and classified according to birth-place of mother. The correlation thus obtained for these two diseases is certainly significant:

$$\rho = \cdot 8609 \pm \cdot 0552, \quad \rho' = \cdot 5442 \pm \cdot 1501.$$

That these values are not due to errors of random sampling is shown by their probable errors, for the odds are some millions to one against such an occurrence. Nor does it seem likely, that they are entirely due to "spurious correlation" in the ρ''s and to an entirely different source in the case of the ρ's. To what then are these correlations due? It seemed possible that the agreement of rate might be due to certain cities having more efficient registration, although the remarks of the Registrar did not lend much support to this theory. He writes: "The 'registration area'—that is to say, the States having laws, the results of whose operation have been accepted as giving practically complete mortality returns, together with the cities in non-registration States where deaths are satisfactorily registered under local authority—remains substantially the same from 1900 to 1905. The geographic distribution is shown in the accompanying map" (*Mortality Statistics* 1905, *U.S.A.*: see our Fig. 3).

I have already referred to the special returns of deaths made by the Census Department in 1900. The small value of the Coefficient of Variation (2·225 %) in the "percentage error of registration" is I think sufficient evidence that the correlation values are not due to errors in registration. If these high correlations were due to varying efficiency in registration, then other diseases might be expected to show a similar value when correlated with cancer. It will be seen further on that there is no significant correlation between cancer or diabetes and diseases of "ill-defined or uncertain cause." Probably one of the most suitable diseases as a control would be nephritis, were it not that its age frequency is so similar to cancer, thus the age factor will give rise to errors to an even greater extent than occurs in the case of diabetes.

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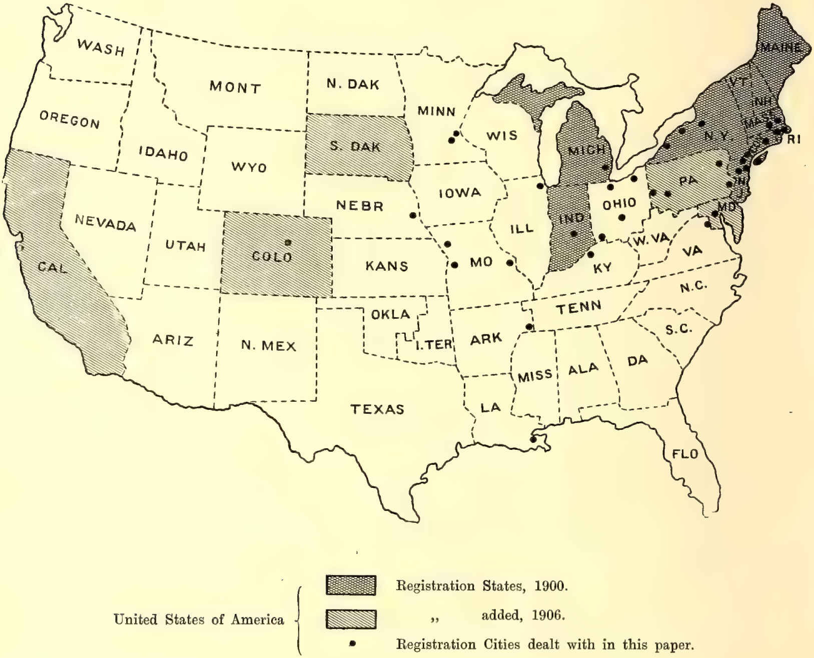
* [I believe I have found a satisfactory method of making *both* age and population corrections free from spurious correlation. This method will shortly be published. ED.]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GA
TEXAS
LA
FLO
United States of America
▩ Registration States, 1900.
▨ ,, added, 1906.
● Registration Cities dealt with in this paper.

FIG. 3.

The values of ρ and ρ' are given for this disease as they may serve as a useful contrast to the other correlation values. In calculating the value of ρ' the cancer correction factor was used for the nephritis deaths as the age frequencies of the two diseases are not very dissimilar. Cancer and Nephritis, 40 Cities:

$$\rho = \cdot 3792 \pm \cdot 0913, \quad \rho' = \cdot 1571 \pm \cdot 1040.$$

These values are of a quite different order from those found for cancer and diabetes.

Unfortunately no figures were obtainable to allow of a correction being made for those deaths which occurred in hospital and did not belong to the district. Although this correction is most desirable, yet it is not likely to very greatly affect the correlation values for cancer and diabetes, as the latter disease is not one that bulks very largely in hospital deaths. It is not a disease very prevalent among the class of patient that frequent hospital, and from its nature would not be admitted unless combined with some acute affection. This of course will occur in a certain number of cases but to a small extent as compared with cancer. This objection will not apply to the returns from the States, and all the cities here dealt with, except a few eastern ones, will draw from fairly large rural areas.

Before coming to the conclusion that the correlation found between cancer and diabetes is of real significance, one must as far as possible exclude all sources of error that might lead to false deductions. The question of the inaccuracy in the records from incomplete registration has already been discussed, but there is another form of inaccuracy that must be considered, viz. that arising from want of care and knowledge on the part of the certifying medical practitioner. In *Mortality Statistics U.S.A.* 1907, the following remark occurs: "In Table IV. may be found two columns headed 'ill-defined causes' and 'cause unknown' which are of no special significance in themselves, but which are nevertheless important, because they show the margin of understatement that must be considered in examining the rates for more definitely stated causes of death." It seemed therefore desirable to correlate cancer and diabetes death-rates with a five-yearly average rate of these groups, for the 40 cities occurring in the other tables. The values found are as follows:

Cancer and diseases of ill-defined and unknown cause, $\rho = \cdot 1020 \pm \cdot 1055$.

Diabetes and diseases of ill-defined and unknown cause, $\rho = \cdot 0773 \pm \cdot 1060$.

It does not seem likely therefore that any appreciable correlation will have arisen from this cause.

It has been suggested to me that in towns well equipped with competent medical men these diseases would figure less in the death returns, because by means of operative treatment in the case of cancer, and dietetic treatment in the case of diabetes, death might eventually be due to some other cause; whereas where medical skill was inferior, or the population not sufficiently educated to take suitable precautions in regard to diet, or submit to operation, a larger number of cases of both diseases would appear in the records. In the first place I do not suppose that the average medical skill would vary very greatly in cities of the size dealt with in this paper. It is no doubt true that in both diseases, life may be prolonged by skilful treatment but a cure is not effected, except in very rare instances. When death occurs from an intercurrent disease the older complaint is included on the death certificate. At least this should be done and it is less likely to be omitted where the medical skill is of a high order. In classifying causes of death it is the rule to select the diseases ordinarily known as constitutional in preference to those known as local diseases. This will be made clear by an extract from the New Tables issued by the L.G.B. and the Schedules of causes of death: "Thus Cancer should be selected in preference to Pneumonia, and Diabetes in preference to Heart-disease." Further the groups already dealt with, viz. "Ill-defined and unknown causes" are, perhaps, the best statistical guide to the efficiency of the local medical profession.

Professor Pearson has kindly drawn my attention to another possible source of error. If it were shown that both diseases were more likely to attack persons in the same social class, or engaged in the same occupations, an artificial correlation might then be set up, owing to the inhabitants of the different cities being engaged in occupations of a diverse nature. It will be seen later that cancer and diabetes are probably negatively correlated in respect to the occupational groups. There is I think no doubt that diabetes is a disease especially prone to attack the wealthy and sedentary classes, but in the U.S.A. this does not seem to be true of cancer. This question is of importance because Mr Heron has by a study of the London statistics arrived, by indirect methods, at an opposite conclusion. From the U.S.A. Census Reports it was possible to obtain the numbers of men employed in various occupations, together with their age distribution, and the cancer and diabetes death-rates. These are given first in eight inclusive groups and then in sub-groups. For the present purpose correction factors for the death-rates were calculated for the eight large groups and for 28 sub-groups as shown in the table. The various occupations were then arranged in order according to their social status, but as it was unlikely that two people would place them in exactly the same order I asked a friend to rearrange them for me and the results of both orders are given. The correlation was found by the method of ranks as given by Pearson*, $r = 2 \sin \frac{\pi}{6} \rho_{12}$, where $\rho_{12} = 1 - 6S(\nu_1 - \nu_2)^2/N(N^2 - 1)$. Correlating high death-rates with superior social occupations the values are as follows:

Eight Inclusive Groups.

Cancer and Occupations $r = -\cdot 5176 \pm \cdot 1746$.

Diabetes and Occupations $r = \cdot 9374 \pm \cdot 0289$.

28 *Sub-groups.*

	1st grouping	2nd grouping
Cancer and Occupations	$r = -\cdot 4574 \pm \cdot 1007$,	$r = -\cdot 3680 \pm \cdot 1102$.
Diabetes and Occupations	$r = \cdot 6716 \pm \cdot 0700$,	$r = \cdot 6748 \pm \cdot 0694$.

To check the results thus found four "four-fold tables" were constructed, the division of the death-rates being taken at their mean, and the occupations being divided at Nos. 9 and 17 of the order as shown in the table.

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* "On Further Methods of Measuring Correlation," *Drapers' Research Memoirs*, Dulau and Co.

There seems to be no doubt that in America there is a negative correlation between cancer and better social class, as measured by occupation, while there is a high positive correlation between diabetes and better social conditions. Thus

Cancer and Occupation.

Death-rate.

Social Status		H.	L.	
	High	2	7	9
	Low	11	8	19
		13	15	28

$r = -{\cdot}56 \pm {\cdot}09$,

	H.	L.	
High	5	12	17
Low	8	3	11
	13	15	28

$r = -{\cdot}63 \pm {\cdot}08$.

Diabetes and Occupation.

Death-rate.

Social Status		H.	L.	
	High	9	0	9
	Low	3	16	19
		12	16	28

$r = 1{\cdot}0$,

	H.	L.	
High	11	6	17
Low	1	10	11
	12	16	28

$r = {\cdot}82 \pm {\cdot}04$.

there does not seem any reason for supposing that an artificial correlation will have arisen between cancer and diabetes death-rates owing to prevalence of varying occupations in the different cities.

If the increased rates observed in the cases of cancer and diabetes were due to a common cause then it is probable that their rates of growth will be found to be fairly highly correlated. Not being able to obtain rates for a sufficient period of time, from the United States reports, the rates given in the Registrar's Report for England and Wales, in five-yearly groups, for the 35 years 1871 to 1905 were used, and here $\rho = {\cdot}8060 \pm {\cdot}0893$. This high correlation shows I think a strong probability that there is a common factor influencing the increase of both diseases. That this value is explicable on the assumption that the increased rates are merely apparent and due to more careful diagnosis is in view of the facts already mentioned almost inconceivable. It is equally clear that the increase is not due to alteration in the age constitution of the population. For example had the same cancer death-rate been operating in 1851—60 as was so acting from 1891—00 the death-rate would have been 743 per million, and in 1871—80, 749, while in 1891—00 it was 758. Thus in 50 years the increased ratio of old people would have led to a gain in rate of only 15 deaths per million of the population, whereas the recorded increase in the last 35 years 1871—05 is 534 per million. It has been previously pointed out, that no great increase in our knowledge of either disease has recently been made, that would aid the general practitioner in more accurate diagnosis in the final stages of the disease.

If then we have succeeded in excluding artificial sources of correlation, it remains to find, if possible, the factor that is producing the increasing rates in these two diseases, and giving rise to their correlation in the various cities and districts.

Only one cause, it seems to me, will fit the facts as we know them, viz.: the pressure of modern civilisation and the strain of modern competition, or some factor closely associated with these. Radio-activity and various electric phenomena have from time to time been accused of producing cancer. The increased use of high tension currents is an undoubted fact in modern city life, but a glance at the varying rates as applying to different occupations will suffice to show that the explanation does not lie here. Another marked feature of modern life is the increasing use of preserved foods. But again the death-rate as evidenced by the occupation groups seems to negative this as a probable cause. For instance the agricultural rate is higher than that for the military and police.

The increasing pressure of competition with its attendant nervous strain appears to be the factor in the aetiology of both diseases that will account for the phenomena. It has long been recognised that nervous shock and worry may be a determining cause in diabetes.

At once another disease becomes associated with this factor in modern life. Insanity, also on the increase, should show a positive correlation with cancer.

Unfortunately figures of the insane rates for the various cities in the United States are not obtainable, so on the suggestion of Dr Troup I have correlated suicides as being the nearest approach to a measure of the prevalence of insanity. Doubtless suicides form some guide to the stress and strain of life in the various districts, but other factors have probably also an effect in determining the prevalence of suicide. Religious beliefs, and moral training, may have a marked influence. The correlation found for cancer death-rates and suicides for the 40 cities, for the five-year groups, is:

$$\rho = \cdot 3962 \pm \cdot 0899,$$

and the 15 States for the year 1906:

$$\rho = \cdot 4412 \pm \cdot 1402,$$

and for the Races according to birth-place of mother, 1900:

$$\rho = \cdot 6010 \pm \cdot 1362.$$

Although these values are not high they are possibly significant; and in the case of the races of the same order as that found to exist between the insane enumerated in hospital and suicides for the 15 States which is:

$$\rho = \cdot 6062 \pm \cdot 1101.$$

In the case of correlations found for the States and Races, it must be remembered that much greater fluctuations may be expected owing to the values being calculated on the returns for only one year, while in the case of the Cities it was possible to obtain a steadier result by averaging the returns for five years.

From the special report on *The Insane and Feeble Minded in U.S.A.*, published by the Census Office and dealing with the returns of 1904, I was able to obtain figures for the insane as they apply to the various States.

Particular caution is required in dealing with these figures, and a few extracts from the introduction to the report may enable the reader to estimate better the true value of the results. The report deals only with the insane in institutions. "No institution for the insane, except a very few small private institutions, refused to give information. Therefore so far as all classes of institutions are concerned the present enumeration is practically complete," p. 6. "To offer any estimate of the probable number of the insane that would have been disclosed by an investigation which included those not in institutions in addition to those in hospital on 31/12/03 would be a mere guess. But it is certainly within the truth to say that there were as many on that date as were found outside of hospitals in 1890, some 30,000," p. 8. This number refers to the whole of the United States and not alone to the 15 States dealt with in this paper. "In many cases the high rank given some States, for instance, Massachusetts, New York, Connecticut, New Jersey, California, &c., probably approximate actual conditions," p. 4.

Two correlation tables were prepared so that the matter might be viewed in somewhat different lights, and as much information as possible obtained from the figures. It seemed probable that yearly admission might be an even better guide to the prevalence of insanity than the actual number in hospital on any given day; because where accommodation was insufficient for the pressure occasioned by cases waiting for admission, it would be possible to mitigate this by discharging chronic and convalescent patients somewhat earlier than would be done by hospital authorities where there was more ample accommodation. If this were so then the admissions would be a better guide to the prevalence of the disorder than the actual number in the institutions. The values of the correlations for cancer death-rates with the insane are as follows:

Insane enumerated 31/12/03 $\rho = \cdot 8711 \pm \cdot 0420$.
Insane admissions 1904 $\rho = \cdot 9173 \pm \cdot 0278$.
Diabetes and Insane admissions $\rho = \cdot 8074 \pm \cdot 0608$.

These values seem to give additional support to the theory of a nervous factor in the aetiology of cancer, and it would be interesting to find an independent measure of the stress of life in the different districts, that could be used as an index to the intensity of the struggle for existence.

In a general way one would expect that in the larger cities the competition would be greater than in the smaller ones, although this would probably not be true of cities with only small differences in population, and other factors besides mere size would have to be taken into consideration. No small cities occur in the tables here used, and the correlations between population and cancer or diabetes death-rates are scarcely significant:

Cancer death-rate and population $\rho = \cdot 1049 \pm \cdot 1055$.
Diabetes death-rate and population $\rho = \cdot 3007 \pm \cdot 0970$.

I thought that a rough measure of the social and economic conditions prevailing in a district might be obtained by reducing the number of periodicals and newspapers printed in the various States to the rate per head of population, and correlating with cancer death-rate; the value thus found is:

$$\rho = \cdot 6182 \pm \cdot 1076.$$

I record these observations for any interest that they may have, but do not think that the deductions should be pushed too far.

Another aspect of the question that requires investigation is the death-rate among the various occupations. No reliable insanity figures can be obtained that correspond to the grouping for occupations as given in *Vital Statistics*, Vol. I., U.S.A. Census, 1900. In fact the relation of insanity to occupation has not been worked out for the United States. As the Editor of the Report above referred to says "On the whole, the question of the relation of insanity to occupation is one that still awaits systematic treatment. The available means of enquiry do not suffice even for a perfect classification of occupations, much less for obtaining data which permit exact comparisons with the total number of the insane drawn from any given occupation with the total number of the same age, &c., engaged in that particular occupation."

Considerable difficulty arises in correlating cancer and diabetes in respect to occupations owing to the large value of the Coefficient of Variability of the correction factor for age distributions, in the various occupations. An inspection of these factors will show that an appreciable positive correlation may be expected from this cause, on the other hand the value of ρ' will contain a spurious correlation of unknown size. Taking into consideration these facts and also that the negative sign of the correlation as found for the eight groups is unchanged even when uncorrected deaths are used, it will probably be safe to conclude that the correlation between cancer and diabetes death-rates with occupation groups is negative.

Occupations. Cancer and Diabetes.

8 Inclusive Groups $\rho = -\cdot 0645 \pm \cdot 2375, \quad \rho' = -\cdot 6719 \pm \cdot 1308.$

28 Sub-groups $\rho = \cdot 2866 \pm \cdot 1170, \quad \rho' = -\cdot 4832 \pm \cdot 0977.$

At first sight this may seem unexpected, but I do not think it is really surprising. The rates of both diseases vary considerably in the different occupations. It must be remembered that certain occupations consist of highly selected individuals, and thus the unstable in certain respects may have been weeded out. Whereas a similar condition imposed on a city or district may lead to an increase in both rates, conditions of work &c. may determine which form the manifestation of instability shall take. For instance occupations associated with a sedentary life probably predispose to diabetes.

The use of an analogy will better explain my meaning. The death-rates from Typhoid Fever and Bullets during time of war would be much increased, and both in relation to time and place would show positive correlation with war. But if these rates were to be compared in relation to deaths in occupations—soldiers at the front with men in garrison—no positive correlation would be observed. Thus under the heading of occupations we may be measuring rather the determining causes of the type of breakdown, than the fundamental cause of the disease.

I may perhaps be allowed briefly to refer to the theory which has been adopted as a working hypothesis in the present investigation. As far as this investigation is concerned there does not seem to be any evidence as to the infectious nature of cancer. Nor has any evidence been adduced that would lead to this belief with the possible exception of cancer-houses, while on the other hand, all the improvement brought about by public health reform, in the way of reducing over-crowding, abolition of back-to-back houses, &c. has not been followed by any reduction of the disease, on the contrary an actual increase has been taking place. That nervous centres do exercise an extraordinary influence over tissue cells is a matter of common knowledge and that some nervous disturbance may affect the cell metabolism in diabetes, and the cell growth in cancer does not seem to me impossible, even if a novel view. This suggestion does not necessarily exclude the presence of an organism as the determining cause, and several analogies will occur to the reader. For instance in a disease like scurvy, which is almost certainly of infectious origin, the organism does not usually obtain a footing until the soil has been prepared by errors of diet.

It must be understood that the strain occasioned by modern competition is only regarded as an exciting factor, in so far as it leads to a breakdown of nervous origin. This breakdown of course may occur without any undue strain, but will in the aggregate do so more frequently when this factor is present. The heredity factor, i.e. the constitutional weakness, is of course of primary importance.

Problems connected with the occurrence of cancer in animals are at present difficult to deal with statistically. That malignant tumours occur in vertebrates is well known, as the following extract from the Third Report of the Cancer Commission shows:—"Cancer is ubiquitous in man and vertebrate animals. It has been shown that cancerous tissue of a species of animal retains the characters of that species, whereas in infective tumours (e.g. tubercle) occurring naturally in separate species, the biological characters of the newly formed tissue are determined by the common infective agent and not by the tissues of the affected animal."

Again, the question of cancer in African Natives requires more study, and the increasing rates for these races as observed in America are of interest. The phlegmatic nature of the Bantu in Africa is well known. The prevalence of Diabetes among Jews is another problem of interest that awaits explanation. This race, however, seems particularly liable to certain nervous complaints, as for instance various forms of insanity, and Amaurotic Family Idiocy.

Since writing the above I have been looking through the small literature of insanity at my disposal and have been struck with the following sentences:—"It has been said that Diabetes, or Glycosuria at least is frequent among the insane. There is, however, some connection between Diabetes and Insanity, as we shall proceed to point out. Dr Maudsley has recorded his experience that it is not uncommon to find a history of Diabetes in the parents or near ancestors of insane patients; this is also our experience, especially among the affluent classes." G. H. Savage.

"Dr Snow informs me that independently of actual brain-deposits, insanity is not uncommon among cancer patients, and that the incurable wards of the Cancer Hospital are seldom without two or three cases." (*Dict. Med. Psych.*, Hack Tuke.)

Several considerations have led me to offer this paper for publication in its present form. Firstly, I venture to hope that the subject will be considered of sufficient importance to induce other biometrical workers to assist in its elucidation; secondly, the time at my disposal is limited while much remains to be done; and thirdly, much of the work is impossible to a resident living away from the big libraries, as many books of reference are unobtainable. Even such districts as have been dealt with could be better handled by workers conversant with the districts and their local conditions. Lastly, the publication of a preliminary paper may bring helpful criticism and so enable future work to be more profitably planned.

It would much enhance the value of the Death Reports, if the Registrars could see their way to add special tables of the causes of death, in cases where the diagnosis has been confirmed by post-mortem examination. A list of associated factors found in each case might be added, much on the lines adopted by the London Asylums Board. Thus there would in time grow up a body of statistical material of the utmost value for determining the true age distributions of the various diseases, and for obtaining many other important pathological facts.

I would take this opportunity of thanking Professor Pearson for much kind advice, and also Dr J. M'D. Troup and Mr G. Clark for their assistance during the compilation of this paper.

Conclusions.

1. That recorded differences in cancer and diabetes death-rates, as applying to different Districts and Cities of the U.S.A., as well as the increased rates observed in recent years, do indicate real differences in the prevalence of the disease.

2. The correlations found to exist between cancer, diabetes, and insanity are not fortuitous and due merely to errors of observation or record.

3. The statistics dealt with in this paper do not give any support to the suggestion that cancer is of infectious origin.

4. That whatever theory as to the causation of cancer is adopted, some explanation of the remarkable correlations between cancer, diabetes and insanity is required.

5. The explanation suggested to account for the facts as disclosed by statistical analysis is, that conditions of modern life, acting on physiologically unsound material, may account for the correlations existing between these three diseases, as well as for their increasing rates.

6. If this suggestion be correct, then the study and teaching of Eugenics has an additional purpose. Although no hope for the sick can be derived from this source, yet the future of the race will not be so hopeless, when children are taught the eugenic law, that marriage with the physiologically unfit is as anti-social, as marriage with their social inferiors is at present held to be.

TABLE I.

Cancer Death-Rates.

Age	MALE			FEMALE	
	Observations		Calculated Areas	Observations Record	Calculated Areas
	Record	Altered			
2·5	12	2	1	11	0
7·5	6	4	2	7	0
12·5	6	6	6	7	4
17·5	10	10	12	14	22
22·5	32	32	26	33	69
27·5	48	48	54	116	178
32·5	78	78	110	313	371
37·5	196	196	210	709	680
42·5	374	374	380	1200	1127
47·5	672	672	656	1909	1705
52·5	1120	1120	1068	2483	2418
57·5	1642	1642	1640	3229	3225
62·5	2278	2278	2338	3781	4330
67·5	3100	3100	3064	4549	4854
72·5	3772	3772	3728	5043	5472
77·5	4050	4050	4020	7137	5777
82·5	3730	3730	3796	5654	5621
87·5	2920	2920	3014	4000	4916
92·5	3920	2000	1886	3927	3563
97·5	1600	800	810	1818	1607
102·5	0	200	194	0	0
107·5	0	0	10	0	0
Totals	29566	27034	27025	45940	45940

Male Curve $y = 4050\left(1 + \frac{x}{52{\cdot}3713}\right)^{45{\cdot}6100}\left(1 - \frac{x}{7{\cdot}0852}\right)^{6{\cdot}1704}$.

Female Curve $y = 5818\left(1 + \frac{x}{15{\cdot}2550}\right)^{4{\cdot}6559}\left(1 - \frac{x}{4{\cdot}7721}\right)^{1{\cdot}4564}$.

TABLE II.

Diabetes Death-Rates.

Constants of Curves.

	Total Data	Suggested Old Age Mortality
Mean	66·9786	70·7810
Mode	86·0008 (Type J)	75·5300
S. D.	3·2984	2·2603
$\sqrt{\beta_1}$	1·3989	0·7403
β_2	5·1345	3·6254

Age	Observations	Mid-Ordinates	
		Old Age Component	Total Curve
2·5	13	0	8
7·5	19	0	19
12·5	36	0	29
17·5	38	1	38
22·5	36	2	45
27·5	51	5	51
32·5	57	11	57
37·5	65	24	67
42·5	102	49	86
47·5	117	93	122
52·5	204	163	182
57·5	238	269	277
62·5	443	403	403
67·5	579	553	553
72·5	627	662	662
77·5	685	676	676
82·5	585	528	528
87·5	201	252	252
92·5	39	29	29
97·5	3	3	3

Total Curve $y=2280\left(1+\frac{x}{30\cdot9147}\right)^{6\cdot1835}\left(1-\frac{x}{0\cdot1309}\right)^{-0\cdot0262}$.

Suggested Old Age Component $=691\cdot7\left(1+\frac{x}{34\cdot1585}\right)^{26\cdot5162}\left(1-\frac{x}{4\cdot0781}\right)^{3\cdot1657}$.

TABLE III.

England and Wales. 1891—1900.

Age	CANCER DEATH-RATE (per Million)		DIABETES DEATH-RATE (per Million)	
	Male and Female	Male	Male and Female	Male
0	30 ⎫ 18	33 ⎫ 24	4	5
5	16 ⎭	18 ⎭	12	12
15	37	41	33	37
25	139 ⎫ 360	99 ⎫ 223	49	57 ⎫ 70
35	646 ⎭	384 ⎭	71	87 ⎭
45	1833 ⎫ 2567	1300 ⎫ 2032	136 ⎫ 201	161 ⎫ 234
55	3662 ⎭	3160 ⎭	300 ⎭	347 ⎭
65	5604 ⎫ 5766	5325 ⎫ 5461	460 ⎫ 438	559 ⎫ 536
75	6145 ⎭	5824 ⎭	380 ⎭	474 ⎭
Total Rate	757	600	75	86

TABLE III *bis*.

Death-Rate (per Million).

England and Wales.

Year	Cancer	Diabetes
1871—75	446	36
1876—80	494	40
1881—85	548	51
1886—90	632	62
1891—95	711	69
1896—00	800	81
1901—05	864	89

TABLE IV.

Meteorological Conditions. Average of about 30 *years.*

City	Hours Sunshine	Mean Temperature	Mean Maximum Temperature	Mean Minimum Temperature	Rainfall in Inches	Days of Rain ·01 and over	Absolute Humidity in grs.
Baltimore	219	55	63	47	43	131	3·71
Boston	208	49	57	41	44	130	3·15
Buffalo	191	47	54	40	37	170	3·03
Chicago	218	48	55	41	33	126	3·17
Cincinnati	216	55	64	47	38	134	3·46
Cleveland	173	49	57	42	36	159	3·21
Columbus	206	52	61	43	37	144	3·37
Denver	251	50	63	37	14	80	2·03
Detroit	202	48	56	41	32	138	3·11
Indianapolis	207	55	61	44	42	136	3·38
Kansas City	212	54	63	45	36	104	3·63
Louisville	217	57	66	48	44	128	3·67
Memphis	227	62	70	53	51	115	6·15
Milwaukee	155	45	53	38	31	129	3·06
New Haven	231	50	58	41	47	134	3·30
New Orleans	198	69	76	62	58	128	5·81
New York	209	52	59	45	45	132	3·48
Omaha	228	50	60	41	31	100	3·17
Philadelphia	215	54	62	46	41	130	3·60
Pittsburg	179	53	62	44	37	154	3·46
Rochester	185	47	55	39	34	178	3·05
St Louis	227	56	64	48	37	115	3·78
St Paul	206	45	53	35	29	116	2·68
San Francisco	239	56	62	50	23	69	3·82
Toledo	201	50	58	42	31	135	3·20
Washington	212	55	64	46	43	126	3·78
Atlantic City	229	52	58	46	42	125	3·96
Mean	209·67	52·41	60·52	44·15	37·63	128·37	3·53
σ	20·34	5·09	5·11	5·32	8·56	22·87	·80

TABLE V.

Cancer.

City		Correction Factor	Crude Death-Rate 1900—04	Corrected Death-Rate 1900—04	Population per 1000's 1903
Allegheny	...	1·2349	435	537	138
Baltimore	...	1·0543	752	793	531
Boston	...	1·0456	896	937	581
Buffalo	...	1·1940	687	820	367
Chicago	...	1·3224	635	840	1873
Cincinnati	...	1·0384	781	811	332
Cleveland	...	1·2073	582	703	414
Columbus	...	1·0783	651	702	135
Denver	...	1·0876	655	712	147
Detroit	...	1·1383	709	807	310
Fall River	...	1·2389	608	753	105
Indianapolis	...	1·0313	578	596	200
Jersey City	...	1·2151	516	627	222
Kansas City	...	1·1791	553	652	173
Louisville	...	1·0908	594	648	216
Memphis	...	1·3327	373	497	114
Milwaukee	...	1·2289	678	833	303
Minneapolis	...	1·2289	587	721	238
New Haven	...	·9882	783	774	115
New Orleans	...	1·1035	736	812	301
Brooklyn	...	1·1557	627	725	1280
Manhattan	...	1·2701	743	944	2001
Queens	...	1·1921	474	565	180
Richmond	...	·9947	821	816	71
Newark	...	1·1215	686	769	267
Omaha	...	1·2701	519	659	113
Paterson	...	1·1401	631	719	109
Philadelphia	...	1·0341	699	723	1368
Pittsburg	...	1·3211	519	686	345
Providence	...	·9793	850	832	189
Rochester	...	1·0027	806	808	174
St Joseph	...	1·4585	306	446	110
St Louis	...	1·1610	622	722	612
St Paul	...	1·3051	555	724	183
San Francisco	...	·9546	1240	1184	356
Scranton	...	1·3351	420	561	110
Syracuse	...	·9668	760	735	114
Toledo	...	1·1383	574	653	146
Washington	...	·9439	745	703	293
Worcester	...	1·0385	728	756	124
Mean	...	—	—	735·05	374·00
σ	...	—	—	128·01	449·24

TABLE VI.

City	Diabetes Correction Factor	Diabetes		Suicides 1900—04	Nephritis 1900—04
		Crude Death-Rate 1900—04	Corrected Death-Rate 1900—04		
Allegheny ...	1·1381	52	59	145	574
Baltimore ...	1·0173	99	101	113	1446
Boston ...	1·0013	151	151	136	858
Buffalo ...	1·1194	146	163	111	1012
Chicago ...	1·1773	85	100	228	1145
Cincinnati ...	1·0081	90	91	217	1256
Cleveland ...	1·1261	59	66	158	840
Columbus ...	1·0260	83	85	139	702
Denver ...	1·0331	121	125	239	1098
Detroit ...	1·0807	113	122	112	751
Fall River ...	1·1450	114	131	61	1059
Indianapolis ...	·9907	96	95	110	668
Jersey City ...	1·1346	83	94	174	1384
Kansas City ...	1·0791	65	70	161	1080
Louisville ...	1·0388	75	78	127	1043
Memphis ...	1·1867	46	55	79	1370
Milwaukee ...	1·1556	94	109	215	623
Minneapolis ...	1·1322	79	89	135	721
New Haven ...	·9817	151	148	158	966
New Orleans ...	1·0699	61	65	137	2065
Brooklyn ...	1·0885	113	128	19	1740
Manhattan ...	1·1504	152	175	225	2218
Queens ...	1·1261	94	106	216	1993
Richmond ...	·9855	144	142	152	1663
Newark ...	1·0623	112	119	238	1533
Omaha ...	1·1468	64	73	142	602
Paterson ...	1·0791	93	100	90	1005
Philadelphia ...	1·0000	98	98	135	1492
Pittsburg ...	1·1811	47	55	155	767
Providence ...	·9628	162	156	110	1324
Rochester ...	·9817	135	132	137	885
St Joseph ...	1·2542	46	58	105	486
St Louis ...	1·0885	68	74	258	1355
St Paul ...	1·1811	74	87	125	724
San Francisco ...	·9317	176	164	550	1070
Scranton ...	1·2136	93	113	76	972
Syracuse ...	·9566	116	111	105	890
Toledo ...	1·0823	106	115	139	645
Washington ...	·9282	107	99	149	1103
Worcester ...	1·0067	131	132	101	713
Mean ...	—	—	107·02	154·44	1117·15
σ ...	—	—	32·61	80·82	436·58

TABLE VII.

States Registration.

	CANCER			DIABETES			Suicides
	Correction Factor	Death-Rate, 1906		Correction Factor	Death-Rate, 1906		
		Crude	Corrected		Crude	Corrected	
Maine ...	·7081	862	610	·7622	168	128	104
New Hampshire ...	·7135	892	636	·7653	169	129	86
Vermont ...	·7049	853	601	·7622	183	139	140
Massachusetts ...	·9087	903	820	·9135	161	147	96
Rhode Island ...	·9415	783	737	·9387	163	153	153
New York ...	·9510	762	725	·9422	160	151	144
Connecticut ...	·8731	806	703	·8897	188	167	138
New Jersey ...	1·0079	661	666	·9894	128	127	154
Michigan ...	·9334	676	631	·9434	137	129	98
Indiana ...	·9780	537	525	·9740	110	107	116
Maryland ...	1·0384	601	624	1·0176	96	98	89
Pennsylvania ...	1·0485	607	636	1·0218	108	110	113
South Dakota ...	1·2308	354	436	1·1645	88	102	69
Colorado ...	1·1828	513	607	1·1029	84	93	190
California ...	·8310	920	764	·8532	161	137	289
Mean ...	—	—	648·07	—	—	127·80	131·93
σ ...	—	—	91·94	—	—	21·30	52·26

TABLE VIII.

States Registration.

	INSANE (per million)		Newspapers to Persons
	Enumerated 31/12/03	Admitted 1904	
Maine ...	125	47	·920
New Hampshire ...	117	83	·522
Vermont ...	255	77	·561
Massachusetts ...	289	133	2·270
Rhode Island ...	235	92	·500
New York ...	339	86	4·510
Connecticut ...	293	89	·505
New Jersey ...	238	65	1·612
Michigan ...	216	58	·988
Indiana ...	166	53	·841
Maryland ...	202	66	·573
Pennsylvania ...	173	50	1·837
South Dakota ...	141	32	·586
Colorado ...	129	86	·984
California ...	361	98	1·025
Mean ...	218·60	74·13	1·22
σ ...	75·50	24·17	1·02

TABLE IX.

Registration Districts. Death-Rate according to Birthplace of Mother in United States of America.

	CANCER			DIABETES			Suicides
	Correction Factor	Crude Death-Rate	Corrected Death-Rate	Correction Factor	Crude Death-Rate	Corrected Death-Rate	
United States of America ...	·9276	483	448	·8380	96	80	68
Ireland	·8149	764	623	·7239	104	75	61
Germany	·8592	782	672	·7653	122	93	193
England and Wales ...	·7400	720	533	·6818	121	82	104
Canada	1·3591	403	548	1·0653	63	67	65
Scandinavia	1·4474	311	450	1·1161	33	37	107
Scotland	·6901	818	565	·6649	123	82	116
Italy	1·6935	228	386	1·2438	26	32	51
Russia and Poland	2·0626	257	530	1·4296	33	47	58
France	·6421	928	596	·6000	129	77	220
Mean	—	—	535·10	—	—	67·20	—
σ	—	—	82·67	—	—	19·96	—

TABLE X.

Occupations. United States of America, 1900. *Registered States.*

	CANCER			DIABETES		
	Correction Factor	Crude Death-Rate	Corrected Death-Rate	Correction Factor	Crude Death-Rate	Corrected Death-Rate
Professional	·6764	517	350	·6771	222	150
Officials, Clerks, etc. ...	·8167	377	308	·7762	181	140
Mercantile Trading, etc. ...	·7472	520	388	·7257	194	139
Public Entertainers ...	·7642	353	270	·7251	159	115
Service, Military, Police ...	·7075	429	303	·6986	87	61
Manufacturing	·7653	533	408	·7401	112	83
Agriculture	·5484	692	379	·5960	133	79
Labourers not Agricultural ...	·8054	665	536	·7678	79	61
Mean	—	—	350·22	—	—	104·44
σ	—	—	99·54	—	—	36·19

TABLE XI.

	CANCER			DIABETES		
	Correction Factor	Death-Rate	Corrected Death-Rate	Correction Factor	Death-Rate	Corrected Death-Rate
Bankers ...	·3617	414	150	·4350	207	90
Law ...	·5234	525	275	·5669	315	179
Physicians ...	·5355	675	361	·5741	169	97
Clergy ...	·4282	1022	438	·4903	341	167
Officials ...	·8168	377	308	·7762	181	140
Hotel Keepers ...	·4483	601	269	·5059	250	126
Book Keepers ...	1·3172	280	369	1·0238	158	162
Collectors ...	·5344	581	310	·5780	216	125
Merchants ...	·5539	813	450	·5898	310	183
Butchers ...	·9094	445	405	·8159	78	64
Bakers ...	·9422	510	481	·8350	128	107
Police ...	·4814	603	290	·5328	93	50
Printers ...	1·2668	221	280	·9965	37	37
Tailors ...	·8402	584	491	·7804	143	112
Barbers ...	1·2119	275	333	·9556	50	48
Plumbers ...	1·4966	82	123	1·0777	62	67
Bootmakers ...	·6656	341	227	·6788	62	42
Carpenters ...	·4841	777	376	·5375	83	45
Farmers ...	·4525	877	397	·5206	165	86
Servants ...	1·2513	443	554	·9817	74	73
Sailors ...	·5942	1131	672	·6255	168	105
Blacksmiths ...	·5828	1073	625	·6143	123	76
Masons ...	·5210	980	511	·5665	181	103
Iron-workers ...	1·0422	315	328	·8848	29	26
Draymen ...	1·0106	226	228	·8687	59	51
Labourers ...	·7792	691	538	·7511	79	59
Factory Operators ...	1·3636	345	470	1·0565	40	42
Pedlars ...	·7416	627	465	·7257	60	44

TABLE XII.

Correlation Values.

Corrected Cancer Death-Rate and Meteorological Factor.
In 27 Cities, 1906.

Sunshine ... ... $r = -{\cdot}0142 \pm {\cdot}1323$
Mean Temperature ... ... $r = -{\cdot}1478 \pm {\cdot}1294$
Mean Maximum Temperature $r = -{\cdot}2236 \pm {\cdot}1257$
Mean Minimum Temperature $r = -{\cdot}0146 \pm {\cdot}1323$
Rainfall ... ... $r = -{\cdot}2517 \pm {\cdot}1239$
Days of Rain ... ... $r = -{\cdot}1621 \pm {\cdot}1288$
Absolute Humidity ... ... $r = -{\cdot}2014 \pm {\cdot}1269$

TABLE XII.—(*continued*).

In 17 Cities, 1906.

Sunshine	$r=\cdot1268\pm\cdot1659$
Rainfall	$r=\cdot3163\pm\cdot1518$
Mean Temperature	$r=-\cdot0032\pm\cdot1685$
Mean Minimum Temperature	$r=\cdot1254\pm\cdot1659$
Mean Maximum Temperature	$r=-\cdot1509\pm\cdot1600$
Relative Humidity 8 a.m.	$r=\cdot3101\pm\cdot1519$
Relative Humidity 8 p.m.	$r=\cdot1603\pm\cdot1600$
Range of Humidity	$r=\cdot0724\pm\cdot1685$

Correlations, 15 *States,* 1906.

Cancer and Diabetes, 1906	$\rho=\cdot9088\pm\cdot0303$,	$\rho'=\cdot8258\pm\cdot0554$
Cancer and Suicides	$\rho=\cdot4412\pm\cdot1402$,	$\rho'=\cdot3801\pm\cdot1490$
Cancer and Insane Enumerated, 31/12/03	$\rho=\cdot8711\pm\cdot0420$,	$\rho'=\cdot8222\pm\cdot0564$
Cancer and Insane Admissions, 1904	$\rho=\cdot9173\pm\cdot0278$,	$\rho'=\cdot9179\pm\cdot0274$
Cancer and Newspapers to persons, 1900	$\rho=\cdot6182\pm\cdot1076$,	$\rho'=\cdot5686\pm\cdot1178$
Diabetes and Insane Admissions	$\rho=\cdot8074\pm\cdot0608$,	$\rho'=\cdot7906\pm\cdot0653$
Suicides and Insane Enumerated	$\rho=\cdot6062\pm\cdot1101$	

Correlations, Cities, 1900—1904.

Cancer with Diabetes, 1900—04	$\rho=\cdot6896\pm\cdot0559$,	$\rho'=\cdot7325\pm\cdot0494$
Cancer with Nephritis, 1900—04	$\rho=\cdot3792\pm\cdot0913$,	$\rho'=\cdot1571\pm\cdot1040$
Cancer with Suicides, 1900—04	$\rho=\cdot3962\pm\cdot0899$,	$\rho'=\cdot6895\pm\cdot0559$
Cancer with Population, 1900	$\rho=\cdot1049\pm\cdot1055$,	$\rho'=\cdot4792\pm\cdot0822$
Diabetes with Population, 1900	$\rho=\cdot3007\pm\cdot0970$,	$\rho'=\cdot4084\pm\cdot0889$
Cancer with Ill-defined Causes	$\rho=\cdot1020\pm\cdot1055$,	$\rho'=-\cdot3453\pm\cdot0939$
Diabetes with Ill-defined Causes	$\rho=\cdot0773\pm\cdot1060$,	$\rho'=-\cdot0193\pm\cdot1066$
Cancer with Percentage Error of Registration	$\rho=\cdot0526\pm\cdot1064$	

Cancer Death-Rate according to Birthplace of Mother, United States of America, 1900, *with*

Diabetes	$\rho=\cdot8609\pm\cdot0552$,	$\rho'=\cdot5442\pm\cdot1501$
Suicides	$\rho=\cdot6010\pm\cdot1362$,	$\rho'=\cdot6731\pm\cdot1166$

Cancer Death-Rate according to Occupation, United States of America, 1900, *with*

Diabetes (8 Large Groups)	$\rho=-\cdot0645\pm\cdot2357$,	$\rho'=-\cdot6719\pm\cdot1308$
„ (28 Sub-groups)	$\rho=\cdot2866\pm\cdot1170$,	$\rho'=-\cdot4832\pm\cdot0977$

Occupation (Social Rank) with

Cancer (8 groups)	$r=-\cdot5176\pm\cdot1746$
Diabetes (8 groups)	$r=\cdot9374\pm\cdot0289$
Cancer (28 groups)	$r=-\cdot4574\pm\cdot1007$
Diabetes (28 groups)	$r=\cdot6716\pm\cdot0700$

Death-Rate of Cancer by 5-yearly periods, 1871—75 *to* 1901—05, *England and Wales, with*

Diabetes Death-Rate	$\rho=\cdot8060\pm\cdot0893$

A QUANTITATIVE STUDY OF THE MORPHOLOGY OF THE FRUIT OF THE BLOODROOT, SANGUINARIA CANADENSIS.

By J. ARTHUR HARRIS, Ph.D., Carnegie Institution of Washington.

I. Introduction.

Heretofore investigations in variation and correlation carried out by means of modern quantitative methods have been largely for the purpose of throwing light upon the problems of evolution. But for several years I have been convinced that the methods of higher statistics should find an extensive application in the fields of morphology and physiology.

The present memoir is one of a series in course of preparation in which an attempt has been made to apply the statistical methods to the problems of the morphology and physiology of the inflorescence and fruit. The results set forth will not show their full significance until the constants obtained for other forms are laid beside them for comparison. These data will be ready shortly, but it seems best not to anticipate the results from material not completely worked up.

II. Material and Methods.

(*a*) *The Morphology of the Fruit of Sanguinaria.*

Sanguinaria, a monotypic genus of the Papaveraceae, widely distributed over the eastern portion of North America, is so familiar to botanists that a detailed discussion of the superficial form of the fruit borne on its slender pedicel is hardly necessary. Gray* has given the essentials when he writes:—

"Ovary oblong, one-celled, with two parietal placentae; style short, columnar; stigma broad, sulcate—two-lobed, the lobes alternate with the placentae. Ovules very numerous, horizontal in several rows on the two placentae, anatropous.

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* Gray, A., *Gen. Pl. U. S.* Vol. i. p. 115, 1848.

"Capsule oblong, somewhat compressed, herbaceo-coriaceous, many-seeded, pointed by the short, persistent style, two-valved; the valves separating from the replum formed by the intervalvular filiform placentae."

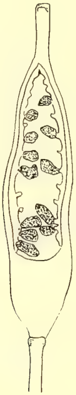

FIG. 1. Diagram of fruit of *Sanguinaria*, opened in the upper part on one side, showing two placentae with matured seeds and abortive ovules.

I am unaware of any investigation of the embryological development of the ovary of *Sanguinaria*. Until such data are available we shall not be able to say more concerning the morphology of the fruit than Gray did sixty years ago. The essential points to be remembered in the present work are the following. The fruit is borne on a slender pedicel sent up from the root-stalk. It is one-celled with two parietal placentae on which the seeds are arranged in two or three irregular rows. Judging by analogy with other ovaries whose embryology is known, it would seem probable that each placenta is composed of the margins of two distinct carpels.

(*b*) *Collection of Material and Characters considered.*

In the spring of 1906 I collected at Meramec Highlands, near St Louis, Mo., a series of 1000 fruiting stalks of *Sanguinaria*. At the time, the stalks seemed to have their full growth, so far as could be judged by general criteria, but the seeds were not fully mature.

In the spring of 1907 another lot of fruits was secured from the same habitat. Only 400 were gathered, and in this case the peduncle was not taken. This second collection was made considerably later in the season than the first, and the fruits were sensibly larger, as will appear when the means of fruit lengths are considered.

From these collections the following measurements and countings were made:—

1. Length of Peduncle.
2. Length of Fruit.
3. Seeds developing on first placenta.
4. Ovules formed on first placenta.
5. Seeds developing on second placenta.
6. Ovules formed on second placenta.
7. Aborted ovules on first placenta.
8. Aborted ovules on second placenta.
9. Total seeds developing per fruit.
10. Total ovules formed per fruit.
11. Total aborted ovules per fruit.

Some words of explanation concerning these characters are necessary.

1. *Length of Peduncle.* A study of the length of the peduncle was not a primary purpose of this investigation, but it was taken up in a rough and ready manner to gain some general appreciation of the influence which it might have upon the magnitude of the characters of the fruit. It is idle to expect to attain great precision in the investigation of a character of this kind.

Sanguinaria is a perennial plant and we do not know the age of the individual root-stalks which produced these peduncles and fruits. The plants were very probably of different ages; all that we can say with certainty is that they were mature enough to produce fruiting stalks. It is quite possible that the age of the individual has a material influence upon the character of the flowering stalk which it produces.

There is unquestionably some error in the determination of the length of the stalks due to the method of collecting. They were simply carefully pulled from the rhizome. Considering the nature of the material with which we are dealing, I think the determinations of stalk length were quite accurate enough for practical purposes.

2. *Length of Fruit.* In both collections the length of the fruit was taken to the nearest millimetre.

As a character for investigation the length of the fruit presents some serious difficulties. The fruit is not infrequently somewhat curved or otherwise irregular in form, and there is relatively a large possible error in measurement. The fruits collected in 1906 had not quite attained their full size, but I was afraid to leave the collection until later because of the difficulty of counting the seeds which very easily become detached. In 1907 the material had more nearly attained its full development.

To reduce the distributions to somewhat more workable limits the fruits have been combined in groups of 3 mm. range. These have proved very satisfactory working units. Constants calculated from them are essentially the same as those calculated from the ungrouped distributions, as I have assured myself by actually carrying out the arithmetic.

3—11. *Meristic Characters of the Fruit.* The counting of the number of ovules formed, the number of seeds developing, and the number of abortive ovules is not at all difficult in this species. The fruits were of slightly different ages in the two collections, but in both cases they were old enough for the determination of the number of seeds developing to be entirely trustworthy.

In recording these characters the numbers on each placenta were noted separately. In the following pages the placentae are referred to as "first" and "second." The two are not differentiated at all, the "first" placenta being merely the first one counted when the fruit was opened. The constants for the distribution of ovules, seeds and aborted ovules should be the same for the two placentae within the limits of the probable error of random sampling. In investigating many relationships the tables may be made symmetrical, that is to say each placenta may be used once as the "first" and once as the "second" number of a pair. The advantage of using these symmetrical tables has been several times pointed out in biometric literature*.

I have found it rather an advantage to work with ovules, seeds and aborted ovules. Naturally, the seeds developing and the aborted ovules per fruit equal the number of ovules formed, and the tables may easily be verified if all three characters be used. Characters 9, 10 and 11 are obtained by adding 3 and 5, 4 and 6, 7 and 8.

(*c*) *Method of Reduction of Data.*

Sheppard's correction was used throughout in the calculation of the standard deviations.

In calculating the probable errors where more than one organ from each individual is measured, there is always some question as to what value of N shall be used. I have taken N as the actual number of flowering stalks in all cases in which one of the characters involved in the constant was measured only a single time for each inflorescence. For instance in correlating between length of fruit and number of seeds per placenta, or between the fertility characters of the two placentae of the same fruit, N was used as 1000 and 400 in the two collections. But when the constants are calculated for the repeated organs it seems logical to calculate the probable errors on the basis of the actual number of such organs, and this was done for the constants calculated for the characters of the individual placentae.

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* For instance, by Pearson and his assistants in their memoir on homotyposis in the vegetable kingdom (*Phil. Trans. Roy. Soc. Lond.* A, Vol. CXCVII. pp. 285—379); by Pearl on Paramecium (*Biometrika*, Vol. v. pp. 249—251, 1907); by Pearson and Barrington in their memoir on the inheritance in vision (*Francis Galton Lab. for Nat. Eugenics*, 5, 1909); by Harris (*Biometrika*, Vol. VII. pp. 214—218, 1909).

III. PROBLEMS CONSIDERED.

1. *Type and Variability of the Characters Investigated in the Two Collections.*

The fundamental physical constants are given in Tables 1 and 2. It has seemed best to table each collection separately and to put them side by side for comparison in a third table. The distributions upon which these constants are based will be found as the totals of the correlation tables. The differences of the constants to be compared are given with their probable errors in Table 3.

TABLE 1.

Constants for 1906 *Collection of Sanguinaria.*

Character	Average and Probable Error	Standard Deviation and Probable Error	Coefficient of Variation
Length of Stalk in 5 cm. Units*	17·090 ± ·058	2·736 ± ·041	16·01
Length of Fruit in 3 mm. Units†	39·703 ± ·134	6·307 ± ·095	15·88
Ovules on First Placenta	14·980 ± ·097	4·569 ± ·069	30·50
Ovules on Second Placenta	14·924 ± ·099	4·658 ± ·070	31·21
Ovules on First and on Second Placenta	14·952 ± ·070	4·614 ± ·049	30·86
Total Ovules per Fruit	29·904 ± ·193	9·054 ± ·136	30·28
Seeds on First Placenta	11·584 ± ·083	3·872 ± ·058	33·42
Seeds on Second Placenta	11·471 ± ·085	3·977 ± ·060	34·67
Seeds on First and on Second Placenta	11·527 ± ·059	3·925 ± ·042	34·05
Total Seeds per Fruit	23·055 ± ·161	7·531 ± ·114	32·66
Aborted Ovules on First Placenta...	3·396 ± ·060	2·827 ± ·043	83·25
Aborted Ovules on Second Placenta	3·453 ± ·060	2·802 ± ·042	81·14
Aborted Ovules on First and on Second Placenta	3·424 ± ·042	2·812 ± ·030	82·12
Total Aborted Ovules per Fruit	6·849 ± ·110	5·162 ± ·078	75·36

* Constants tabled in units of 1 cm.

† Constants tabled in units of 1 mm.

Another constant which is of much service in studies of fertility is not given in either of these tables. This is the ratio of the total seeds developing to the total ovules formed.

Several years ago Pearson‡ defined fecundity as the ratio of the number of offspring actually produced to those which might have come into existence under the circumstances. His definition was formulated for use in a study of inheritance in thoroughbred horses. It seems to me a good one and I shall use it throughout a series of studies on fertility in plants, modifying the terms involved as may be necessary but keeping always to the general idea of the definition. Mayer[*] used the same ratio in a study of symmetry and fertility in a Medusa. He used the term "coefficient of fertility." Coefficient of fecundity is perhaps a more suitable term, agreeing with Pearson's terminology. The coefficient of fecundity is a very useful constant. Its value lies between 0 and 1, and its form is therefore similar to that of some other statistical constants.

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‡ *Phil. Trans. Roy. Soc. Lond.* A, Vol. CXCII. p. 261, 1899.

TABLE 2.

Constants for 1907 *Collection of Sanguinaria.*

Character	Average and Probable Error	Standard Deviation and Probable Error	Coefficient of Variation
Length of Fruit in 3 mm. Units[†]	51·865 ± ·268	7·948 ± ·190	15·32
Ovules on First Placenta	12·812 ± ·137	4·052 ± ·097	31·63
Ovules on Second Placenta	12·792 ± ·136	4·024 ± ·096	31·46
Ovules on First and on Second Placenta ...	12·802 ± ·096	4·038 ± ·068	31·54
Total Ovules per Fruit	25·605 ± ·265	7·850 ± ·187	30·66
Seeds on First Placenta	9·717 ± ·134	3·980 ± ·095	40·96
Seeds on Second Placenta	9·795 ± ·135	3·991 ± ·095	40·74
Seeds on First and on Second Placenta	9·756 ± ·095	3·986 ± ·067	40·85
Total Seeds per Fruit	19·512 ± ·258	7·660 ± ·183	39·26
Aborted Ovules on First Placenta	3·095 ± ·086	2·559 ± ·061	82·68
Aborted Ovules on Second Placenta	2·997 ± ·088	2·610 ± ·062	87·09
Aborted Ovules on First and on Second Placenta	3·046 ± ·062	2·585 ± ·044	84·86
Total Aborted Ovules per Fruit	6·092 ± ·161	4·785 ± ·114	78·55

[†] Constants tabulated in units of 1 mm.

In the present case the coefficient of fecundity is the ratio of the total seeds developing in the population to the total ovules produced.

For the present series of data we have:—

$$\text{For 1906 series, C.F.} = \frac{23055}{29904} = \cdot7710 \pm \cdot0016\text{[‡]},$$

$$\text{For 1907 series, C.F.} = \frac{7805}{10242} = \cdot7621 \pm \underline{\cdot0042},$$

$$\text{Difference,} \qquad -\cdot0089 \pm \cdot0045.$$

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[*] Mayer, A. G., *Sci. Bull. Brook. Mus.* Vol. I. 1901; see also C. B. Davenport, *Biometrika*, Vol. I. pp. 255, 256, 1902.

[‡] It is known that if y_s be the frequency of any class of individuals in a population of m individuals, the probable error of the frequency is given (*Biometrika*, Vol. II. p. 274, 1903) by the formula

$$Ey_s = \cdot67449\sqrt{y_s \times \left(1 - \frac{y_s}{m}\right)}.$$

In the present case y_s is the number of seeds developing in the population m, or total number of ovules. From the absolute probable error of the number of seeds developing calculated from this formula, the probable error of the coefficient of fecundity was obtained by taking the ratio of the absolute probable error to m, the total number of ovules.

In the present paper the study of variation has been a means to an end. The standard deviations have been necessary in the calculation of the coefficients of correlation, and the consideration of the interrelationships between different parts of the organism as indicated by the coefficient of correlation has been the main purpose of the paper.

TABLE 3.

Comparison of Constants for 1906 *and* 1907 *Collections of Sanguinaria.*

Character	Average and Probable Error	Standard Deviation and Probable Error	Coefficient of Variation
Length of Fruit, 1906	39·703 ± ·135	6·307 ± ·095	15·89 ± ·23
" " 1907	51·865 ± ·268	7·948 ± ·190	15·33 ± ·37
Difference	+ 12·162 ± ·300	+ 1·641 ± ·212	− ·56 ± ·43
Ovules per Placenta, 1906	14·952 ± ·070	4·613 ± ·049	30·86 ± ·51
" " 1907	12·803 ± ·096	4·038 ± ·068	31·64 ± ·83
Difference	− 2·149 ± ·119	− ·575 ± ·084	+ ·78 ± ·98
Total Ovules per Fruit, 1906	29·904 ± ·193	9·055 ± ·137	30·28 ± ·50
" " " 1907	25·605 ± ·265	7·850 ± ·187	30·66 ± ·80
Difference	− 4·299 ± ·328	− 1·205 ± ·232	+ ·38 ± ·94
Seeds per Placenta, 1906	11·527 ± ·059	3·925 ± ·042	34·05 ± ·57
" " 1907	9·756 ± ·095	3·986 ± ·067	40·85 ± 1·13
Difference	− 1·771 ± ·112	+ ·061 ± ·079	+ 6·80 ± 1·26
Total Seeds per Fruit, 1906	23·055 ± ·161	7·531 ± ·114	32·66 ± ·54
" " " 1907	19·513 ± ·258	7·660 ± ·183	39·26 ± 1·07
Difference	− 3·542 ± ·304	+ ·129 ± ·215	+ 6·60 ± 1·20
Aborted Ovules per Placenta, 1906 ...	3·424 ± ·042	2·812 ± ·030	82·12 ± 2·69
" " " 1907 ...	3·046 ± ·062	2·585 ± ·044	84·87 ± 3·17
Difference	− ·378 ± ·075	− ·227 ± ·053	+ 2·75 ± 4·15
Total Aborted Ovules per Fruit, 1906 ...	6·849 ± ·110	5·161 ± ·078	75·36 ± 2·43
" " " " 1907 ...	6·093 ± ·161	4·785 ± ·114	78·54 ± 2·73
Difference	− ·756 ± ·195	− ·376 ± ·141	+ 3·18 ± 3·65

The constants for the two years considered individually require no separate discussion. The means are not of interest except in comparison with the same constants for other races of *Sanguinaria*. The standard deviations and coefficients of variation are of about the order generally found for plant characters.

The comparison table shows that some of the differences are several times their probable errors, and hence clearly significant. For length of fruit this is quite obviously due to the somewhat greater maturity of the 1907 series. Here I do not care to discuss in detail the interpretation to be assigned to these differences. I must point out, however, that the fact that our probable errors show that there are significant differences in the samples with which we are dealing does not justify us in concluding that there are differences in the populations which they represent unless we are quite sure that our samples have been drawn in precisely the same way. Rocks, steep banks and gentle slopes, fallen logs and drifts of leaves are possibly all factors which make for heterogeneity in the environment of the individual organism in a plant formation which the ecologist would consider quite homogeneous. In collecting two samples a and b from such a habitat we might get constants differing sensibly with regard to their probable error merely by failing to have the several subhabitats contribute the same proportional number of individuals in each case.

These points are emphasized because I do not wish to have these results interpreted as indicating that there is a real difference between the whole population of *Sanguinaria* at Meramec Highlands in the two years in question due to some peculiarity in the seasons themselves. There *may* be such a difference, but there is no strong evidence for such a conclusion in our data which were not collected in a way, nor for the purpose, to determine whether there is a real difference between the whole populations in different seasons.

Turning aside from the contrasting of the constants for individual characters and taking a general view of the whole comparison table, I think the impression must be that the agreement between the two collections is very close.

I think we may consider the two series of material upon which we propose to investigate certain morphological relationships as on the whole very similar in nature. Tests for the similarity of series of material used in morphological work have rarely been applied, but it would be desirable if some attention could be given to points like this.

2. *The Correlation between the Length of the Peduncle and the Length of the Fruit.*

It seems important to obtain some idea of the relationship between the degree of development of the fruit-stalk and the size attained by the fruit. The data appear in Table I, 1906, and I have calculated both the coefficient of correlation, r, and the correlation ratio, η.

This latter constant, recently introduced by Pearson[*], furnishes an adequate description of the interdependence of two variables when the rate of change of the dependent variable cannot be expressed by the slope of a straight line.

The two constants have the values

$$\begin{aligned} r &= \cdot 335 \pm \cdot 019 \\ \eta &= \cdot 379 \pm \cdot 018^{\dagger} \\ \hline \eta - r &= \cdot 044 \end{aligned}$$

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[*] Pearson, K., *Drapers' Co. Research Memoirs*, Biometric Series, No. 2, Dulau & Co., 1905.

[†] Here E_η was calculated by Pearson's short formula, *loc. cit.* p. 20, 1905.

It appears, therefore, that there is a very substantial interdependence between the length of the peduncle and the length of the fruit. It will be borne in mind, however, that the fruit in this series had not quite reached its maximum size, and that owing to difficulties inherent in the species, constants involving peduncle length and fruit length will always be open to some question.

Blakeman* has shown that, if $\xi = \eta^2 - r^2$, a good test for linearity of regression is given by using the formula

$$\frac{\xi}{E_\xi} = \frac{\sqrt{N}}{0{\cdot}67449} \cdot \frac{1}{2}\sqrt{\xi}\, \frac{1}{\sqrt{1 + (1-\eta^2)^2 - (1-r^2)^2}}.$$

How large the value of ξ/E_ξ shall be to be considered significant must be decided by each writer for himself, but when we find $\xi/E_\xi > 2{\cdot}5$ we may begin to be suspicious of the linearity of our regressions. From our data we find

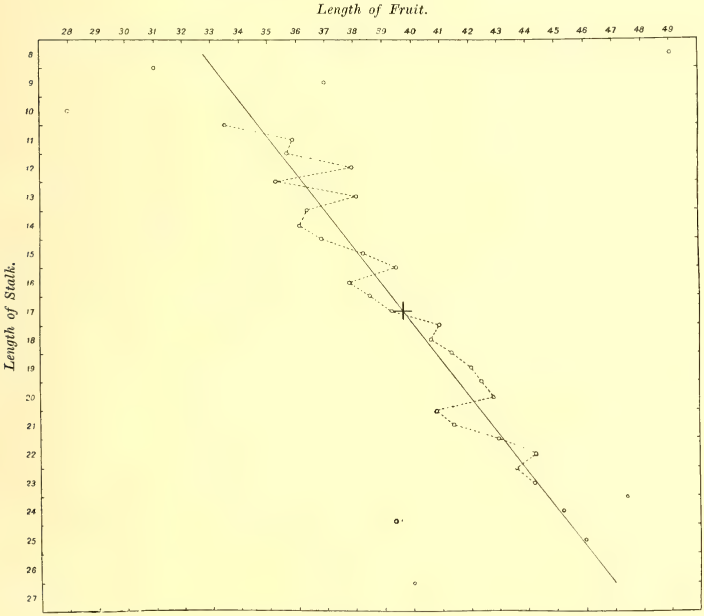

Diagram I. Regression of fruit length on peduncle length. (Means depending on a few isolated observations not joined up.)

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[*] Blakeman, J., *Biometrika*, Vol. iv. pp. 335—350, 1905.

$\xi = {\cdot}03149$, and $\xi/E_\xi = 4{\cdot}279$. It seems rather questionable, therefore, whether our regression line can be regarded as strictly linear within the limits of the probable errors of random sampling.

The equation of the regression straight line is

$$y = 26{\cdot}517 + {\cdot}772x.$$

I have plotted the means of arrays and find that they are scattered with some irregularity on either side of the theoretical line, but considering the nature of our material the fit is perhaps as good as could be expected. There is no suggestion in the graph that any other curve would give better results.

The biological significance of this constant showing the interdependence between length of flowering stalk and length of fruit seems to me to be chiefly descriptive. We cannot say that the length of the stalk determines to some extent the length of the fruit, or *vice versa*. In short, we have not "explained" anything, as we biologists are so wont to do. But we have measured a relationship and recorded it in a form that may be directly compared with other similar measurements when they shall have been made, and when the series so described are enough, comparison will permit of pushing problems further.

3. *The Relationship between the Length of the Peduncle and the Fertility of the Fruit.*

We have little definite knowledge of the relationship between the physical development of an organism and its fertility. In man Pearson[*] has brought forward evidence to show that tall women are more fertile than short ones. I am not aware of further published evidence for animals, though Professor Pearson informs me that data for other mammals are in existence and will shortly be published.

In plant material it has been shown in an earlier paper[†] that there is a correlation of about ·5 or over for length of flowering stalk and number of flowers per umbel in *Nothoscordium* and *Allium*. In the case of *Cercis*[‡] and *Celastrus*[§], however, there seems to be no correlation between the size of the inflorescence as measured by the number of flowers or fruits produced and the fertility of the ovaries. Before we can go very far into the problem of the relationship between the degree of development of the vegetative and reproductive characters we must have together a sufficient number of series of reduced data to enable us to make extensive comparisons.

In the table of correlation constants I have given the relationship between the length of the peduncle and the fertility characters of "first" and "second" locules separately. The constants were worked out largely to furnish an extra safeguard against arithmetical blunders, but the results seem worth publishing as this may indicate to the non-statistical reader something of the amount of divergence between constants which may arise purely as a result of chance.

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[*] Pearson, K., *Proc. Roy. Soc.* Vol. LIX. pp. 301—305, 1896.

[†] Harris, J. Arthur, *Ann. Rept. Mo. Bot. Gard.* Vol. XX. 1909.

[‡] Harris, J. Arthur, *Biometrika.* In press.

[§] Harris, J. Arthur, *Ann. Rept. Mo. Bot. Gard.* Vol. XX. 1909.

The correlation coefficients and their probable errors are given in Table 4. In two cases I have also compared the correlation ratio, η. See Table 5. In these two cases Blakeman's formula cited above gives:—

$$\text{For length and ovules, } \xi/E_\xi = 4{\cdot}125,$$

$$\text{For length and seeds, } \xi/E_\xi = 4{\cdot}131.$$

Here again there is some question concerning the linearity of our regression. The equations to the regression lines are:—

$$\text{For ovules per placenta, } y = 5{\cdot}863 + {\cdot}532x,$$

$$\text{For seeds per placenta, } y = 2{\cdot}987 + {\cdot}500x.$$

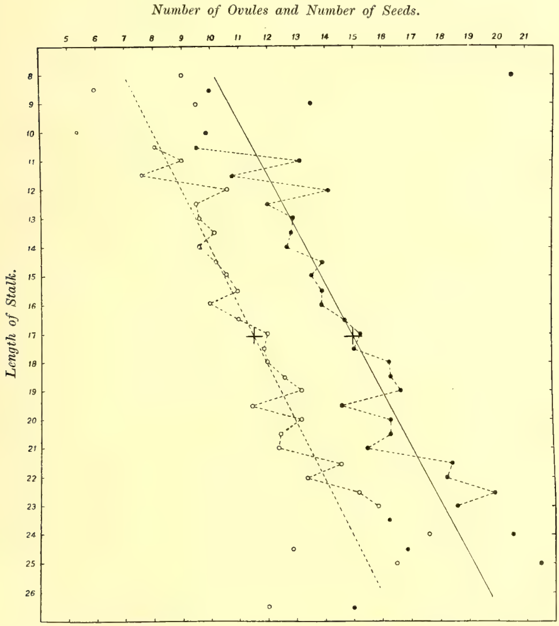

DIAGRAM II. Regression of number of ovules per placenta on peduncle length and of number of seeds per placenta on peduncle length. Solid dots and line = ovules; circles and dotted line = seeds. (Means depending on a few isolated observations not joined up.)

In neither case can the straight line be considered very satisfactory, but at present I do not see the advantage of seeking for further regression equations. When more biological data are available it may be profitable to go further with the statistical analysis.

TABLE 4.

Correlation between Length of Peduncle and the Fertility of the Fruit in Sanguinaria.

Relationship between Length of Stalk and	Correlation and Probable Error
Ovules on First Placenta	·313 ± ·019
Ovules on Second Placenta	·321 ± ·019
Ovules on First and on Second Placenta	·315 ± ·019
Total Ovules per Fruit	·323 ± ·019
Seeds on First Placenta	·341 ± ·019
Seeds on Second Placenta	·356 ± ·019
Seeds on First and on Second Placenta	·348 ± ·019
Total Seeds per Fruit	·363 ± ·019
Aborted Ovules on First Placenta	·039 ± ·021
Aborted Ovules on Second Placenta	·029 ± ·021
Aborted Ovules on First and on Second Placenta...	·034 ± ·021
Total Aborted Ovules per Fruit	·037 ± ·021

TABLE 5.

Relationship	r	η	$\eta - r$
Length and Ovules per Placenta ...	·315 ± ·019	·361 ± ·019	·046
Length and Seeds per Placenta ...	·348 ± ·019	·388 ± ·018	·040

These constants show that there is essentially the same degree of interdependence between the length of the fruit-stalk and the number of ovules formed and the number of seeds developing as between the length of the fruit-stalk and the length of the fruit. In the case of both ovules and seeds the correlation between length of peduncle and the total number per fruit is slightly, though *not significantly*, higher than for the numbers per placenta.

A question in which the biologist is interested is this:—Is the relationship between the length of the peduncle and the number of seeds developing more intimate than that between the length of the peduncle and the number of ovules formed?

Suppose for a moment that the degree of development of the peduncle affords some rough measure of the "vigor" of the plant. I think no physiologist can object to it as a working hypothesis, and I dare say that most would take it for granted. Now it is important to know whether the vigor of the plant shows itself more in the number of seeds developing—a character which is not definitely and unchangeably fixed until after a considerable period of physiological activity—than in the number of ovules formed—a character definitely and unchangeably established relatively early in the development of the organ we are studying.

If vigor finds fuller expression in the number of seeds developing than in the number of ovules formed we should expect to find a higher coefficient of correlation between the length of stalk and the number of seeds than between the length of stalk and the number of ovules. The comparison is made in Table 6.

TABLE 6.

Comparison of Relationship between Stalk-Length and Number of Ovules and Stalk-Length and Number of Seeds.

Relation between Stalk-Length and	Correlation and Probable Error
Ovules on First and on Second Placenta ...	·315 ± ·019
Seeds on First and on Second Placenta ...	·348 ± ·019
Difference	+ ·033 ± ·027
Total Ovules per Fruit	·323 ± ·019
Total Seeds per Fruit	·363 ± ·019
Difference	+ ·040 ± ·027

It appears that while actually the correlation between length of stalk and number of seeds is slightly higher than that between length of stalk and number of ovules, the difference is so small as to be insignificant in comparison with its probable error.

4. *The Relationship between the Length of the Fruit and its Fertility.*

That there should be a fairly close interdependence between the size of a fruit like that of *Sanguinaria* and the number of ovules formed and the number of seeds developing is to be expected. The placental space in a fruit of this species is pretty well filled up with ovules, and if there are many ovules the placentae must be longer than if there are only a few. The demonstration by means of a "formidable mathematical apparatus" of this rather obvious fact is not in itself a matter of much weight, but it is of importance to measure the degree of resemblance instead of merely asserting that a relationship exists, and to express the result of the measurement in such terms that it may easily and directly be compared with like measures in other species.

The constants for both lots of data are compared in Table 7.

TABLE 7.

Correlation between Length of Fruit and Number of Ovules Formed and Seeds Developing per Placenta and per Fruit.

Relationship between Length of Fruit and	Correlation and Probable Error		
	1906	1907	Difference
Ovules on First and on Second Placenta	·517 ± ·016	·638 ± ·020	+ ·121 ± ·025
Total Ovules per Fruit	·527 ± ·015	·656 ± ·019	+ ·131 ± ·025
Seeds on First and on Second Placenta	·552 ± ·015	·661 ± ·019	+ ·109 ± ·024
Total Seeds per Fruit	·576 ± ·014	·688 ± ·018	+ ·112 ± ·023
Aborted Ovules on First and on Second Placenta	·078 ± ·021	·023 ± ·034	− ·106 ± ·040
Total Aborted Ovules per Fruit	·085 ± ·021	·025 ± ·034	− ·110 ± ·040

The equations to the regression straight lines for length of pod and ovules per placenta and for length of pod and seeds developing per placenta in the two series are given in Table 8. Table 9 affords a comparison of r and η for these relationships.

TABLE 8.

Equations to Regression Straight Lines for Fertility Characters on Pod Length.

Regression of Fertility Characters on Pod Length	Equation to Straight Line
Ovules per Placenta on Pod Length, 1906 ...	$y = \cdot063 + \cdot378x$
Seeds per Placenta on Pod Length, 1906 ...	$y = -2\cdot115 + \cdot344x$
Ovules per Placenta on Pod Length, 1907 ...	$y = -4\cdot000 + \cdot324x$
Seeds per Placenta on Pod Length, 1907 ...	$y = -7\cdot438 + \cdot332x$

TABLE 9.

Comparison of r and η and Blakeman's Test for the Linearity of Regression for Fruit Length and Fertility Characters.

Relationship	Coefficient of Correlation, r	Correlation ratio, η	$\eta^2 - r^2 = \xi$	ξ / E_ξ
Ovules and Length, 1906	·517 ± ·016	·532 ± ·015	·016	2·971
Seeds and Length, 1906	·552 ± ·015	·552 ± ·015	—	—
Ovules and Length, 1907	·638 ± ·020	·637 ± ·020	—	—
Seeds and Length, 1907	·661 ± ·019	·677 ± ·018	·021	2·160

In both of the cases in which ξ/E_ξ was calculated it is found to have a value not greatly exceeding 2·5 in the first case and falling somewhat short in the second. Judging merely from the statistics in hand, the deviations of the means of arrays from a straight line may be due to nothing more than the probable errors of random sampling. In the case of length and seeds for 1906 and length and ovules for 1907, ξ/E_ξ was not calculated, since η is slightly less than r. This theoretically impossible result is explained as follows. In calculating η I followed the conventional practice of using the standard deviation for the population calculated without Sheppard's modification. In calculating r, however, Sheppard's correction was applied to both standard deviations. Had standard deviations *without* Sheppard's correction been used, r would have been lowered somewhat by the increase in the value of the product $\sigma_1\sigma_2$, and would then be less than η. It seems rather pedantic under the circumstances to recalculate the values of r, for biologically the results could have little increased value. For one year it is the correlation for length and ovules and for the other year it is the correlation for length and seeds which give such nearly identical values of r and η. There is, therefore, no reason to think that the regression may be non-linear for ovules and not for seeds, or *vice versa*.

For both ovules and seeds the constants for 1907 are about 0·110 ± ·025 higher than in 1906.

Possibly the more mature condition of the fruits gathered in 1907 may have as one of its results the raising of the coefficient of correlation for length of fruit and number of ovules or seeds.

These constants are purely descriptive. They express the degree of interdependence between length of fruit and number of ovules or seeds but do not explain anything concerning the cause of this interdependence. The correlation coefficients in themselves cannot even be considered adequate to establish the existence of a causal nexus inherent in the fruit primordium and finding expression in the relationship observed between length of fruit and number of ovules or seeds. There are two ways in which a correlation between fruit length and fertility might arise independently of any cause for such a relationship innate in the fruit.

(*a*) Heterogeneity in the individuals from which the fruits were taken.

(*b*) Correlation between both the length and the fertility of the fruit and some other organ of the plant, say length of peduncle.

The first of these possible sources of correlation is not open to investigation on the basis of our material. The second may more easily be taken into account.

Both length of fruit and number of ovules and seeds are correlated with length of peduncle. What influence will this have upon the correlation between the length of the fruit and its fertility? The best way of getting at this problem seems to be to determine the correlation between the length of the pod and its fertility characters for constant length of stem. In doing this we strip off, so to speak, the influence of the length of the stalk upon the coefficient of correlation between length and fertility of fruit and have our relationship uninfluenced by this character.

The familiar formula for the partial correlation coefficient is

$$\rho_{xy} = \frac{r_{xy} - r_{xz}\, r_{yz}}{\sqrt{1 - r_{xz}^2}\,\sqrt{1 - r_{yz}^2}}.$$

In the present series x is length of fruit, y the number of ovules or seeds, as the case may be, and z, whose influence on the gross correlation is to be eliminated, is the length of the peduncle.

The partial correlation coefficients for the total ovules and total seeds for the 1906 series calculated by the above formula appear in Table 10.

TABLE 10.

Correlation between Length of Fruit and Number of Ovules and Seeds per Fruit for Variable and Constant Lengths of Peduncle.

Relationship between Length of Fruit and	Correlation
Total Ovules per Pod	
For Variable Length of Peduncle ...	·527
For Constant Length of Peduncle ...	·470
Difference	·057
Total Seeds per Pod	
For Variable Length of Peduncle ...	·576
For Constant Length of Peduncle ...	·517
Difference	·059

The differences show that there is a small amount of the relationship between the length of the fruit and the number of ovules or seeds which it produces which is due to the correlation of both of the characters of the associated pair with peduncle length. But this influence is comparatively small.

A question of very real interest is that concerning the relative magnitude of the correlation between the length of the fruit and the number of ovules and the length of the fruit and the number of seeds. The results may be extracted from Table 7 for comparison in Table 11.

While the correlation for length and seeds is the higher in both series, the excess above the relationship for length and ovules cannot be given much weight. I was at first somewhat surprised at this result, for I had rather expected that the length of a soft fruit of this kind was much more dependent upon the number of seeds developing than upon the number of ovules formed. We are dealing, however, with a form in which the correlation between the number of ovules formed and the number of seeds developing is high. Under such circumstances these results would be expected. The problem should be worked out on some form in which there is only a moderate correlation between the number of ovules and number of seeds.

TABLE 11.

Comparison of Correlation for Fruit Length and Number of Ovules and Fruit Length and Number of Seeds.

Relationship	Correlation in 1906	Correlation in 1907
Length and Ovules	·527 ± ·015	·656 ± ·019
Length and Seeds	·576 ± ·014	·688 ± ·017
Difference ...	·049 ± ·021	·032 ± ·026

5. *The Relationship between the Number of Ovules Formed and the Number of Seeds Developing.*

Our knowledge of the physiology of seed production is very limited. We know practically nothing of why one ovule in a locule develops into a mature seed and another beside it fails to develop. General explanations, such as "lack of nutrition," "failure to be fertilized," etc., are thoroughly unsatisfactory. Experimental evidence would be the most desirable, but until it is forthcoming I think we shall have to depend upon the biometric analysis of large bodies of data to enable us to get some insight into the problems.

TABLE 12.

Correlation for Fertility Characters in Sanguinaria.

Relationship between Number of Ovules Formed and Number of Seeds Developing	Correlation and Probable Error		
	1906	1907	Difference
For First Placenta	·7901 ± ·0080	·7998 ± ·0121	—
For Second Placenta	·8037 ± ·0076	·7905 ± ·0127	—
For First and Second Placenta ...	·7970 ± ·0055	·7951 ± ·0088	− ·0019 ± ·0104
For the Fruit	·8141 ± ·0072	·8106 ± ·0116	− ·0035 ± ·0136

The first question which suggests itself for investigation is whether an ovule formed in a pod producing a large number of ovules is more, or less, likely to

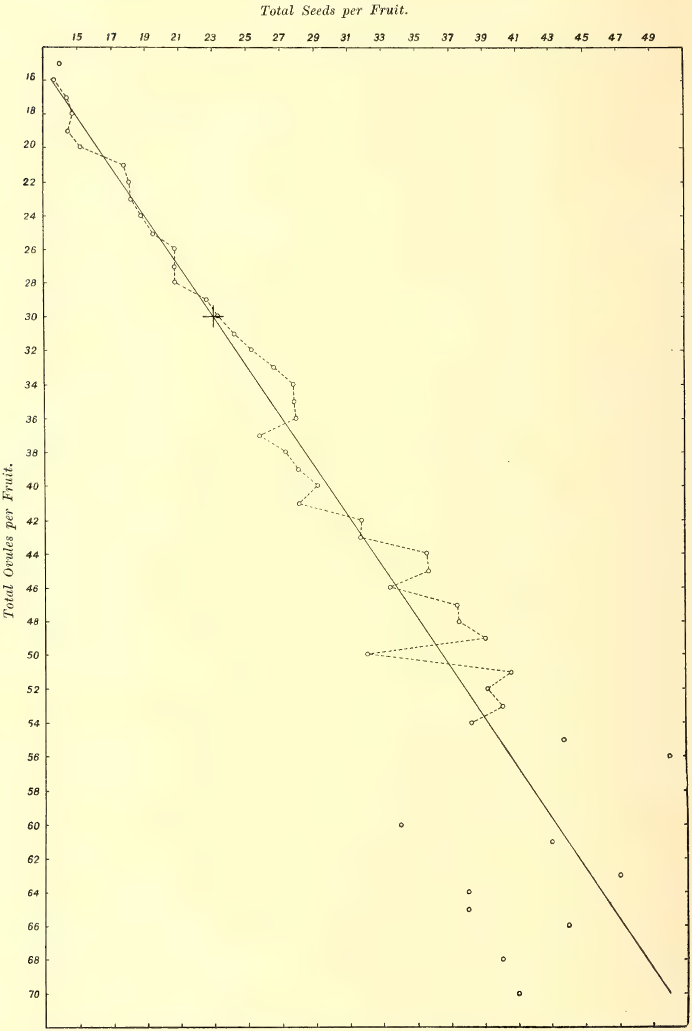

Diagram III. Regression of total seeds per fruit on total ovules per fruit. (Means depending on a few isolated observations not joined up.)

develop than one which is produced in a fruit with a smaller number of ovules. This will "explain" nothing when we arrive at the answer to this question. If we find that it has a better chance of developing in a fruit of some particular type we shall not know *why* it has this advantage over an ovule formed in a different type of pod. But we shall have definitely isolated one of the factors—or complexes of factors—which have an influence upon the development of an ovule into a seed and shall have quantitatively determined the amount of influence which it wields.

The ordinary coefficients of correlation between the number of ovules and the number of seeds developing per pod are shown in Table 12.

Two points in this table are noteworthy. First, the correlations are high. Second, the agreement between the two series of material is very close. The constants for the two collections differ by less than the probable error of their difference.

The equations to the regression straight lines for total ovules and total seeds per fruit are:—

$$\text{For 1906 series,}\quad y = 2{\cdot}808 + {\cdot}677x,$$

$$\text{For 1907 series,}\quad y = -{\cdot}741 + {\cdot}719x.$$

The comparison of η with the coefficients of correlation and the ξ/E_ξ of Blakeman's test for the linearity of regression are given in Table 13.

TABLE 13.

Tests for Linearity of Regression in Correlation between Total Ovules and Total Seeds.

Constant	1906 Series	1907 Series
Coefficient of Correlation, r ...	$\cdot 8141 \pm \cdot 0072$	$\cdot 8106 \pm \cdot 0116$
Correlation Ratio, η	$\cdot 8317 \pm \cdot 0066$	$\cdot 8420 \pm \cdot 0098$
$\xi, = \eta^2 - r^2$	·02896	·05199
Blakeman's Criterion, ξ/E_ξ ...	4·028	3·438

I am rather surprised to find ξ/E_ξ so high as it is in these two cases. The indications are that the divergence of the means from a straight line is rather larger than one would expect to be accounted for by the errors of sampling. From a graph, however, there is no clear indication that any curve would give any better fit than the straight line. Certainly the data in hand do not justify any further analysis along these lines. The two constants are so nearly identical that for any practical purpose it is immaterial which is employed to describe the relationship between number of ovules and number of seeds per fruit.

While the coefficient of correlation shows the degree of interdependence of number of ovules and seeds it does not yield one other item of information which is essential to our understanding of fertility. This is a knowledge of the influence of the number of ovules per fruit upon the chance of an individual ovule developing into a seed. This is quite a different problem from that of the relationship between the total number of ovules per pod and the total number of seeds developing.

As pointed out in an earlier paper*, the constant which we need is the correlation between the number of ovules and the deviation of the number of seeds from their probable number. The coefficients calculated by the method described in that paper are given in Table 14.

TABLE 14.

Correlation between Number of Ovules and Deviation of Number of Seeds from the Probable Number on the Assumption of the same Proportional Fecundity throughout the Population.

Relationship between Ovules and Seeds	Correlation and Probable Error		
	1906	1907	Difference
For both Placentae ...	−·178 ± ·015	·038 ± ·024	+·216 ± ·028
For the Fruit	−·191 ± ·021	·051 ± ·034	+·241 ± ·039

These results show how important it is to have more than one series of material upon which to base conclusions when the correlations are of a low order of magnitude. Considering only the 1906 series we see constants with a negative sign and about ten times their probable error. It would seem from these results alone that as the number of ovules per placenta or per fruit increased the capacity of the fruit for maturing its ovules into seeds decreased. But when the 1907 series is taken into consideration also we note that the correlations are hardly twice their probable errors but with the positive sign. Of course it may be possible that a larger sampling in the 1907 plants would have given a sensible negative correlation. But all that we can conclude is that while there may be a relationship between the number of ovules per fruit and the capacity of the fruit for maturing its seeds there is no very satisfactory argument for such a theory in our present collections.

These conclusions agree with the results announced for one sample of 10,000 pods of *Cercis* given as an illustration in the paper on the method of determining the relationship between ovules and seeds, but do not agree with the results for the general sample of *Robinia*. The factors underlying differences of this kind deserve investigation.

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* Harris, J. Arthur, *Biometrika*, Vol. VI. pp. 438—443, 1909.

6. *The Correlation between the two Placentae of the Fruit.*

The correlations between organs of the two sides are now available for a considerable number of bilaterally symmetrical animal organisms but practically nothing has as yet been put into print concerning the similarity of the two sides of bilaterally symmetrical plant organs. The fruit of *Sanguinaria* furnishes a very suitable object for the determination of such relationships.

Since the two placentae are quite undifferentiated the correlations may be determined from symmetrical tables. Our tables as published have not been rendered symmetrical. Anyone who desires may determine the constants from them as they stand. I have worked by both methods and find no sensible difference in results. The coefficients for symmetrical tables can easily be calculated from them by use of proper formulae*. For the constants see Table 15.

TABLE 15.

Organic Correlations for Fruits of Sanguinaria.

Characters of the two Placentae	Correlation and Probable Error		
	1906	1907	Difference
Ovules of "First" Placenta and Ovules of "Second" Placenta	·9239 ± ·0031	·8868 ± ·0072	− ·0371 ± ·0078
Seeds of "First" Placenta and Seeds of "Second" Placenta	·8014 ± ·0076	·8443 ± ·0097	+ ·0429 ± ·0123
Aborted Ovules on "First" Placenta and Aborted Ovules on "Second" Placenta	·6793 ± ·0115	·7070 ± ·0169	+ ·0277 ± ·0204
Ovules on "First" Placenta and Seeds Developing on "Second" Placenta	·7356 ± ·0098	·7191 ± ·0163	− ·0165 ± ·0193

The equation of the regression straight line for the relationship for ovules is

$$y = 1{\cdot}1384 + {\cdot}9239\,x.$$

A graphic representation of the slope and the empirical means shows that except at the ends, where the observations are few, the observed and the theoretical means show a close agreement. We also note:—

Ovules of first and ovules of second placenta,

$$r = {\cdot}9239 \pm {\cdot}0031,$$

$$\eta = {\cdot}9229 \pm {\cdot}0032,$$

$$\eta - r = -{\cdot}0010.$$

Seeds of first and seeds of second placenta,

$$r = {\cdot}8014 \pm {\cdot}0076,$$

$$\eta = {\cdot}8015 \pm {\cdot}0076,$$

$$\eta - r = {\cdot}0001.$$

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* See R. Pearl, *Biometrika*, Vol. v. pp. 250, 251.

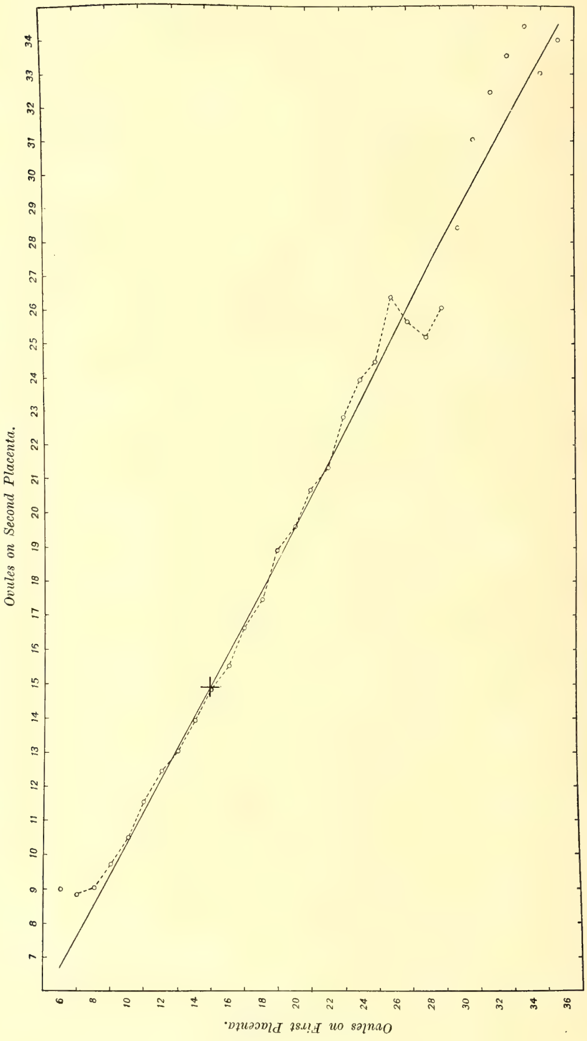

Diagram IV. Regression of number of ovules on "second" placenta on number of ovules on "first" placenta. (Means depending on a few isolated observations not joined up.)

The agreement is remarkably close. The condition $r > \eta$ for the ovules is due to the use of Sheppard's correction in the calculation of r. Without Sheppard's correction for the standard deviations we find $r = \cdot 9203$, where the theoretical condition $\eta > r$ is fulfilled. For the direct correlation for both ovules and seeds developing the differences in the constants for the two seasons are about four times their probable errors. But considering the possibilities of error in the sampling of the habitat in the two years I would not attach much significance to them as indicating real biological differences in the two years. The other two correlations differ in the two years by an amount of about the magnitude of its probable error.

The significance of these constants seems to me to be twofold: (*a*) descriptive, (*b*) as a means of gaining a somewhat deeper insight into the physiology of seed development.

The constant showing the degree of interdependence between the number of ovules on the two placentae taken in connection with the means and standard deviations, is a terse description of these features in the fruits of a given population. As it stands it is interesting as showing the high degree of perfection in the morphogenetic process giving rise to the ovary*. After studying a considerable mass of reduced data, some of which I hope to publish before long, it seems to me that a comparative study of these interplacental or interlocular correlation coefficients presents some problems of considerable interest.

These organic correlation coefficients seem to me to illustrate very well the way in which statistical methods may be applied to certain physiological problems.

The direct correlation for number of ovules per placenta is higher than that for either number of seeds developing or number of abortive ovules. Since the ovules are uninfluenced by the accidents of nutrition or fertilization which affect the number of seeds or the number of aborted ovules, the correlation between the number of ovules on the two placentae is quite independent of that for the other fertility characters; but the converse is not at all true.

We may ask to what extent the correlation between the number of ovules per placenta influences the correlation for number of seeds per placenta. The number of seeds developing per placenta is closely correlated with the number of ovules, hence a correlation between the number of seeds on the first and on the second placenta would necessarily be found even though there were no *physiological* relationship whatever between the development of the seed on the two sides of the fruit. Our object now is to remove the influence of the correlation for number of ovules on first and on second placenta from the correlation for seeds developing. If there is a physiological interdependence between the two placentae of the fruit depending on some factor other than the number of ovules present, we should expect to find a significant correlation between the number of seeds developing after the contribution of the ovules has been removed.

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* Since the tables are symmetrical the slope of the regression line is the same as the coefficient of correlation.

Again we have recourse to the partial correlation coefficient as in the investigation of the influence of peduncle length upon the relationship between the length of the fruit and its fertility characters. Here the more complex formula for four variables must be used.

The partial correlation, $\mathbf{r}_{s_1s_2}$, for constant values of o_1 and o_2 is given by the formula*

$$\mathbf{r}_{s_1s_2}=\frac{r_{s_1s_2}(1-r_{o_1o_2}{}^2)-r_{o_1s_1}r_{o_1s_2}-r_{o_2s_1}r_{o_2s_2}+r_{o_1o_2}(r_{o_1s_1}r_{o_2s_2}+r_{o_1s_2}r_{o_2s_1})}{\sqrt{(1-r_{o_1o_2}{}^2-r_{o_2s_1}{}^2-r_{o_1s_1}{}^2+2r_{o_1o_2}r_{o_1s_1}r_{o_2s_1})}\sqrt{(1-r_{o_1o_2}{}^2-r_{o_2s_2}{}^2-r_{o_1s_2}{}^2+2r_{o_1o_2}r_{o_1s_2}r_{o_2s_2})}},$$

where o_1 and o_2, s_1 and s_2 are the ovules and seeds on the first and second placenta. But since our tables have been rendered symmetrical $o_1=o_2$, $s_1=s_2$ and we may write

$$\mathbf{r}_{s's'}=\frac{r_{s's'}(1-r_{o'o'}{}^2)-2r_{os}r_{o's'}+r_{o'o'}(r_{os}{}^2+r_{o's'}{}^2)}{1-r_{o'o'}{}^2-r_{o's'}{}^2-r_{os}{}^2+2r_{o'o'}r_{os}r_{o's'}},$$

where the correlations without dashes are those for the same placenta and those with dashes are for the relationships between different placentae.

Substituting constants for these formulae we find :—

For 1906, $\mathbf{r}_{s's'}=\cdot 590$,

For 1907, $\mathbf{r}_{s's'}=\cdot 714$.

Remembering that our gross correlations were $r=\cdot 801$ for 1906 and $r=\cdot 844$ for 1907 it is clear that :—

(*a*) A certain amount of the correlation between the number of seeds developing is due to the correlation between the number of ovules formed on the two placentae.

(*b*) The correlation is mainly due, however, to the physiological factors upon which the development of ovules into seeds depends.

Under the term physiological as used here are comprehended, (*a*) the ecological factors which determine whether an ovule shall receive a sperm, and the availability of food material and other requisites for growth, (*b*) the innate vigor or other physiological characters of the individual which determine whether a fertilized ovule shall develop into a seed.

The foregoing analysis shows, I believe, that the main bulk of the correlation between the number of seeds on the two placentae is due to a complex of ecological and physiological causes, and is not merely a necessary statistical result of the correlation for ovules. Some of the biological factors may have an influence on both the number of ovules formed and the proportion of these ovules which shall develop into seeds. Experimental data in quantity and form for statistical treatment are desirable.

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* Pearson, K., *Phil. Trans.* A, Vol. cc. p. 31, 1902.

IV. Recapitulation.

The present paper embodies the results of a quantitative study of the inflorescence and fruit of *Sanguinaria* with special reference to morphological and physiological problems. The results presented are chiefly of value as descriptive quantitative data. The constants will gain in significance as others from organs of distinct morphological type are laid beside them for comparison. The following points may be noted.

1. The variation and correlation constants for two collections of *Sanguinaria* made in the same habitat for two successive years show very close agreement for some characters and significant divergence for others. The significant differences do not necessarily indicate that the populations in the two years are really different for the material was not collected in a manner to secure the same proportional sampling of subhabitats in the two seasons, and these may differ enough among themselves to introduce a slight heterogeneity into the material. These results furnish another illustration of the difficulty of establishing "place constants" for an organism easily affected by environment. On the whole, the two series may be regarded as very satisfactorily similar as material for a morphological and physiological investigation.

2. The length of the peduncle and the length of the fruit are to some extent interdependent, the magnitude of the relationship being measured by a coefficient of correlation of about ·350. Number of ovules formed, number of seeds developing and number of aborted ovules per fruit are also correlated with peduncle length. For both ovules and seeds developing the correlation is about ·300—·350, the intensity of the relationship being slightly but not significantly higher for length of fruiting stalk and number of seeds developing than for length of fruiting stalk and number of ovules formed.

Since our correlation coefficient is purely descriptive, we cannot say that the length of the peduncle has any influence upon the length of the fruit or upon its fertility. All that we can do at present is to measure the degree of interdependence of these pairs of characters. Possibly neither has any real influence upon the development of the other and their measurable interdependence is due merely to the age or physiological vigor of the plants which produced them. Interdependence of their magnitude would then be merely the result of their mutual dependence upon other factors. Perhaps questions of this sort can be solved only by appropriate experiments, but we sorely need quantitative data of the kind presented here for numerous species to guide us in the intelligent planning of experiments.

3. It is not surprising to find that length of fruit and number of ovules formed and length of fruit and number of seeds developing are interdependent, but it is important to measure the degree of correlation between them in such a way that comparison with other forms will be possible. As indicated by our material the correlations for length and ovules or seeds developing lie between ·500 and ·700.

The correlation between the length of fruiting stalk and both length of fruit and fertility characters of the fruit would necessarily bring about a statistical correlation between length of the fruit and its fertility characters, whether there is any other biological reason for such a correlation or not. Calculation for the relationship between length of fruit and its fertility for constant stem length shows that the removal of the influence of this latter character reduces the correlation but slightly.

Clearly, then, the correlation between length and fertility of fruit is due mainly to causes other than the length of peduncle.

In both series the correlation between the length of the fruit and the number of seeds developing is slightly but hardly significantly higher than that between the length of the fruit and the number of ovules formed.

4. The correlation between the number of ovules formed and the number of seeds developing per placenta or per fruit is high, the constants falling immediately above or below ·800. This is obviously a necessary condition when the ratio seeds/ovules is high, as it is in this case, and the quantitative statement of the degree of the relationship is chiefly of interest in comparison with similar constants from other material. It tells us nothing concerning the influence of the number of ovules formed per fruit upon the probability of an individual ovule developing into a seed. In other words, the coefficient of correlation between number of ovules formed and number of seeds developing is a measure of the influence of the number of ovules upon the *absolute* fertility as measured by the number of seeds developing, but not upon *relative* fertility.

A suitable constant for the measurement of this latter relationship indicates that in the 1906 series as the number of ovules per fruit increases the capacity of the fruit for maturing its ovules into seeds decreases slightly, while in the 1907 collection there seems to be no significant influence of the number of ovules per fruit upon the chances of an ovule developing into a seed.

It has been suggested to me that the fruits in which the larger number of ovules are formed are the more vigorous, and that in consequence they will be able to develop a higher per cent. of their ovules into seeds. The discordant results secured in this study as well as those announced earlier for *Cercis* and *Robinia* indicate that extreme caution must be used in asserting that the number of ovules formed in a fruit has anything to do with the proportion of them which shall develop into seeds.

5. The correlations between the two placentae are all high, the direct correlation for the number of aborted ovules being actually the lowest, although the cross correlation for ovules of the "first" placenta and seeds developing on the "second" placenta is perhaps not significantly higher. The values for the direct correlations for seeds developing are ·80 and ·84 and those for ovules ·89 and ·92 in the two series. These constants show that there is a high degree of bilateral symmetry in the fruit of *Sanguinaria*.

Since the number of ovules formed and the number of seeds developing on a placenta are correlated, the correlation between the number of ovules per placenta must bring about a correlation between the number of seeds developing on the two placentae. Evidence is adduced to show that the correlation for seeds is not entirely due to the correlation for ovules, but that the larger part of the interrelationship for the seeds developing must be due to ecological or physiological factors.

London, December 7, 1909.

V. APPENDIX OF CORRELATION TABLES OF DATA.

The tables for the relationships between the two placentae are not rendered symmetrical except for the cross correlation for ovules of "first" placenta and seeds of "second" placenta where it is obviously necessary to use each placenta once as a "first" and once as a "second" placenta or publish two tables.

It is unnecessary to publish tables for the following four relationships:—length of peduncle and total ovules, length of peduncle and total seeds, length of fruit and total ovules, length of fruit and total seeds. The product moments for these can be easily derived from those for ovules and seeds per placenta by taking the origin at 0 in calculating the first moments of the arrays of number of ovules or number of seeds per placenta. Since both placentae of a fruit were entered under the same grade of peduncle length or fruit length the first moment for the array gives at once the total number of ovules or seeds formed by that grade of peduncle or fruit. This is the same for the number per placenta or the total number per fruit. The origin of the other character—peduncle length or fruit length in the present case—may then be taken wherever we please, and the product moment calculated at once by multiplying up the first moments of the arrays by the deviations of the classes of the other character from their origin. The means of arrays for tests of linearity of regression may be easily obtained. The standard deviations for total ovules and total seeds per fruit are available from other tables.

The correlation tables for number of ovules failing per placenta and per fruit are also omitted, for if their standard deviations be known the more important correlations can be derived from the tables for ovules formed and seeds developing.

TABLE I., 1906.

Length of Peduncle and Length of Fruit.

Length of Peduncle.

Length of Fruit.	8	8·5	9	9·5	10	10·5	11	11·5	12	12·5	13	13·5	14	14·5	15	15·5	16	16·5	17	17·5	18	18·5	19	19·5	20	20·5	21	21·5	22	22·5	23	23·5	24	24·5	25	25·5	26	26·5	Totals
21—23	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
24—26	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
27—29	—	—	—	—	3	—	—	—	—	1	1	1	3	2	—	—	7	3	2	1	—	1	—	—	1	1	—	1	—	—	—	—	—	—	—	—	—	—	28
30—32	—	1	—	—	—	2	3	3	2	3	2	6	5	6	6	3	9	8	3	5	3	3	4	—	—	3	1	—	—	—	—	—	—	—	—	—	—	—	81
33—35	—	—	—	—	—	3	—	4	—	5	4	6	8	13	15	13	13	17	17	7	7	9	4	5	6	5	2	1	—	—	1	1	1	1	—	—	—	—	168
36—38	—	—	1	—	—	1	4	—	4	4	4	6	7	14	10	9	18	9	16	12	17	9	9	4	8	4	5	3	1	—	2	—	—	2	—	—	—	—	183
39—41	—	—	—	—	—	—	—	—	3	2	6	5	5	4	4	14	17	13	9	10	11	12	11	7	5	5	4	1	4	3	2	—	—	—	1	—	—	1	159
42—44	—	—	—	—	—	—	—	1	2	—	2	2	4	3	7	8	11	10	12	14	12	16	19	7	9	6	4	—	—	2	1	1	1	—	—	—	—	—	154
45—47	—	—	—	—	—	—	1	—	—	1	3	2	—	3	6	7	6	10	4	6	7	8	12	5	4	5	3	3	—	3	5	—	—	—	—	—	—	—	104
48—50	1	—	—	—	—	—	—	1	—	—	—	—	1	1	2	2	3	2	5	8	7	6	6	3	7	7	3	1	2	1	1	—	—	1	—	—	—	—	71
51—53	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1	4	1	1	1	2	6	—	1	—	2	—	1	1	2	—	1	—	—	—	25
54—56	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	—	1	1	—	—	1	2	1	—	—	—	1	—	—	—	—	—	—	—	—	—	—	10
57—59	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	1	1	—	2	—	—	1	—	—	—	—	—	1	—	—	—	—	—	—	—	9
60—62	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	—	—	—	—	—	—	2
63—65	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	1	1	1	0	3	6	8	9	13	16	22	28	33	47	51	59	85	75	71	69	67	68	68	34	47	36	23	12	9	9	14	4	4	4	2	0	0	1	1000

TABLE II., 1906.

Length of Peduncle and Number of Ovules on First and on Second Placenta.

Length of Peduncle.

Ovules on First Placenta and on Second Placenta.

	8	8·5	9	9·5	10	10·5	11	11·5	12	12·5	13	13·5	14	14·5	15	15·5	16	16·5	17	17·5	18	18·5	19	19·5	20	20·5	21	21·5	22	22·5	23	23·5	24	24·5	25	25·5	26	26·5	Totals
6	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1
7	–	–	–	–	–	1	–	2	–	–	–	–	1	1	–	–	–	1	–	1	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	8
8	–	–	–	–	1	2	1	1	–	2	–	1	1	2	3	2	6	2	–	3	1	1	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	30
9	–	–	–	–	3	5	–	1	3	4	4	7	5	3	5	10	8	5	3	4	3	5	3	3	–	–	–	2	–	–	–	–	–	–	–	–	–	–	86
10	–	2	–	–	–	2	2	4	2	6	8	11	11	8	8	12	13	15	12	7	5	3	2	4	3	5	4	3	1	–	1	–	–	1	–	–	–	–	155
11	–	–	–	–	1	–	2	5	1	6	8	9	7	15	19	11	28	11	12	11	9	8	8	11	5	7	4	–	2	–	–	–	1	1	–	–	–	–	202
12	–	–	–	–	–	–	1	2	7	3	2	8	10	14	14	15	20	26	15	19	11	9	11	8	10	5	7	1	1	–	3	2	1	1	–	–	–	–	226
13	–	–	1	–	1	2	4	1	1	4	5	6	10	14	11	16	20	19	9	14	15	14	12	7	8	9	7	1	–	2	2	1	–	–	–	–	–	–	216
14	–	–	1	–	–	–	2	1	1	–	5	1	5	6	8	12	15	11	14	23	13	12	13	5	3	5	3	1	–	–	–	1	–	1	–	–	–	–	162
15	–	–	–	–	–	–	1	–	1	–	3	2	4	3	7	11	16	12	21	8	14	12	16	4	13	5	4	–	–	–	2	2	–	–	–	–	–	2	163
16	–	–	–	–	–	–	1	–	1	3	3	4	5	5	10	9	10	6	6	6	6	15	14	7	12	5	4	1	1	1	3	–	–	–	–	–	–	–	138
17	–	–	–	–	–	–	–	1	2	2	1	2	1	2	6	1	5	8	9	6	12	9	16	3	12	6	2	4	2	1	2	–	–	–	–	–	–	–	115
18	–	–	–	–	–	–	1	–	3	1	1	1	3	5	2	2	11	7	7	7	17	8	4	2	4	3	1	2	1	4	1	–	–	2	–	–	–	–	100
19	1	–	–	–	–	–	1	–	2	1	3	–	1	6	1	2	1	4	14	6	4	11	5	6	6	5	1	3	3	2	1	–	–	–	–	–	–	–	90
20	–	–	–	–	–	–	–	–	1	–	1	2	–	5	4	3	6	4	5	3	5	8	6	2	1	3	1	–	3	4	3	–	–	–	2	–	–	–	72
21	–	–	–	–	–	–	–	–	1	–	–	–	2	1	–	3	2	4	6	5	2	5	3	3	4	4	–	–	1	1	1	–	1	–	–	–	–	–	49
22	1	–	–	–	–	–	–	–	–	–	–	–	–	2	–	2	6	3	3	3	6	3	8	1	2	2	1	–	1	1	2	–	1	–	1	–	–	–	49
23	–	–	–	–	–	–	–	–	–	–	–	–	–	1	2	2	–	6	3	3	2	2	5	1	3	–	2	–	–	–	–	–	1	–	–	–	–	–	33
24	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	2	–	1	3	1	–	3	1	–	2	2	1	–	–	–	2	1	2	–	1	–	–	–	23
25	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	2	–	2	–	2	1	1	1	1	2	2	1	–	–	–	3	1	–	–	–	–	–	–	20
26	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	1	2	2	2	–	–	1	1	2	1	–	2	–	–	2	–	–	–	–	17
27	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	3	–	1	1	2	1	–	–	–	–	1	–	–	–	–	–	10
28	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	1	–	1	–	1	1	1	1	–	1	2	–	1	1	–	–	–	–	–	–	–	–	–	12
29	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1
30	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	2	1	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	5
31	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	2
32	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	–	2	1	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	–	5
33	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	2
34	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–	2	–	–	–	–	1	–	1	–	–	–	–	–	–	–	–	5
35	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1
36	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	2
Totals	2	2	2	0	6	12	16	18	26	32	44	56	66	94	102	118	170	150	142	138	134	136	136	68	94	72	46	24	18	18	28	8	8	8	4	0	0	2	2000

TABLE III., 1906.

Length of Peduncle and Number of Seeds Developing on First and on Second Placenta.

Length of Peduncle.

Seeds Developing on First and on Second Placenta.

	8	8·5	9	9·5	10	10·5	11	11·5	12	12·5	13	13·5	14	14·5	15	15·5	16	16·5	17	17·5	18	18·5	19	19·5	20	20·5	21	21·5	22	22·5	23	23·5	24	24·5	25	25·5	26	26·5	Totals
0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	1	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	4
2	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	1	1	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	6
3	—	—	—	—	1	—	—	—	—	—	—	—	1	—	—	1	1	2	—	1	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	9
4	—	—	—	—	1	—	2	2	1	—	—	—	1	2	1	—	2	—	—	1	2	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	16
5	—	—	—	—	2	1	—	—	—	1	—	—	1	3	3	—	6	9	—	—	1	1	—	—	—	1	—	1	—	—	—	1	—	—	—	—	—	—	31
6	—	2	—	—	1	3	—	3	3	2	5	2	4	4	5	4	9	5	3	6	5	3	1	2	—	1	—	2	—	—	—	1	—	—	—	—	—	—	76
7	—	—	—	—	—	1	2	5	1	5	7	8	6	10	8	5	15	9	7	7	5	5	3	3	1	—	1	1	—	—	—	—	—	—	—	—	—	—	115
8	—	—	—	—	—	4	1	3	2	5	6	5	6	12	10	17	14	14	7	13	7	3	8	2	4	3	3	1	—	—	—	—	—	—	—	—	—	—	150
9	2	—	1	—	1	1	4	1	2	2	9	11	13	13	15	18	21	17	19	10	9	14	11	11	12	6	6	—	2	1	1	—	1	2	—	—	—	—	236
10	—	—	1	—	—	—	2	1	4	7	4	11	16	14	14	15	31	15	17	16	16	15	14	7	5	8	6	1	1	2	—	—	1	2	—	—	—	—	246
11	—	—	—	—	—	—	3	3	4	2	2	7	2	9	10	14	31	15	14	19	15	11	10	13	15	7	5	—	3	—	2	4	—	—	—	—	—	1	221
12	—	—	—	—	—	—	1	—	3	3	1	1	3	5	11	12	11	18	16	14	14	17	21	9	10	7	8	2	3	1	2	—	—	1	2	—	—	—	196
13	—	—	—	—	—	2	1	—	—	4	5	4	5	6	7	8	9	11	17	9	15	17	8	4	7	7	5	1	1	2	5	—	—	1	—	—	—	1	162
14	—	—	—	—	—	—	—	—	3	1	2	3	2	6	10	5	5	10	14	9	10	11	11	2	6	5	4	2	2	2	—	—	—	—	—	—	—	—	125
15	—	—	—	—	—	—	—	—	—	—	2	—	4	2	3	6	4	4	6	10	10	9	14	6	6	7	—	5	2	2	3	—	—	—	—	—	—	—	105
16	—	—	—	—	—	—	—	—	1	—	—	2	1	—	—	1	4	7	7	6	10	4	9	2	8	3	1	2	1	—	2	—	—	—	—	—	—	—	71
17	—	—	—	—	—	—	—	—	2	—	1	—	—	5	1	3	1	7	5	3	5	3	7	2	3	5	1	1	—	3	4	—	—	—	—	—	—	—	62
18	—	—	—	—	—	—	—	—	—	—	—	1	—	2	1	2	2	2	5	4	2	6	7	2	8	1	—	—	1	3	3	—	2	1	—	—	—	—	55
19	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	3	2	2	3	5	6	5	3	—	3	1	2	—	1	—	3	—	1	—	—	—	—	—	42
20	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	2	1	2	—	2	1	1	2	1	2	2	2	1	1	—	—	—	1	—	1	—	—	—	23
21	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	3	1	1	—	2	2	—	—	—	—	1	—	—	—	—	—	—	12
22	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	2	—	3	—	—	1	1	—	—	—	2	1	—	1	1	1	—	—	—	15
23	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	1	—	—	1	1	—	—	—	—	—	—	5
24	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	1	—	—	4	—	1	—	—	1	—	—	1	—	—	—	—	—	—	—	10
25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
26	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	2
27	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	2
28	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	2	2	2	0	6	12	16	18	26	32	44	56	66	94	102	118	170	150	142	138	134	136	136	68	94	72	46	24	18	18	28	8	8	8	4	0	0	2	2000

TABLE IV., 1906. *Length of Fruit and Number of Ovules on First and on Second Placenta.*

Ovules on First and on Second Placenta.

Length of Fruit.	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	Totals
21—23	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
24—26	—	1	—	1	—	—	4	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	8
27—29	—	2	4	10	10	10	6	4	3	3	—	—	1	1	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	56
30—32	—	2	7	19	32	27	24	21	6	7	7	1	3	2	1	1	—	—	1	—	—	—	1	—	—	—	—	—	—	—	—	162
33—35	—	2	11	28	41	56	41	35	22	28	12	9	9	16	9	4	2	4	2	3	1	1	—	—	—	—	—	—	—	—	—	336
36—38	1	—	3	13	40	51	49	45	35	25	19	14	21	17	16	6	5	1	—	1	2	—	—	—	1	1	—	—	—	—	—	366
39—41	—	1	3	10	16	26	47	46	41	28	26	24	10	10	8	4	7	5	2	1	—	1	—	—	2	—	—	—	—	—	—	318
42—44	—	—	—	2	8	19	34	38	33	35	30	20	27	12	7	17	9	4	4	2	2	—	2	1	—	—	—	—	2	—	—	308
45—47	—	—	2	2	4	7	12	19	16	18	25	21	10	14	12	3	11	11	3	2	5	3	3	—	1	—	2	1	1	—	—	208
48—50	—	—	—	—	3	4	5	6	4	16	16	19	13	6	12	9	8	5	4	5	4	—	3	—	—	—	—	—	—	—	—	142
51—53	—	—	—	—	—	2	4	2	1	3	1	6	2	7	4	1	4	3	2	1	1	4	1	—	1	—	—	—	—	—	—	50
54—56	—	—	—	—	—	—	—	—	—	—	—	1	3	2	2	1	2	—	3	3	1	1	1	—	—	—	—	—	—	—	—	20
57—59	—	—	—	—	—	—	—	—	—	—	1	—	1	3	—	3	—	—	1	1	1	—	1	—	—	1	1	1	1	1	1	18
60—62	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	1	—	1	4
63—65	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	2
Totals	1	8	30	86	155	202	226	216	162	163	138	115	100	90	72	49	49	33	23	20	17	10	12	1	5	2	5	2	5	1	2	2000

TABLE IV., 1907. *Length of Fruit and Number of Ovules on First and on Second Placenta.*

Ovules on First and on Second Placenta.

Length of Fruit.	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	Totals
24—26	—	1	1	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
27—29	—	—	—	1	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
30—32	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
33—35	—	—	2	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
36—38	2	3	3	1	—	2	1	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	14
39—41	1	1	6	7	5	3	3	2	1	2	—	—	2	1	—	—	—	—	—	—	—	—	—	—	—	34
42—44	—	1	5	16	16	13	8	5	2	2	2	2	2	—	—	—	—	—	—	—	—	—	—	—	—	74
45—47	—	1	6	11	11	21	19	13	13	8	2	5	—	—	—	1	—	—	1	—	—	—	—	—	—	112
48—50	—	—	2	9	18	20	24	20	11	6	5	2	1	—	—	—	—	—	—	—	—	—	—	—	—	118
51—53	—	—	1	—	7	18	26	11	8	12	16	3	2	2	2	—	—	—	—	—	—	—	—	—	—	108
54—56	—	—	—	—	6	8	17	17	13	15	14	14	9	7	4	4	1	1	1	—	1	—	—	—	—	132
57—59	—	—	—	—	1	4	10	10	14	6	4	6	4	5	1	2	5	—	—	—	1	1	—	—	—	74
60—62	—	—	—	1	2	1	5	5	8	8	3	4	—	2	2	5	—	2	1	1	—	—	—	—	—	50
63—65	—	—	—	—	—	2	2	2	2	3	2	4	3	1	—	1	2	1	1	—	—	1	1	—	2	30
66—68	—	—	—	—	—	—	—	1	1	1	2	1	5	2	3	2	3	1	1	1	—	—	—	—	2	26
69—71	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	2	—	—	—	—	—	4
72—74	—	—	—	—	—	—	—	—	—	—	—	1	—	1	2	2	1	—	—	1	—	—	—	—	—	8
75—77	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	1	1	—	4
78—80	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	2
Totals	3	7	26	47	67	94	115	86	73	64	50	43	29	22	15	18	13	5	5	5	2	4	2	1	4	800

TABLE V., 1906.

Length of Fruit and Number of Seeds Developing on First and on Second Placenta.

Seeds Developing on First and on Second Placenta.

Length of Fruit.	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	Totals
21—23	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
24—26	—	—	—	—	2	2	2	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	8
27—29	1	2	1	2	2	6	6	10	8	8	6	2	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	56
30—32	—	—	—	2	4	6	21	22	26	32	22	12	6	4	1	2	—	—	1	1	—	—	—	—	—	—	—	—	—	162
33—35	—	2	1	1	4	5	18	34	41	55	44	42	29	27	11	6	4	2	5	3	—	2	—	—	—	—	—	—	—	336
36—38	—	—	1	—	4	9	10	22	36	51	62	48	36	26	15	14	11	11	5	2	1	—	1	—	1	—	—	—	—	366
39—41	—	—	—	2	—	1	10	10	18	46	53	37	41	34	24	14	5	7	7	3	4	—	1	1	—	—	—	—	—	318
42—44	—	—	2	—	—	1	5	9	9	28	33	38	42	30	31	26	18	12	10	5	2	2	3	—	1	—	1	—	—	308
45—47	—	—	—	—	—	—	3	5	8	10	10	24	26	23	21	17	13	14	13	8	4	3	2	1	2	—	1	—	—	208
48—50	—	—	—	1	—	1	1	2	3	4	15	9	10	10	17	17	12	10	5	12	6	3	4	—	—	—	—	—	—	142
51—53	—	—	—	—	—	—	—	1	1	—	1	7	3	4	4	5	4	3	4	2	4	1	3	1	—	1	—	—	1	50
54—56	—	—	—	—	—	—	—	—	—	—	—	1	1	1	—	3	3	2	2	2	1	—	1	—	3	—	—	—	—	20
57—59	—	—	—	—	—	—	—	—	—	—	—	1	—	3	1	1	1	1	3	2	1	—	—	1	2	—	—	1	—	18
60—62	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	1	—	—	1	—	4
63—65	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	2
Totals	1	4	6	9	16	31	76	115	150	236	246	221	196	162	125	105	71	62	55	42	23	12	15	5	10	1	2	2	1	2000

TABLE V., 1907.

Length of Fruit and Number of Seeds Developing on First and on Second Placenta.

Seeds Developing on First and on Second Placenta.

Length of Fruit.	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	Totals
24—26	2	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
27—29	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
30—32	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
33—35	—	1	1	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
36—38	1	—	—	6	2	3	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	14
39—41	—	—	—	4	6	6	4	6	2	2	—	1	1	—	—	1	1	—	—	—	—	—	—	—	—	—	—	34
42—44	—	2	5	6	8	12	9	14	4	3	6	1	1	1	1	1	—	—	—	—	—	—	—	—	—	—	—	74
45—47	—	—	—	2	2	8	13	13	19	16	15	13	1	4	1	2	2	1	—	—	—	—	—	—	—	—	—	112
48—50	—	—	—	—	—	5	12	20	20	21	22	7	9	—	1	—	1	—	—	—	—	—	—	—	—	—	—	118
51—53	—	—	1	2	2	2	7	11	14	16	13	16	10	5	7	1	1	—	—	—	—	—	—	—	—	—	—	108
54—56	—	1	—	—	2	1	—	7	8	14	18	17	22	12	11	13	1	2	1	2	—	—	—	—	—	—	—	132
57—59	—	—	—	—	—	—	2	5	7	10	9	10	12	4	3	3	3	1	1	1	1	—	1	—	—	1	—	74
60—62	—	—	—	—	—	1	—	2	4	4	6	8	9	5	—	1	1	2	5	2	—	—	—	—	—	—	—	50
63—65	—	—	—	—	—	—	1	—	5	2	4	3	—	6	2	—	1	—	—	1	1	—	1	—	1	1	1	30
66—68	—	—	—	—	—	—	—	—	—	—	1	1	3	1	6	3	2	4	3	1	1	—	—	—	—	—	—	26
69—71	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	1	—	—	1	—	—	—	4
72—74	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	2	—	3	1	—	—	—	—	—	—	—	8
75—77	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	1	—	—	1	—	—	—	—	—	—	4
78—80	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	2
Totals	4	5	7	23	23	38	50	78	83	88	94	77	69	40	32	26	16	12	13	8	5	0	2	2	2	2	1	800

TABLE VI., 1906.

Ovules Formed and Number of Seeds Developing per Placenta.

Seeds Developing per Placenta.

Ovules per Placenta.	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	Totals
6	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
7	—	—	—	—	—	—	3	5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	8
8	—	—	—	1	3	2	4	11	9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	30
9	—	—	2	1	1	6	14	19	25	18	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	86
10	—	1	1	1	3	5	16	24	33	49	22	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	155
11	—	—	—	1	2	3	15	23	26	50	45	37	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	202
12	—	1	—	1	2	6	6	9	28	42	52	52	27	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	226
13	1	—	—	2	1	4	8	9	17	35	40	41	41	17	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	216
14	—	—	3	1	2	—	2	8	4	16	25	26	35	27	13	—	—	—	—	—	—	—	—	—	—	—	—	—	—	162
15	—	1	—	—	—	—	2	1	2	4	27	33	32	32	19	10	—	—	—	—	—	—	—	—	—	—	—	—	—	163
16	—	—	—	—	—	2	1	2	2	4	12	12	19	32	25	21	6	—	—	—	—	—	—	—	—	—	—	—	—	138
17	—	—	—	—	—	—	1	—	1	6	4	2	12	15	27	20	15	12	—	—	—	—	—	—	—	—	—	—	—	115
18	—	—	—	—	1	1	1	—	1	5	8	8	10	10	11	17	13	10	4	—	—	—	—	—	—	—	—	—	—	100
19	—	—	—	1	—	1	1	3	1	4	4	3	9	8	12	13	10	8	10	2	—	—	—	—	—	—	—	—	—	90
20	—	—	—	—	—	—	1	1	—	1	6	4	6	11	8	9	5	10	6	3	1	—	—	—	—	—	—	—	—	72
21	—	—	—	—	—	—	—	—	1	—	—	1	1	1	5	8	9	10	7	4	1	1	—	—	—	—	—	—	—	49
22	—	—	—	—	—	1	—	—	—	2	1	2	3	3	3	2	6	5	6	10	3	1	1	—	—	—	—	—	—	49
23	—	1	—	—	—	—	—	—	—	—	—	—	1	1	1	2	5	3	7	6	6	—	—	—	—	—	—	—	—	33
24	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	1	4	4	3	4	4	—	1	—	—	—	—	23
25	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	—	—	4	3	3	2	3	2	—	—	—	—	—	20
26	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	3	5	1	2	2	—	2	—	—	—	—	17
27	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	2	1	1	1	—	3	—	1	—	—	—	10
28	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	1	2	1	2	—	3	—	1	—	—	12
29	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
30	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	2	—	—	—	1	5
31	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	2
32	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2	—	—	1	—	—	—	—	1	—	5
33	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	—	—	—	2
34	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1	—	1	—	1	1	—	5
35	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
36	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1	—	—	—	—	2
Totals	1	4	6	9	16	31	76	115	150	236	246	221	196	162	125	105	71	62	55	42	23	12	15	5	10	1	2	2	1	2000

TABLE VI., 1907.

Ovules Formed and Number of Seeds Developing per Placenta.

Seeds Developing per Placenta.

Ovules per Placenta.	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	Totals
5	—	—	—	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
6	—	—	—	3	4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	7
7	1	—	—	7	3	6	7	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	26
8	2	—	1	3	3	9	8	12	9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	47
9	—	2	1	2	2	9	9	17	17	8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	67
10	1	1	1	3	3	5	8	15	16	27	14	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	94
11	—	—	3	1	2	3	7	11	15	25	32	16	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	115
12	—	—	—	1	3	2	3	9	11	8	21	18	10	—	—	—	—	—	—	—	—	—	—	—	—	—	—	86
13	—	—	—	1	—	1	5	7	3	6	12	16	17	5	—	—	—	—	—	—	—	—	—	—	—	—	—	73
14	—	1	—	—	1	1	1	2	2	7	10	14	11	11	3	—	—	—	—	—	—	—	—	—	—	—	—	64
15	—	—	1	—	—	—	—	1	2	4	3	4	10	9	14	2	—	—	—	—	—	—	—	—	—	—	—	50
16	—	1	—	—	—	1	—	2	3	3	1	4	8	7	5	7	1	—	—	—	—	—	—	—	—	—	—	43
17	—	—	—	1	—	—	2	—	2	—	1	1	7	4	1	6	2	2	—	—	—	—	—	—	—	—	—	29
18	—	—	—	—	—	—	—	—	1	—	—	—	5	1	1	4	7	1	2	—	—	—	—	—	—	—	—	22
19	—	—	—	—	—	—	—	—	—	—	—	1	—	1	2	—	2	4	3	2	—	—	—	—	—	—	—	15
20	—	—	—	—	1	—	—	—	1	—	—	2	—	—	1	3	3	1	6	—	—	—	—	—	—	—	—	18
21	—	—	—	—	—	—	—	—	1	—	—	1	1	—	1	1	1	1	—	3	3	—	—	—	—	—	—	13
22	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	1	—	1	1	—	—	—	—	—	—	5
23	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	2	1	—	—	—	—	—	—	—	—	5
24	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2	1	—	—	1	—	—	—	5
25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1	—	—	—	—	2
26	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	1	—	4
27	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1	2
28	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
29	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	1	1	—	4
Totals	4	5	7	23	23	38	50	78	83	88	94	77	69	40	32	26	16	12	13	8	5	0	2	2	2	2	1	800

TABLE VII., 1906. *Total Number of Ovules Formed and* Total Seeds Developing

Total Ovules per Fruit.

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*
15	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	—	—	—	—	—	—	—	—	—	—	—
16	—	—	—	—	—	—	—	—	—	—	1	—	4	—	2	1	—	—	—	—	—	—	—	—	—	—
17	—	—	—	—	—	—	—	1	—	—	1	1	1	1	4	3	2	—	—	—	—	—	—	—	—	—
18	—	—	—	—	—	—	—	1	1	—	—	1	1	2	1	2	5	1	—	—	—	—	—	—	—	—
19	—	—	—	—	1	—	—	1	—	—	2	2	2	3	7	4	5	1	1	—	—	—	—	—	—	—
20	—	—	—	—	—	1	—	1	—	1	—	2	3	3	9	2	5	4	2	2	—	—	—	—	—	—
21	—	—	—	—	—	—	1	—	—	1	—	1	—	—	5	4	5	9	10	8	4	—	—	—	—	—
22	—	—	—	—	—	—	—	—	—	1	—	—	1	2	9	3	10	3	12	5	6	6	—	—	—	—
23	—	—	—	—	—	—	—	—	—	1	1	1	2	1	1	6	8	6	7	7	11	2	—	—	—	—
24	—	—	—	—	—	—	—	—	—	2	—	2	1	2	3	5	5	4	4	9	6	11	3	1	—	—
25	—	—	—	—	—	—	—	1	—	2	—	—	1	3	—	3	3	4	5	8	5	8	9	—	3	—
26	—	—	1	—	—	—	—	—	—	—	—	—	1	1	1	—	4	1	3	6	10	10	6	5	5	—
27	—	—	—	—	—	—	—	—	—	—	1	—	—	1	2	3	2	3	4	6	6	4	12	4	2	3
28	1	—	—	1	—	—	—	—	—	—	—	—	—	—	1	—	—	2	6	5	6	2	7	1	4	2
29	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	2	—	4	2	6	2	5	4	6	5
30	—	—	—	—	—	—	—	—	2	—	—	—	1	—	—	—	—	1	1	1	—	1	10	7	8	2
31	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	2	2	—	2	2	4	5	6	2
32	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1	—	3	3	1	2	3	2	5
33	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	—	—	—	1	—	5	2	5
34	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	2	1	—	4	2
35	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	1	—	1	—	1	2
36	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	1	1	1	—	2	4
37	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	2	—	—	1	—	—	—	3	2
38	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1	—	1	1	1	—	2	—
39	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1	—	—	2	2	—	—	—
40	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	1	2	2
41	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1	—	—	—	—	1	—	—	—
42	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—
43	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	1
44	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
45	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
46	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—
47	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
48	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
49	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
50	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
51	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
52	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
53	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
54	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
55	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
56	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
57	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
58	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
59	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
60	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
61	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
62	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
63	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
64	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
65	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
66	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
67	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
68	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
69	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
70	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Totals	1	0	1	1	2	1	1	5	3	9	6	12	22	21	47	38	58	49	64	63	70	56	68	38	53	39

Total Number of Seeds Developing per Fruit.

per Fruit.

27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	Totals
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	8
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	14
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	15
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	29
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	35
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	48
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	58
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	54
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	58
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	55
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	54
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	53
2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	40
3	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	42
1	4	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	40
5	4	3	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	42
4	1	3	3	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	35
2	5	4	3	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	32
6	2	3	4	3	3	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	32
3	2	1	3	5	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	24
—	4	4	1	2	1	2	1	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	28
1	1	1	3	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	17
3	—	—	—	1	1	2	2	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	17
—	2	4	3	2	1	1	1	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	22
1	1	3	—	1	3	3	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	21
2	—	—	—	—	1	—	—	2	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10
1	1	—	3	1	2	1	2	3	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	19
—	1	—	—	1	—	2	1	3	1	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	15
—	1	—	—	—	2	—	—	1	2	1	1	1	—	—	1	—	—	—	—	—	—	—	—	—	10
—	—	—	—	1	—	—	—	—	2	—	—	2	—	—	1	—	—	—	—	—	—	—	—	—	7
—	—	—	—	1	—	1	2	—	—	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	8
—	—	—	—	—	—	—	—	—	2	2	1	—	1	—	—	—	—	—	—	—	—	—	—	—	6
—	—	—	—	—	—	—	1	1	—	—	1	1	—	1	—	—	—	—	—	—	—	—	—	—	5
—	—	—	—	—	—	—	—	1	—	1	1	1	2	—	—	—	1	—	—	—	—	—	—	—	7
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1	—	—	—	3
—	—	—	—	1	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	2	—	—	—	—	4
—	—	—	—	—	—	—	1	—	1	1	—	—	1	1	—	2	—	—	—	—	—	—	—	—	7
—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	2
—	—	—	—	—	—	—	—	—	1	—	—	2	—	2	—	—	—	1	—	—	—	—	—	—	7
—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	1	—	—	3
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	2
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2
34	30	27	25	24	19	14	15	15	12	12	7	9	6	4	3	4	2	1	1	3	1	1	2	1	1000

TABLE VII., 1906. *Total Number of Ovules Formed and Total Number of Seeds Developing per Fruit.*

Total Seeds Developing per Fruit.

Total Ovules per Fruit.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
15	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	—	—	—	—	—	—	—	—	—	—	—
16	—	—	—	—	—	—	—	—	—	—	1	—	4	—	2	1	—	—	—	—	—	—	—	—	—	—
17	—	—	—	—	—	—	—	1	—	—	1	1	1	1	4	3	2	—	—	—	—	—	—	—	—	—
18	—	—	—	—	—	—	—	1	1	—	—	1	1	2	1	2	5	1	—	—	—	—	—	—	—	—
19	—	—	—	—	1	—	—	1	—	—	2	2	2	3	7	4	5	1	1	—	—	—	—	—	—	—
20	—	—	—	—	—	1	—	1	—	1	—	2	3	3	9	2	5	4	2	2	—	—	—	—	—	—
21	—	—	—	—	—	—	1	—	—	1	—	1	—	—	5	4	5	9	10	8	4	—	—	—	—	—
22	—	—	—	—	—	—	—	—	—	1	—	—	1	2	9	3	10	3	12	5	6	6	—	—	—	—
23	—	—	—	—	—	—	—	—	—	1	1	1	2	1	1	6	8	6	7	7	11	2	—	—	—	—
24	—	—	—	—	—	—	—	—	—	2	—	2	1	2	3	5	5	4	4	9	6	11	3	1	—	—
25	—	—	—	—	—	—	—	1	—	2	—	—	1	3	—	3	3	4	5	8	5	8	9	—	3	—
26	—	—	1	—	—	—	—	—	—	—	—	—	1	1	1	—	4	1	3	6	10	10	6	5	5	—
27	—	—	—	—	—	—	—	—	—	—	1	—	—	1	2	3	2	3	4	6	6	4	12	4	2	3
28	1	—	—	1	—	—	—	—	—	—	—	—	—	—	1	—	—	2	6	5	6	2	7	1	4	2
29	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	2	—	4	2	6	2	5	4	6	5
30	—	—	—	—	—	—	—	—	2	—	—	—	1	—	—	—	—	1	1	1	—	1	10	7	8	2
31	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	2	2	—	2	2	4	5	6	2
32	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1	—	3	3	1	2	3	2	5
33	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	—	—	—	1	—	5	2	5
34	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	2	1	—	4	2
35	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	1	—	1	—	1	2
36	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	1	1	1	—	2	4
37	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	2	—	—	1	—	—	—	3	2
38	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1	—	1	1	1	—	2	—
39	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1	—	—	2	2	—	—	—
40	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	1	2	2
41	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1	—	—	—	—	1	—	—	—
42	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—
43	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	1
44	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
45	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
46	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—
47	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
48	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
49	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
50	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
51	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
52	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
53	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
54	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
55	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
56	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
57	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
58	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
59	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
60	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
61	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
62	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
63	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
64	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
65	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
66	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
67	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
68	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
69	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
70	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Totals	1	0	1	1	2	1	1	5	3	9	6	12	22	21	47	38	58	49	64	63	70	56	68	38	53	39

Total Ovules per Fruit.	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	Totals
15	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
16	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	8
17	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	14
18	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	15
19	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	29
20	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	35
21	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	48
22	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	58
23	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	54
24	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	58
25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	55
26	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	54
27	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	53
28	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	40
29	3	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	42
30	1	4	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	40
31	5	4	3	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	42
32	4	1	3	3	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	35
33	2	5	4	3	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	32
34	6	2	3	4	3	3	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	32
35	3	2	1	3	5	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	24
36	—	4	4	1	2	1	2	1	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	28
37	1	1	1	3	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	17
38	3	—	—	—	1	1	2	2	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	17
39	—	2	4	3	2	1	1	1	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	22
40	1	1	3	—	1	3	3	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	21
41	2	—	—	—	—	1	—	—	2	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10
42	1	1	—	3	1	2	1	2	3	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	19
43	—	1	—	—	1	—	2	1	3	1	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	15
44	—	1	—	—	—	2	—	—	1	2	1	1	1	—	—	1	—	—	—	—	—	—	—	—	—	10
45	—	—	—	—	1	—	—	—	—	2	—	—	2	—	—	1	—	—	—	—	—	—	—	—	—	7
46	—	—	—	—	1	—	1	2	—	—	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	8
47	—	—	—	—	—	—	—	—	—	2	2	1	—	1	—	—	—	—	—	—	—	—	—	—	—	6
48	—	—	—	—	—	—	—	1	1	—	—	1	1	—	1	—	—	—	—	—	—	—	—	—	—	5
49	—	—	—	—	—	—	—	—	1	—	1	1	1	2	—	—	—	1	—	—	—	—	—	—	—	7
50	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1	—	—	—	3
51	—	—	—	—	1	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	2	—	—	—	—	4
52	—	—	—	—	—	—	—	1	—	1	1	—	—	1	1	—	2	—	—	—	—	—	—	—	—	7
53	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	2
54	—	—	—	—	—	—	—	—	—	1	—	—	2	—	2	—	—	—	1	—	—	—	—	—	—	7
55	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	1	—	—	3
56	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1
57	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
58	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
59	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
60	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
61	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
62	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
63	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
64	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
65	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
66	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
67	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
68	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	2
69	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
70	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2
Totals	34	30	27	25	24	19	14	15	15	12	12	7	9	6	4	3	4	2	1	1	3	1	1	2	1	1000

TABLE VII., 1907. *Total Number of Ovules Formed and*

Total Seeds Developing

Total Ovules per Fruit.

	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*
11	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
12	—	—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—
13	—	—	—	—	—	—	1	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—
14	—	—	—	—	—	—	1	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—
15	1	—	—	—	—	1	—	—	1	1	1	1	2	1	—	1	—	—	—	—	—	—	—	—	—
16	—	—	—	—	—	—	—	2	3	1	—	3	1	1	2	1	—	—	—	—	—	—	—	—	—
17	—	—	—	—	—	—	1	—	—	—	2	1	1	1	4	1	1	1	—	—	—	—	—	—	—
18	—	—	—	—	—	—	1	1	1	—	1	1	3	2	3	1	2	3	1	—	—	—	—	—	—
19	—	—	—	1	—	—	—	—	—	—	1	1	2	—	3	4	3	5	—	—	—	—	—	—	—
20	—	—	—	2	—	—	—	—	—	—	1	3	—	—	1	1	1	3	3	2	—	—	—	—	—
21	—	—	—	—	—	—	—	—	—	—	—	—	2	1	1	1	1	6	1	10	5	—	—	—	—
22	—	1	—	—	—	1	1	—	—	—	—	—	1	—	2	1	2	3	3	4	3	3	3	—	—
23	—	—	—	—	—	—	—	—	1	1	—	—	1	1	—	1	2	2	4	2	4	2	2	1	—
24	—	—	—	—	—	—	—	—	—	—	1	—	—	3	1	4	1	4	3	4	5	5	6	3	—
25	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	2	3	—	1	2	1	6	2	1
26	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	1	—	1	4	3	2	3	1
27	—	—	—	—	—	—	—	—	1	—	1	—	—	—	1	—	2	—	—	—	—	4	1	1	3
28	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	1	1	2
29	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	1	—	2	—	—	—	—	3	1
30	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	1	—	—	—	2
31	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—
32	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	1	1	—	—
33	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1	—	2	—	1	—	—	—	1	—
34	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1	—
35	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
36	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—
37	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—
38	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
39	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
40	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
41	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—
42	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
43	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
44	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
45	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
46	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
47	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
48	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
49	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
50	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
51	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
52	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
53	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
54	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
55	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
56	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
57	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
58	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Totals	1	1	0	3	0	2	6	4	8	4	11	12	14	13	21	17	20	34	17	26	26	19	22	17	10

Total Number of Seeds Developing per Fruit.

per Fruit.

25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	Totals
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	14
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	13
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	20
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	20
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	17
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	28
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	28
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	24
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	40
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	20
2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	19
—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	15
2	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	9
7	1	4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	21
3	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10
—	4	—	1	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	8
—	—	2	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	8
1	1	—	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	11
1	—	4	—	—	2	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10
—	2	—	2	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5
—	—	—	1	—	1	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5
—	—	—	—	1	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
—	1	—	—	—	1	—	—	—	1	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	5
1	—	—	1	—	—	—	—	1	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	5
1	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	3
—	1	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	3
—	1	—	—	—	—	—	—	—	—	1	—	—	—	2	—	—	—	—	—	—	—	—	—	—	4
—	—	—	—	—	1	—	—	1	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	3
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
—	—	—	1	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	3
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
—	—	—	—	—	1	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	2
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2
18	12	11	12	3	7	2	2	3	4	3	5	3	0	2	0	0	0	1	0	0	0	2	1	1	400

TABLE VII., 1907. *Total Number of Ovules Formed and Total Number of Seeds Developing per Fruit.*

Total Seeds Developing per Fruit.

Total Ovules per Fruit.	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	Totals
11	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
12	—	—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
13	—	—	—	—	—	—	1	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
14	—	—	—	—	—	—	1	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
15	1	—	—	—	—	1	—	—	1	1	1	1	2	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10
16	—	—	—	—	—	—	—	2	3	1	—	3	1	1	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	14
17	—	—	—	—	—	—	1	—	—	—	2	1	1	1	4	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	13
18	—	—	—	—	—	—	1	1	1	—	1	1	3	2	3	1	2	3	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	20
19	—	—	—	1	—	—	—	—	—	—	1	1	2	—	3	4	3	5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	20
20	—	—	—	2	—	—	—	—	—	—	1	3	—	—	1	1	1	3	3	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	17
21	—	—	—	—	—	—	—	—	—	—	—	—	2	1	1	1	1	6	1	10	5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	28
22	—	1	—	—	—	1	1	—	—	—	—	—	1	—	2	1	2	3	3	4	3	3	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	28
23	—	—	—	—	—	—	—	—	1	1	—	—	1	1	—	1	2	2	4	2	4	2	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	24
24	—	—	—	—	—	—	—	—	—	—	1	—	—	3	1	4	1	4	3	4	5	5	6	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	40
25	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	2	3	—	1	2	1	6	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	20
26	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	1	—	1	4	3	2	3	1	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	19
27	—	—	—	—	—	—	—	—	1	—	1	—	—	—	1	—	2	—	—	—	—	4	1	1	3	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	15
28	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	1	1	2	2	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	9
29	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	1	—	2	—	—	—	—	3	1	7	1	4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	21
30	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	1	—	—	—	2	3	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10
31	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	4	—	1	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	8
32	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	1	1	—	—	—	—	2	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	8
33	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1	—	2	—	1	—	—	—	1	—	1	1	—	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	11
34	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1	—	1	—	4	—	—	2	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10
35	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	2	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5
36	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1	—	1	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5
37	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
38	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	—	—	—	1	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	5
39	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	—	1	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	5
40	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	3
41	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	3
42	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1	—	—	—	2	—	—	—	—	—	—	—	—	—	—	4
43	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	3
44	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
45	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	3
46	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
47	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	2
48	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
49	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
50	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
51	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
52	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
53	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1
54	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
55	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
56	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
57	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
58	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2
Totals	1	1	0	3	0	2	6	4	8	4	11	12	14	13	21	17	20	34	17	26	26	19	22	17	10	18	12	11	12	3	7	2	2	3	4	3	5	3	0	2	0	0	0	1	0	0	0	2	1	1	400

TABLE VIII., 1906.

Number of Ovules on First Placenta and Number of Ovules on Second Placenta.

Ovules on Second Placenta.

Ovules on First Placenta.

	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	Totals
6	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
7	—	—	1	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
8	—	1	3	8	2	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	16
9	—	3	5	10	13	2	3	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	37
10	—	—	3	12	27	15	7	5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	69
11	—	—	2	6	25	37	23	9	6	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	110
12	—	—	—	5	9	18	37	23	7	6	4	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	110
13	—	—	—	5	7	10	21	30	19	9	5	3	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	110
14	—	—	—	—	2	5	14	18	15	9	6	5	2	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	78
15	—	—	—	—	—	2	8	10	21	21	15	5	3	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	87
16	—	—	—	—	—	1	2	7	7	15	16	12	7	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	69
17	—	—	—	—	—	—	—	1	4	8	13	15	7	13	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	63
18	—	—	—	—	—	—	1	2	4	3	6	9	11	8	4	2	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	52
19	—	—	—	—	—	—	—	—	—	1	3	3	8	9	6	4	4	1	—	1	—	—	—	—	—	—	—	—	—	—	—	40
20	—	—	—	—	—	—	—	1	—	3	—	—	4	10	9	3	4	1	2	1	—	—	—	—	—	—	—	—	—	—	—	38
21	—	—	—	—	—	—	—	—	—	—	—	—	3	5	1	8	5	1	—	1	—	—	—	—	—	—	—	—	—	—	—	24
22	—	—	—	—	—	—	—	—	—	—	—	—	1	2	5	5	4	2	1	—	1	—	—	—	—	—	—	—	—	—	—	21
23	—	—	—	—	—	—	—	—	—	—	—	—	—	1	4	2	3	3	1	2	2	1	—	—	—	—	—	—	—	—	—	19
24	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	3	3	1	3	—	1	2	—	—	—	—	—	—	—	—	14
25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	1	2	—	2	1	1	—	1	—	—	—	—	—	—	10
26	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	2	—	2	—	—	—	—	—	—	—	—	5
27	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	2	1	—	—	—	—	—	—	—	—	5
28	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	3	—	—	—	—	—	—	—	—	—	—	6
29	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
30	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	1	—	—	—	—	—	3
31	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
32	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	2
33	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
34	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	2	4
35	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
Totals	0	4	14	49	86	92	116	106	84	76	69	52	48	50	34	25	28	14	9	10	12	5	6	0	2	1	3	2	1	0	2	1000

TABLE VIII., 1907.

Number of Ovules on First Placenta and Number of Ovules on Second Placenta.

Ovules on Second Placenta.

Ovules on First Placenta.

	5	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	Totals
6	1	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
7	2	1	1	6	2	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	15
8	—	—	3	6	8	4	1	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	24
9	—	1	4	3	8	9	3	5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	33
10	—	2	—	5	8	11	15	7	2	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	53
11	—	—	2	2	3	5	16	8	11	5	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	55
12	—	—	—	—	3	2	11	13	7	3	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	40
13	—	—	—	—	1	3	9	6	6	6	4	2	—	—	—	—	—	—	—	—	—	—	—	—	—	37
14	—	—	—	—	—	3	1	5	5	2	12	1	1	—	—	—	—	—	—	—	—	—	—	—	—	30
15	—	—	—	—	1	1	2	2	2	4	4	2	1	2	—	—	—	—	—	—	—	—	—	—	—	21
16	—	—	—	—	—	1	1	—	3	5	2	4	3	3	—	—	—	—	—	—	—	—	—	—	—	22
17	—	—	—	—	—	—	—	—	—	3	2	4	2	2	1	2	—	—	—	—	—	—	—	—	—	16
18	—	—	—	—	—	—	—	—	—	1	2	4	2	1	—	2	—	—	—	—	—	—	—	—	—	12
19	—	—	—	—	—	—	—	—	—	—	1	1	2	1	1	2	1	—	—	1	—	—	—	—	—	10
20	—	—	—	—	—	—	—	—	—	—	—	1	—	1	3	1	3	1	—	—	—	—	—	—	—	10
21	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	3	1	—	—	—	—	—	—	—	5
22	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	1	—	—	—	—	3
23	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	—	—	—	—	2
24	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	1	—	—	—	—	—	3
25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
26	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	2
27	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
28	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
29	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	2
Totals	3	4	11	23	34	41	60	46	36	34	29	21	13	10	5	8	8	2	3	2	1	2	2	0	2	400

TABLE IX., 1906.

Number of Seeds Developing on First Placenta and Number of Seeds Developing on Second Placenta.

Seeds Developing on Second Placenta.

Seeds Developing on First Placenta.

	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	Totals
1	1	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
2	—	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
3	—	—	—	—	—	1	—	—	1	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
4	—	—	—	1	2	3	1	1	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10
5	—	1	—	1	—	4	2	1	3	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	13
6	—	—	1	—	2	2	7	6	5	4	2	3	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	33
7	—	—	—	1	—	1	11	8	12	9	5	4	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	53
8	—	—	1	—	1	—	4	16	12	23	8	6	3	4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	78
9	—	—	—	1	—	3	13	10	14	22	23	13	5	2	3	1	—	—	—	—	—	—	—	—	—	—	—	—	110
10	—	—	-	—	—	2	3	12	10	24	20	20	11	12	7	2	2	1	—	—	—	—	—	—	—	—	—	—	126
11	—	—	—	—	—	1	1	3	7	22	24	22	18	8	4	4	1	—	—	1	—	—	—	—	—	—	—	—	116
12	—	—	—	—	—	—	1	4	4	12	14	20	13	23	11	7	—	2	—	—	—	—	—	—	—	—	—	—	111
13	—	—	—	—	—	—	—	1	3	7	12	7	16	8	5	4	5	3	4	1	—	—	—	—	—	—	—	—	76
14	—	—	—	—	—	—	—	—	—	2	2	2	10	12	15	5	5	3	1	—	2	—	—	—	—	—	—	—	59
15	—	—	—	—	—	—	—	—	—	—	5	4	4	9	11	10	3	4	2	2	—	—	1	—	—	—	—	—	55
16	—	—	—	—	—	1	—	—	-	1	—	2	2	1	3	7	7	6	2	4	1	1	—	—	—	—	—	—	38
17	—	—	—	—	—	—	—	—	—	—	2	—	2	3	4	3	4	6	5	1	—	—	1	—	—	—	—	—	31
18	—	—	—	—	—	—	—	—	—	—	—	1	—	3	2	2	2	3	5	3	3	—	2	—	1	—	—	—	27
19	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	2	2	5	3	2	—	1	—	1	—	—	—	18
20	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	1	5	—	—	1	1	—	—	—	1	12
21	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	1	2	2	1	—	—	1	—	—	—	9
22	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	2	1	—	—	1	—	—	1	—	—	6
23	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	1	—	1	—	4
24	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	1	—	—	—	3
25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
26	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
27	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
28	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
Totals	1	2	3	5	6	18	43	62	72	126	120	105	85	86	66	50	33	31	28	24	11	3	9	1	7	1	1	1	1000

TABLE IX., 1907.

Number of Seeds Developing on First Placenta and Number of Seeds Developing on Second Placenta.

Seeds Developing on Second Placenta.

Seeds Developing on First Placenta.

	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	Totals
0	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
1	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
2	—	2	—	—	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5
3	1	—	1	5	3	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	13
4	—	1	—	—	2	2	3	4	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	13
5	—	—	—	2	2	4	3	7	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	20
6	—	—	—	—	1	3	3	5	7	1	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	22
7	—	—	—	3	1	3	7	7	8	8	4	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	42
8	—	—	—	—	—	—	4	4	6	11	6	1	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	35
9	—	—	1	—	—	1	3	2	13	3	10	8	3	2	1	—	—	—	—	—	—	—	—	—	—	—	—	47
10	—	—	—	—	—	1	2	5	7	9	10	6	8	2	—	—	—	—	—	—	—	—	—	—	—	—	—	50
11	—	—	—	—	—	—	1	1	5	5	5	8	8	4	5	—	—	—	—	1	—	—	—	—	—	—	—	43
12	—	—	—	—	—	—	—	—	1	2	4	5	4	6	1	3	1	—	—	—	—	—	—	—	—	—	—	27
13	—	—	—	—	—	—	—	—	—	—	1	1	6	4	1	1	—	—	—	—	—	—	—	—	—	—	—	14
14	—	—	—	—	—	—	—	1	—	—	1	1	4	2	5	1	1	1	—	—	—	—	—	—	—	—	—	17
15	—	—	—	—	—	—	1	—	—	—	—	3	3	3	1	3	—	—	—	—	—	—	—	—	—	—	—	14
16	—	—	—	—	—	—	—	—	—	—	—	1	2	1	1	1	1	—	3	—	—	—	—	—	—	—	—	10
17	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	1	2	—	1	1	2	—	—	—	—	—	—	9
18	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	3	1	—	—	—	—	—	—	—	6
19	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	—	—	—	—	—	—	—	—	—	3
20	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	1	—	—	—	3
21	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
22	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	2
23	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
24	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1
Totals	2	4	2	10	10	18	28	36	48	41	44	34	42	26	15	12	6	3	7	5	2	0	0	1	1	2	1	400

TABLE X., 1906.

Number of Aborted Ovules on First Placenta and Number of Aborted Ovules on Second Placenta.

Aborted Ovules on Second Placenta.

Aborted Ovules on First Placenta.

	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	Totals
0	27	21	23	11	6	1	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	91
1	31	55	32	28	13	8	4	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	173
2	14	37	39	43	24	10	6	4	2	—	1	—	—	1	—	—	—	—	—	—	—	—	—	—	181
3	17	22	37	28	27	12	9	6	2	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	161
4	5	12	19	33	27	18	6	10	6	2	2	—	—	—	—	—	—	—	—	—	—	—	—	—	140
5	1	7	19	13	20	13	9	7	3	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	93
6	—	5	2	6	4	8	9	3	2	1	2	—	1	—	—	—	—	—	—	—	—	—	—	—	43
7	—	1	2	5	2	5	4	8	5	2	2	1	2	—	—	—	—	—	—	—	—	—	—	—	39
8	1	—	—	1	4	3	3	5	—	—	2	1	—	—	—	—	—	—	—	—	—	—	—	—	20
9	—	—	2	2	—	2	1	5	4	2	1	1	1	—	—	—	—	—	—	—	—	—	—	—	21
10	—	—	—	—	—	1	2	2	1	1	1	2	—	1	—	—	1	—	—	—	—	—	—	—	12
11	—	—	—	—	—	2	1	—	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6
12	—	—	—	1	—	—	—	—	1	—	1	1	2	—	—	—	—	—	—	—	—	—	—	—	6
13	—	—	—	—	—	—	2	—	—	—	—	—	1	2	1	1	—	—	—	—	—	—	—	—	7
14	—	—	—	—	—	1	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	3
15	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
16	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
17	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
18	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
19	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
20	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
21	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
22	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1	2
Totals	96	160	175	171	127	84	57	52	29	10	13	6	7	6	2	1	2	0	1	0	0	0	0	1	1000

TABLE X., 1907.

Number of Aborted Ovules on First Placenta and Number of Aborted Ovules on Second Placenta.

Aborted Ovules on Second Placenta.

Aborted Ovules on First Placenta.	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	Totals
0	9	14	10	3	—	1	—	—	—	—	—	—	—	—	—	—	37
1	12	29	25	9	8	3	1	—	—	—	—	—	—	—	—	—	87
2	12	16	24	12	4	6	—	1	—	—	—	—	—	—	—	—	75
3	4	16	17	12	4	3	4	—	1	—	—	—	—	—	—	—	61
4	2	8	11	10	10	4	3	2	1	—	—	—	—	—	—	—	51
5	1	3	2	5	5	6	5	1	—	1	—	—	—	—	—	—	29
6	—	2	1	4	3	4	—	1	2	1	3	—	—	1	—	—	22
7	—	—	2	—	3	2	1	1	1	—	2	1	—	—	—	—	13
8	—	—	—	—	1	2	—	1	1	3	—	—	—	1	—	—	9
9	—	—	—	—	—	1	1	4	2	—	1	—	—	—	—	—	9
10	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
11	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	2
12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
13	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
14	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
15	—	—	—	—	—	—	1	—	—	—	—	—	1	—	—	1	3
16	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
Totals	40	88	92	55	38	33	16	11	8	5	6	1	2	3	1	1	400

TABLE XII., 1906 AND 1907.

Seriation of Total Number of Aborted Ovules per Placenta.

Grade	1906	1907	Grade	1906	1907
0	27	9	*25*	1	1
1	52	26	*26*	3	—
2	92	51	*27*	3	1
3	97	48	*28*	1	—
4	100	51	*29*	—	1
5	107	47	*30*	—	1
6	87	33	*31*	1	—
7	98	25	*32*	—	—
8	64	19	*33*	—	—
9	60	20	*34*	—	—
10	36	12	*35*	—	—
11	34	15	*36*	—	—
12	34	5	*37*	—	—
13	16	4	*38*	1	—
14	18	5	*39*	1	—
15	15	5	*40*	—	—
16	13	8	*41*	—	—
17	9	7	*42*	—	—
18	7	1	*43*	—	—
19	11	2	*44*	—	—
20	3	—	*45*	1	—
21	3	2			
22	1	—			
23	2	1	Totals	1000	400
24	2	—			

TABLE XI., 1906.

Number of Ovules on "First" Placenta and Number of Seeds Developing on "Second" Placenta.

Seeds Developing on "Second" Placenta.

Ovules on "First" Placenta.

	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	Totals
6	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
7	—	—	—	—	—	1	2	1	2	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	8
8	—	—	—	—	1	2	3	7	7	9	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	30
9	—	—	1	1	3	3	12	18	17	13	9	5	—	4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	86
10	—	1	1	2	4	6	15	23	32	21	23	20	7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	155
11	—	—	—	1	2	3	12	24	35	50	28	31	9	6	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	202
12	—	—	1	2	2	5	13	14	13	49	46	44	16	17	3	1	—	—	—	—	—	—	—	—	—	—	—	—	—	226
13	—	1	1	—	—	5	8	14	13	33	37	38	42	11	6	3	3	1	—	—	—	—	—	—	—	—	—	—	—	216
14	—	1	2	—	—	2	6	4	11	21	32	16	24	20	14	7	1	1	—	—	—	—	—	—	—	—	—	—	—	162
15	1	—	—	1	1	1	1	2	11	13	19	22	33	27	11	12	3	3	2	—	—	—	—	—	—	—	—	—	—	163
16	—	—	—	—	2	—	1	—	4	10	21	16	18	10	24	17	9	3	3	—	—	—	—	—	—	—	—	—	—	138
17	—	—	—	—	—	—	—	3	—	3	8	6	15	19	27	13	8	9	2	2	—	—	—	—	—	—	—	—	—	115
18	—	—	—	1	—	1	—	2	2	6	11	8	8	13	15	16	9	5	1	1	—	—	1	—	—	—	—	—	—	100
19	—	—	—	1	—	1	2	1	—	4	3	4	11	12	9	12	7	14	3	4	—	1	1	—	—	—	—	—	—	90
20	—	—	—	—	—	—	1	—	1	—	8	7	5	11	4	4	9	5	9	5	1	—	2	—	—	—	—	—	—	72
21	—	—	—	—	—	—	—	1	—	1	1	1	2	6	3	6	9	5	8	4	1	1	—	—	—	—	—	—	—	49
22	—	—	—	—	—	1	—	1	—	1	—	2	5	—	4	7	7	5	6	5	4	—	1	—	—	—	—	—	—	49
23	—	—	—	—	1	—	—	—	—	—	—	—	1	2	2	—	3	5	7	4	3	3	1	1	—	—	—	—	—	33
24	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	4	—	1	4	4	2	2	1	1	2	1	—	—	—	23
25	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1	1	2	—	5	4	2	—	1	3	—	—	—	—	20
26	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	2	—	2	1	3	1	3	1	2	—	—	—	1	17
27	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3	—	3	1	1	—	—	—	1	—	—	10
28	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	2	—	—	1	4	—	1	1	1	—	—	—	—	—	12
29	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
30	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	1	1	—	—	1	—	—	—	—	—	—	5
31	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	2
32	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	2	—	1	—	—	—	—	—	—	5
33	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	2
34	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	—	2	—	1	—	—	5
35	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
36	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	2
Totals	1	4	6	9	16	31	76	115	150	236	246	221	196	162	125	105	71	62	55	42	23	12	15	5	10	1	2	2	1	2000

TABLE XI., 1907.

Number of Ovules on "First" Placenta and Number of Seeds Developing on "Second" Placenta.

Seeds Developing on "Second" Placenta.

Ovules on "First" Placenta.	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	Totals
5	—	—	—	2	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
6	—	—	—	4	—	—	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	7
7	1	—	1	4	4	5	2	5	3	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	26
8	1	1	—	4	4	6	5	13	2	7	3	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	47
9	—	—	3	4	4	7	6	13	12	7	6	2	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	67
10	1	1	—	1	5	11	10	9	15	11	20	4	6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	94
11	—	1	1	1	3	2	10	11	17	18	24	15	8	2	1	1	—	—	—	—	—	—	—	—	—	—	—	115
12	—	1	1	—	—	2	5	13	12	16	12	14	6	3	1	—	—	—	—	—	—	—	—	—	—	—	—	86
13	—	—	—	—	2	1	3	5	6	11	11	17	9	4	4	—	—	—	—	—	—	—	—	—	—	—	—	73
14	1	—	—	1	—	2	1	4	9	7	7	4	7	9	7	3	1	1	—	—	—	—	—	—	—	—	—	64
15	—	—	1	1	—	—	2	3	1	4	6	5	11	7	4	1	3	1	—	—	—	—	—	—	—	—	—	50
16	—	—	—	1	—	—	3	1	3	3	3	5	9	4	4	5	1	1	—	—	—	—	—	—	—	—	—	43
17	—	—	—	—	—	1	—	—	1	2	1	4	3	5	2	6	1	1	1	1	—	—	—	—	—	—	—	29
18	—	1	—	—	—	—	—	—	—	1	—	4	2	2	1	3	4	2	2	—	—	—	—	—	—	—	—	22
19	—	—	—	—	—	—	—	—	—	—	1	1	2	1	—	3	1	—	4	2	—	—	—	—	—	—	—	15
20	—	—	—	—	—	—	1	—	2	—	—	—	1	1	3	—	3	2	3	1	1	—	—	—	—	—	—	18
21	—	—	—	—	1	—	—	—	—	—	—	1	1	—	1	1	—	2	—	3	3	—	—	—	—	—	—	13
22	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	2	1	—	—	—	—	—	—	—	—	5
23	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	1	—	1	1	—	—	—	—	—	—	—	5
24	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1	—	1	—	1	—	—	1	—	—	—	5
25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1	—	2
26	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	—	1	4
27	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1	—	—	—	—	2
28	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
29	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	1	1	—	4
Totals	4	5	7	23	23	38	50	78	83	88	94	77	69	40	32	26	16	12	13	8	5	0	2	2	2	2	1	800

AN OBJECTIVE STUDY OF MATHEMATICAL INTELLIGENCE*.

By WILLIAM BROWN, M.A., Oxford; Lecturer on Psychology, King's College, London.

I. *Introductory.*

One of the most pressing needs of the psychology of the present day is a theory of reasoning, full, concrete, and capable of practical application. If we turn to the chapters on reasoning in any of the standard text-books, we find a vagueness and diffidence strongly contrasting with the wealth of detail and confident generalisation prominent in the chapters on lower mental processes†. This state of affairs is due in part, doubtless, to the desire not to encroach on the territory of the logician, in part to the complexity of the processes concerned,—most of all, however, to the insufficiency of the method employed, viz. that of unaided introspection. That this is so, is clear from the fact that the psychology of thought has even a longer history than that of most of the simpler mental processes, and yet has made far less progress.

The unsatisfactory character of the generally-accepted introspective account has been recently brought into prominence by considerations, separately conclusive, yet quite independent of one another. These originate from the three distinct realms of normal psychology, psychopathology, and applied psychology of education, respectively.

(1) In normal psychology the evidence is derived from certain experiments, among which those of Thorndike, Woodworth, Winch, Watt, and Ach are of chief importance. They all point to the fact that mental imagery, or, in more general terms, sensory content of any kind, is not an indispensable factor in cognitive process, and, even when present, is of small efficiency. Thorndike‡ compared

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* Read before the British Psychological Society, March 13, 1909. Considerable changes have since been made in the methods of evaluating the results.

† A recent notable exception is J. E. Miller's *The Psychology of Thinking*, Macmillan, 1909.

‡ E. L. Thorndike: "On the Function of Visual Images," *Journal of Philosophy, Psychology, and Scientific Methods*, Vol. iv. No. 12, p. 324, June 6, 1907.

good and bad visualisers with one another with respect to memory of facts of visual perception, and found no superiority in the case of the former class. One of Woodworth's experiments* was to present to the subject a *relation* by means of two terms, a and b, between which it held, and a third term, c, for which a fourth term, x, was to be found bearing the same relation to c that b did to a. The terms were cards of different colours and shapes, and also significant words, such as :—

London : England :: Paris : x,
Mice : cats :: worms : x.

The subjects' introspections showed that, although in difficult cases the relation may come to consciousness in the form of a name or some sort of mental imagery, in some cases the relation is felt without a name or mental image of any kind, and in easy cases the answer comes immediately, with very little consciousness at all.

Winch† tested the memory of school-children for drawings under conditions in which mental "imaging" immediately after the period of exposure was either enjoined or forbidden. No superiority in memorising-power was shown by the children under the former of the two conditions.

Watt‡ and Ach§ reached their conclusions in the course of experiments on reaction-times. It was found by both observers that the reaction was determined in the mind of the subject by the general purpose or known nature of the task set by the experimenter, often at a very low degree of consciousness. According to Ach, image and stimulus are both secondary to the purpose or task.

(2) In the realm of psychopathology there is overwhelming evidence that loss of mental functioning may take other forms than those of loss of sensation or of mental imagery. In particular, the researches of Marie|| and his school¶ into the problem of aphasia seem to point to a priority of thought over sensory content of any kind.

(3) The practical uselessness for education of the prevalent psychological theory of reasoning is becoming more and more apparent in educational circles. Outside the walls of the infant school, education deals primarily with the training and development of reasoning as such, but this cannot be considered, after the manner of the faculty psychologists, as a general capacity functioning independently of the material on and through which it works. Each school subject has potenti-

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* R. S. Woodworth : "The Consciousness of Relation," *Essays Philosophical and Psychological in Honour of William James*, 1908, pp. 489—492.

† W. H. Winch : "The Function of Images," *Journal of Philosophy, Psychology, and Scientific Methods*, Vol. v. No. 13, pp. 337 ff., June 18, 1908.

‡ H. J. Watt : *Experimentelle Beiträge zu einer Theorie des Denkens*, 1905.

§ N. Ach : *Ueber die Willenstätigkeit und das Denken*, Göttingen, 1905.

|| Pierre Marie : *Semaine Medicale*, May 23, Oct. 17, and Nov. 28, 1906. Pierre Marie : "Sur la Fonction du Langage," *Revue de Philosophie*, 1907.

¶ François Montier : *L'Aphasie de Broca* : Paris, Steinheil, 1908. L. Dugas : "Une Théorie Nouvelle de l'Aphasie," *Journal de Psychologie*, Sept. Oct., 1908.

ally a psychology as well as a logic; but this potentiality has not as yet been actualised by the psychologists, the reason being, as is shown by the cases previously mentioned, that introspection alone is insufficient to give us the requisite data*.

Apart from the experiments mentioned earlier in this paper, most of the experimental work on the psychology of reasoning has taken the form of a study of "tests" of elementary physical and mental processes accepted problematically as symptoms of degrees of intelligence. The history of this movement is already recorded with sufficient fulness in Meumann's *Experimentelle Pädagogik*, Bd. I. (Leipzig, 1907). Hitherto the results obtained have been either conflicting or negative. In particular, the hypothesis, put forward by one investigator, that general intelligence and general power of sensory discrimination are identical, has not been confirmed by recent work.

Mention should, however, be made of the fact that the problem of reasoning has been attacked more directly in a recent research by Willis L. Gard, entitled "A Preliminary Study of the Psychology of Reasoning," communicated to the *American Journal of Psychology*† by Prof. Sanford. The experiments took the form of getting the subject to solve simple mathematical puzzles, e.g. to fill up gaps in partially worked out division and multiplication sums. It seems somewhat doubtful whether puzzles are ever a good test of intelligence, since they really involve a good deal of specialisation of interest and ability.

II. *Historical.*

Turning now to the subject-proper of this paper, viz. the study of mathematical intelligence, we find that earlier experimental work is limited to the analysis of the mental processes involved in the simplest forms of arithmetical operation,—counting, adding, subtracting, etc.

(*a*) *Counting.* As regards counting, experiments have been worked to decide the rival claims of the *intuitional method* (of Pestalozzi) and the *counting method* in the teaching of number. These two methods are based on the spatial and temporal groupings of units, respectively, and the experiments therefore fall naturally into two classes. The first includes the experiments of Cattell‡, Lay§, Warren||, Messenger¶ and Helene Nanu**, who all worked with the tachistoscope, exposing to the subjects' view lines or dots, varying in number and arrangement, for very short lengths of time. Experiments belonging to the second class are

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* See William Brown: "Educational Psychology in the Secondary School," *Journal of Philosophy, Psychology, and Scientific Methods*, Vol. VII. No. 1, Jan. 6, 1910.

† Vol. XVIII. Oct. 1907.

‡ Cattell: "Ueber die Zeit des Erkennens u.s.w." *Philos. Studien*, III. 1886.

§ Lay: *Führer durch den ersten Rechenunterricht*, 1 Aufl., 1898.

|| Warren: *Princetown Contributions to Psychology*, II. 1898, Vol. 3.

¶ Messenger: "The Perception of Number," *Psychol. Review*, Monograph Supplement, V. 1903.

** Helene A. Nanu: "Zur Psychologie der Zahlauffassung," *Würzburger Dissertation*, 1904.

those of Dietze*, who employed a metronome and asked the subject to compare series of beats of different lengths with one another (without counting). His purpose was to determine how many beats could be correctly estimated without counting. The result was found to vary with the rapidity of the beats, the most favourable interval between the beats being ·2 to ·3 sec.

(*b*) *Simple Arithmetical Processes.* The psychology of addition, subtraction, and other simple arithmetical operations has been repeatedly studied by introspection, either casual or controlled. An example of the former is to be found in Binet's *Psychologie des Grands Calculateurs et Joueurs d'Échecs*, and of the latter in an article in Vol. XVII. of the *Am. J. P.* (1906)†.

(*c*) *Higher Mathematical Processes.* As regards the psychological analysis of higher mathematical processes, very little work has thus far been done. In almost all cases the data have been school and college marks or class-lists. The method employed has been generally that of correlation, but the coefficients of correlation have been worked out between mathematical ability as a whole and the equally complex mental capacities involved in the other main subjects of the school or college curriculum, e.g. Classics, French, English, etc. No instance of analysis of the *inner* relations of mathematical intelligence is known to the writer.

III. *Collection and Analysis of Data.*

Towards the close of the year 1908, the writer examined in mathematics a group of 83 boys belonging to the middle forms of the classical side of an English public school. The group consisted of five sets or forms, viz. U. V^A, U. V^B, L. VI, U. VI and VII; but all the boys were examined on the same three papers—Geometry, Arithmetic, and Algebra—and they had all been working along the same lines and in the same environment. The examination papers were set by one of the masters, and the writer was expected to return the boys' answers, marked, within the week. Although this arrangement meant that the work had to be done at high pressure, it was on the whole a fortunate one so far as the research was concerned, since it almost certainly rendered the standard of marking much more steady than the latter would have been if the marking had extended over a longer time.

The papers were first marked according to ordinary school standards for the sake of the school examination, and then *according to a differential system of*

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* Dietze: "Ueber den Bewusstseinsumfang," *Philos. Studien*, II. 1885.

† C. E. Browne: "The Psychology of the Simple Arithmetical Processes," *Am. J. P.* Vol. XVII. Jan. 1906; see also Frank D. Mitchell: "Mathematical Prodigies," *Am. J. P.* Vol. XVIII. Jan. 1907.

Recently I have myself had excellent opportunities of studying these arithmetical processes introspectively. The processes seem to go on almost mechanically, but are yet controlled from moment to moment by auditory and kinaesthetic imagery. When any distraction occurs, either from without or from within, visual imagery is immediately called up, and this is sufficiently stable to allow me to pause for a moment and "collect my thoughts." The various forms of imagery present seem to act as controls of the thought, but otherwise to be quite inactive, the "driving power" coming from the *standing purpose* to add, subtract, etc. (Cf. supra, p. 353, Ach's results.)

marking based on an analysis of the intellectual processes involved in the answering of the questions as far as they could be detected by introspection; according to this system, the maximum mark for any one process in any one answer was 10. (For samples of questions, analyses, and actual marks, see Appendices I. and III.)

It was decided that the number of boys examined was not large enough to sustain a mathematical superstructure built on so detailed an analysis. The analyses were therefore subsumed under nine principal headings, viz., A, B, C, D, E, F, G, H, I, distributed as follows:—

Geometry.

A. Memory of definitions and general principles (e.g. principle of superposition) in Geometry.

[1 (iii) + 3 (iii) + 5 (i) + 6 (i) + 8 (i) + 9 (i)]*.

B. Memory of constructions in Geometry.

[2 (i) + 3 (ii) + 4 (i) + 5 (iii) + 9 (ii)].

C. Memory of preceding propositions, and power of applying them. Geometry.

[1 (ii) + 2 (iii) + 3 (i) + 4 (iii + v) + 5 (ii + iv + vi) + 6 (ii) + 7 (i + iv) + 8 (ii + iii)].

D. Recognition of necessity of generality in proof, and power of recognizing general relations in a particular case. Geometry.

[1 (i + iv) + 3 (iv) + 4 (iv + vi) + 5 (v) + 6 (iii) + 8 (iv)].

Arithmetic.

E. Accuracy in Arithmetic.

[1 (ii) + 2 (iii) + 3 (iv) + 4 (iii) + 5 (iv) + 6 (ii) + 7 (ii) + 8 (iii) + 9 (ii) + 10 (iv) + 11 (ii) + 12 (iii) + 13 (ii)].

F. General memory of rules and power of applying them. Arithmetic.

[1 (i) + 2 (i + ii) + 3 (ii) + 4 (i + ii) + 6 (i) + 8 (i + ii) + 10 (i + iii) + 11 (i) + 12 (i + ii) + 13 (i)].

G. Power of doing sums in percentage and proportion. Arithmetic.

[7 (i) + 9 (i) + 12 (i + ii)].

Algebra.

H. Accuracy in Algebra.

[1 (iii) + 2 (iii) + 3 (ii) + 4 (iv) + 5 (ii) + 6 (iii) + 7 (iv) + 8 (iii) + 9 (ii) + 10 (iii)].

I. General memory of rules and power of applying them. Algebra.

[1 (i + ii) + 2 (ii) + 3 (i) + 4 (ii + iii) + 5 (i) + 6 (ii) + 7 (i + ii + iii) + 8 (i)].

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* See Appendix I for meaning of these numbers.

These marks, together with the ages of the boys and their total marks for Geometry, Arithmetic, and Algebra, gave thirteen series of measurements and also thirteen corresponding series of "ranks" or orders. Correlation coefficients were now evaluated, using both actual measurements and also ranks. In the former case, Pearson's full method was used, the measurements were grouped, correlation tables were drawn up, and the product-moment formula, $r_{xy} = \frac{S(xy)}{n\sigma_x\sigma_y}$, was applied*. In the case of ranks, two methods were used, (1) the product-moment formula, which in the case of ranks reduces to $1 - \frac{6S(d^2)}{n(n^2-1)}$ where d = difference of corresponding ranks in the two series, was applied. This gave what Pearson† calls ρ, or "rank-correlation," and the result was then converted into "true variate correlation" by the formula $r = 2\sin\left(\frac{\pi}{6}\rho\right)$; (2) the formula $R = 1 - \frac{6S(g)}{n^2-1}$, where g = gain in rank of second series on first‡, was applied, and again the true variate correlation was obtained by Pearson's formula

$$r = 2\cos\frac{\pi}{3}(1-R) - 1\text{§}$$

The above-mentioned two formulae giving r in terms of ρ and R respectively were deduced by Pearson on the assumption of "normal" or Gaussian distribution. If the correction is not made, in other words if ranks are themselves taken as measurements, the distribution obtained is a rectangle,—not obviously a reasonable assumption.

The coefficients (crude values) obtained in these three ways are given in the following table:—

	$r = \frac{S(xy)}{n\sigma_x\sigma_y}$ [marks]	$r = 2\sin\left(\frac{\pi}{6}\rho\right)$ [ranks]	$r = 2\cos\frac{\pi}{3}(1-R) - 1$ [ranks]
Arith. Alg. ...	·79 ± ·03	·65	·65
Geom. Alg. ...	·66 ± ·04	·63	·59
Geom. Arith. ...	·58 ± ·05	·61	·57
CD ...	·81 ± ·02	·77	·74
EH ...	·69 ± ·04	·55	·57
CG ...	·59 ± ·05	·56	·53
FI ...	·49 ± ·06	·49	·51
GI ...	·49 ± ·06	·54	·55
DG ...	·44 ± ·06	·43	·46
BG ...	·26 ± ·07	·24	·24

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* For a specimen correlation table, see Appendix II.

† Karl Pearson: "On Further Methods of Determining Correlation," *Drapers' Company Research Memoirs*, Biometric Series, IV., London, Dulau & Co., 1907, pp. 11 and 18.

‡ C. Spearman: "'Foot-rule' for measuring Correlation," *Brit. Journ. of Psychology*, Vol. II. Part I, July, 1906, pp. 100—104.

§ Karl Pearson: *Op. cit.* p. 17. Spearman suggested in his "Foot-rule" article the formula $r = \sin\left(\frac{\pi}{2}R\right)$, which he attempts to justify on merely *empirical* grounds. A full criticism of his work on "ranks" will be found in Pearson's article above-mentioned.

The discrepancies between mark-values and grade- (or adjusted rank) values are due to deviations from normal distribution*. This is empirically obvious from the fact that the discrepancies are larger the larger the deviations from normal distribution. This is also the case with the following additional coefficients, calculated by only two of the methods:—

	$r=\frac{S(xy)}{n\sigma_x\sigma_y}$ [marks]	$r=2\cos\frac{\pi}{3}(1-R)-1$ [ranks]
Arith. G ...	·78 ± ·03	·75
Alg. G† ...	·71 ± ·04	·57
Geom. G ...	·67 ± ·04	·60
AC ...	·78 ± ·03	·81
AD ...	·65 ± ·05	·60
FG ...	·64 ± ·05	·63
BC ...	·60 ± ·05	·68
CI ...	·55 ± ·06	·57
CF ...	·51 ± ·06	·50
AB ...	·49 ± ·06	·65
BD ...	·37 ± ·07	·29

Thirty-three other coefficients were worked out by the formula

$$r=2\cos\frac{\pi}{3}(1-R)-1$$

only. They are as follows:—

Geom.	A	·81	HI	·92	AF	·46
"	B	·55	EF	·82	DE	·46
"	C	·94	EG	·76	AE	·44
"	D	·82	AH	·61	DI	·43
Arith.	C	·54	CH	·57	AG	·41
"	D	·43	EI	·57	DH	·40
"	E	·91	FH	·55	BF	·28
"	F	·80	GH	·53	BH	·26
Alg.	C	·59	DF	·49	BE	·23
"	D	·41	AI	·47	BI	·22
"	H	·96	CE	·47		
"	I	·93				

All the above coefficients are "crude," i.e. uncorrected for the effect of extraneous conditions. In the present case the disturbing conditions are principally difference of age and difference of class or form. To correct for the first, coefficients were evaluated between age and each of the other twelve series. For ranks

$$\left[r=2\cos\frac{\pi}{3}(1-R)-1\right]$$

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* Considering the smallness of the sample, and therefore the largeness of the random deviations from possible normality, the two formulae lead to results in strikingly close accordance and suggest normal distribution as being the rule for the psychical characters observed.

† The exceptionally large discrepancy here corresponds with the fact that the characters compared have the two most skew distributions of the series, their distributions being: *Alg.* 26, 16, 19, 9·5, 6·5, 4, 1, 1 (a J-curve). *G* 30, 26, 12, 4, 2, 9 (a U-curve). Unit of grouping, 7 marks.

the values were as follows :—

Correlations with Age (ranks).

Geom.	0·19$^{\times}$	B	0·11	F	0·02
Alg.	0·19$^{\times}$	C	0·06	G	−0·12
Arith.	0·04	D	0·05	H	0·18$^{\times}$
A	0·21$^{\times}$	E	−0·02	I	0·19$^{\times}$

$$\text{P.E. for zero correlation} = \frac{{\cdot}7738}{\sqrt{83}} = 0{\cdot}09.$$

The only values greater than twice the probable error, and therefore possibly appreciable, are those marked with a cross. The corresponding values, using marks, are :—

Geom. 0·06, Alg. 0·23$^{\times}$, A 0·09, H 0·19$^{\times}$, I 0·07,

$$\text{P.E. for zero correlation} = \frac{{\cdot}67449}{\sqrt{83}} = 0{\cdot}075.$$

In the second case, therefore, only Alg. and H are affected appreciably. Owing to the discrepancies between marks and ranks, only the coefficients obtained from marks were corrected. The formula used was, of course, the partial correlation coefficient $\bar{r}_{12}$ for constant age*, viz.

$$\bar{r}_{12} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{(1 - r_{13}^2)(1 - r_{23}^2)}}.$$

Thus, for Geom. Alg., putting Geom. 1, Alg. 2, Age 3,

$$\bar{r}_{12} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{(1 - r_{13}^2)(1 - r_{23}^2)}} = \frac{{\cdot}66 - 0 \times {\cdot}23}{\sqrt{(1 - 0^2)(1 - {\cdot}23^2)}} = \frac{{\cdot}66}{{\cdot}973} = {\cdot}68.$$

The second main disturbing factor was the arrangement of the boys in sets or forms. The boys of the higher sets might have been expected to do better in every part of the work than those of the lower sets, because they had covered more of the syllabus than the others. Instead, however, of calculating correlation coefficients between order of form and each of the series of measurements, the "correlation ratio," η, was evaluated†. In carrying out the correction, η is substituted instead of r in the partial correlation coefficient.

The values of η obtained were the following :—

Correlation Ratios, "Form" and Character.

Geom.	·71	D	·57
Arith.	·59	E	·73
Alg.	·87	F	·59
A	·59	G	·65
B	·28	H	·76
C	·66	I	·80

Age ·41

The fact that η for age is ·41 while the correlation between age and total marks is nil seems to show that the effect of difference of form in the other series is due mainly to the difference in the extent of syllabus covered in the different forms. This, at any rate, seems to be the most satisfactory solution of a difficulty which the writer himself would be the first to admit.

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* See, e.g., G. Udny Yule: "On the Theory of Correlation," *Journal of the Royal Statistical Society*, Vol. LX. Part IV. Dec. 1897, p. 833.

† See Karl Pearson: Biometric Series II, *Drapers' Company Research Memoirs*, pp. 9 ff. For an example of the method, see Appendix II.

The finally corrected values of the correlations are :—

Alg. Arith.	·76 ± ·03	BD	·27 ± ·07	FG	·41 ± ·06
Geom. Arith.	·28 ± ·07	CD	·69 ± ·04	BG	·11 ± ·08
Geom. Alg.	·18 ± ·08	Arith. G	·65 ± ·04	DG	·11 ± ·08
AB	·42 ± ·06	Alg. G	·45 ± ·06	CF	·20 ± ·07
AC	·64 ± ·05	Geom. G	·39 ± ·07	CI	·05 ± ·08
AD	·31 ± ·07	CG	·28 ± ·07	EH	·33 ± ·07
BC	·57 ± ·05	IG	·00 ± ·08	FI	·04 ± ·08

Several interesting results can be obtained from these values by applying the theory of multiple correlation. Thus, limiting our attention for the moment to Geometry, Arithmetic, and Algebra, it is possible for us to deduce the degree of correlation between any two of them on the assumption that the third ability is constant throughout the series. It is given by the formula

$$\bar{r}_{12} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{(1 - r_{13}^2)(1 - r_{23}^2)}}. \quad \text{[See above, p. 359.]}$$

The partial correlation coefficients in the present case are :—

Geom. Alg. − ·05 ± ·08, Geom. Arith. ·23 ± ·07, Alg. Arith. ·75 ± ·03.

We may perhaps infer from these values that geometrical ability is only related to algebraical ability through the mediation of arithmetical ability[*].

Taking now four variates, e.g. A, B, C, and D, it is, in a similar way, possible to deduce the degree of correlation between any two, say C and D, on the assumption that the other two abilities are constant throughout the series. Putting the numbers 1, 2, 3, 4 in the place of A, B, C, D, the partial correlation coefficient

$$\bar{r}_{34} = \frac{r_{34}(1 - r_{12}^2) + r_{31}(r_{12}r_{42} - r_{41}) + r_{32}(r_{41}r_{12} - r_{42})}{\sqrt{1 - r_{12}^2 - r_{14}^2 - r_{24}^2 + 2r_{12}r_{14}r_{24}}\,\sqrt{1 - r_{12}^2 - r_{13}^2 - r_{23}^2 + 2r_{12}r_{13}r_{23}}} \text{ [†]}$$

$$= \cdot 93.$$

There is thus a very close relation indeed between memory of propositions in geometry, and the power of recognising general relations in a particular case in geometry.

Obviously, the method could be extended indefinitely.

A glance at the table of corrected coefficients suggests the following additional results :—

(1) the ability to do percentage and proportion sums in arithmetic is more closely related to essential geometrical ability than to essential algebraical ability;

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[*] This result harmonises with the view that mathematical reasoners fall into two types, the so-called "geometrical" (or "intuitional") and the "analytical" (or "logical") types. See H. Poincaré : *La Valeur de la Science*, pp. 11—15.

[†] See Karl Pearson : "Selection, etc." *Phil. Trans.* Vol. 200 A, p. 31.

(2) the close relation between arithmetical ability and algebraical ability is due mainly to habits of accuracy common to both;

(3) memory of preceding propositions is the ability most closely related to the other forms of geometrical ability in school-boys,—is, in fact, the central ability in school geometry.

It would be interesting to test the results for the existence of a general geometrical ability or a general mathematical ability, but unfortunately the only formula hitherto suggested for such a purpose, viz. that given by Dr Spearman[*], is open to very serious criticism[†]. The writer hopes to return to the problem in a future paper.

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[*] Krueger and Spearman: *Zeitschrift für Psychologie*, Bd. 44, 1906, p. 84.

[†] See Clark Wissler: "The Spearman Correlation Formula," *Science*, N. S. Vol. XXII. No. 558, Sept. 8, 1905, pp. 309—311.

William Brown: "Some Experimental Results in Correlation," *Comptes Rendus du VI^me^ Congrès International de Psychologie*, Genève, Aug. 1909. In this pamphlet mention was made of the assumptions upon which the applicability of Spearman's formula was based, and an alternative proof of the formula based on those assumptions, contributed by Mr G. Udny Yule, was brought forward.

The formula may be written

$$r^4{}_{XY} = \frac{r_{X_1Y_1}\, r_{X_1Y_2}\, r_{X_2Y_1}\, r_{X_2Y_2}}{r^2{}_{X_1X_2}\, r^2{}_{Y_1Y_2}},$$

where X and Y represent the unknown *true* values to be correlated, and X_1, X_2; Y_1, Y_2 are two pairs of *observed* values, vitiated more or less by errors of observation.

Let

$$x_1 = x + \delta_1, \quad y_1 = y + \epsilon_1,$$
$$x_2 = x + \delta_2, \quad y_2 = y + \epsilon_2,$$

where x, y, δ, ϵ represent deviations from means.

The assumption involved in the formula is that the errors of measurement δ_1, δ_2, ϵ_1, ϵ_2 are uncorrelated with each other or with x or y.

I suggested that the justifiability of this assumption might be tested by correlating $X_1 \sim X_2$ and $Y_1 \sim Y_2$, $X_1 \sim X_2$ and X_1, $Y_1 \sim Y_2$ and Y_1 which should all give a zero value for the coefficient. In this statement the sign of subtraction ($-$) should have been used instead of the difference-sign ($\sim$), and only the first of the three coefficients, viz. that between $X_1 - X_2$ and $Y_1 - Y_2$, should be expected to give a zero value.

Thus,

$$r_{\substack{X_1 - X_2 \\ Y_1 - Y_2}} = \frac{S\{(x_1 - x_2)(y_1 - y_2)\}}{\sqrt{S(x_1 - x_2)^2\, S(y_1 - y_2)^2}}$$
$$= \frac{S\{(\delta_1 - \delta_2)(\epsilon_1 - \epsilon_2)\}}{\sqrt{S(\delta_1 - \delta_2)^2\, S(\epsilon_1 - \epsilon_2)^2}}$$
$$= 0, \text{ since the numerator vanishes.}$$

Applying this test to some of the material on which I based my Geneva pamphlet, I found:—

1. In the case of accuracy in bisecting and trisecting lines, where the subjects of the experiment were 43 adults (Group A in the paper)

$$r_{\substack{B_1 - B_2 \\ T_1 - T_2}} = 0{\cdot}30 \pm 0{\cdot}09;$$

2. In the case of speed (S) and accuracy (A) in the addition of series of 10 single digits, where the subjects were 38 elementary school children, girls between the ages of 11 and 12 (Group C)

$$r_{S_1 - S_2,\, A_1 - A_2} = 0{\cdot}35 \pm 0{\cdot}09.$$

Thus, in each case a correlation greater than three times its probable error was found, showing that errors *were* almost certainly correlated among themselves and hence that Spearman's formula could not be applied.

In cases, however, where the correlation does work out to zero we cannot infer that errors are not correlated, but only that

$$S(\delta_1\epsilon_1) + S(\delta_2\epsilon_2) = S(\delta_1\epsilon_2) + S(\delta_2\epsilon_1).$$

The formula may still be quite inapplicable.

The nature of the correlation tables, and the form of the regression curves to which they gave rise, seemed to indicate that the number of boys examined (83) was sufficiently large to give fairly reliable results. On the other hand, the numbers furnished by separate school forms of average size (say 20) are certainly too small. Thus, the correlation coefficient between mathematics and classics was calculated for the Mathematical Eighth Form of the same school for Christmas, 1907, and again for Christmas, 1908. The two values were 0·20 and 0·52 respectively, showing a discrepancy of 160 % of the smaller value! Similarly, the mathematics-classics coefficient for the VIIth, U. VIth, and L. VIth forms of the mathematical division investigated in this paper worked out as 0·23, 0·76, and – 0·25 respectively. This would seem to indicate that the results of Correlational Psychology hitherto obtained, where small numbers of cases have almost invariably been used, are utterly unreliable. In such cases the ordinary formulae for probable errors (the proofs of which rest on the assumption of a large number of observations) do not apply, and consequently no clear appreciation of the significance of the results is possible.

APPENDIX I.

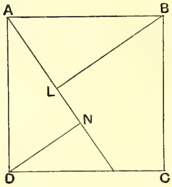
A. *Sample of questions in Geometry.*

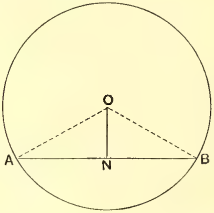
1. (*a*) If two triangles have two angles and a side of one equal respectively to two angles and a side of the other, they will be equal in all respects.

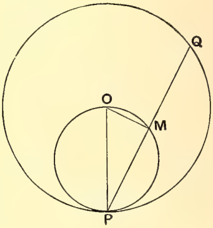
   (*b*) $ABCD$ is a square, perpendiculars BL and DN are drawn to any straight line through A. Prove that $AL = DN$.

4. (*a*) The straight line from the centre of a circle, perpendicular to a chord, bisects the chord.

   (*b*) OP is the radius of a circle. On OP as diameter another circle is described. Prove that all chords of the larger circle drawn through P are bisected by the smaller circle.







B. *Sample of analyses of above two Geometry questions.*

1. (*a*) (i) Recognition of necessity of generality in the proof.

(ii) Memory of general truth that three angles of a triangle equal two right-angles, and application to this case.

(iii) Principle of "proof by superposition": memory, sufficient understanding, and application of it to case under consideration.

(*b*) (iv) Power of recognising general relations in a particular case.

(v) Readiness to note and make use of the more special relations obtaining in the particular case.

4. (*a*) (i) Rational or merely visual memory of construction.

(ii) Connecting equality of lines with equality of triangles, of which the lines are sides.

(iii) Memory of result of a particular theorem (on right-angled triangles).

(*b*) (iv) Power of keeping essential and general relation of above theorem in mind, and of looking at the present case through the medium of it.

(v) Memory of theorem on angle in a semi-circle.

(vi) Power of seeing that the relation is a general one (i.e. power of noting from just what facts or relations the conclusion follows, and of realising that the sequence is not influenced by other facts).

C. *Sample of Marking-Sheet. Upper VIth*

Questions.

Name	Age		*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	Totals
C. E. B.	14·0	Geom.	9, 0 10, 10, 9, 0, 0	3 2, 2, 5	—	10, 0 10, 10, 10, 0, 0, 0	10, 15 10, 10, 10, 10, 10, 10	8 10, 2, 0, 5	0, 0 0, 0, 0, 0	—	—	—	—	—	—	55
		Arith.	10 10, 10	10 0, 0, 10	10 10, 10, 10, 10	1 10, 8, 5	10 10, 10, 10, 10	10 10, 10	0 0, 0	4, 3 10, 9, 8	14 10, 10	12 10, 10, 0, 10	14 10, 10	14 10, 10, 10	—	112
		Alg.	7, 8 10, 10, 10	7 10, 10	4, 5, 6 10, 10	0, 8 10, 10, 10, 7	4 8, 6	—	15 10, 10, 10, 10	5 10, 7, 6	15 10, 10	15 10, 10, 10	—	—	—	99

APPENDIX II.

A. *Evaluation of* $r_{\substack{\text{Geom.}\\\text{Arith.}}}$. *Correlation Table.*

Geometry.

Arithmetic.	*0—15*	*15—30*	*30—45*	*45—60*	*60—75*	*75—90*	*90—105*	*105—120*	*120—135*	Totals
0—15	—	1	2	1	—	—	—	—	—	4
15—30	1·5	7·5	3·5	2	—	—	1	—	—	15·5
30—45	1	7	5·5	7·5	1·5	1	—	—	—	23·5
45—60	—	2	7	5	7·5	—	1	—	—	22·5
60—75	—	—	2	0·5	—	2	—	—	1	5·5
75—90	—	—	1	0·5	—	2	—	1	—	4·5
90—105	—	—	—	2	1·5	1	—	—	1	5·5
105—120	—	—	—	1	—	—	—	—	—	1
120—135	—	—	—	—	—	1	—	—	—	1
Totals	2·5	17·5	21	19·5	10·5	7	2	1	2	83

B. *Evaluation of* η *for Arithmetic and Order of Form.*

Form (x).

Arithmetic (y).	*U. V*^{B}	*U. V*^{A}	*L. VI*	*U. VI*	*VII*	Totals
0—15	2 2	—	2 2	—	—	4
15—30	8 1	2 1	3·5 1	2 1	—	15·5
30—45	5 0	7 0	4·5 0	7 0	—	23·5
45—60	3 1	4 1	5 1	4 1	6·5 1	22·5
60—75	—	2 2	1 2	1 2	1·5 2	5·5
75—90	—	—	1 3	0·5 3	3 3	4·5
90—105	—	—	—	1·5 4	4 4	5·5
105—120	—	—	—	1 5	—	1
120—135	—	—	—	1 6	—	1
Totals (n_x)	18	15	17	18	15	83 (N)
Means (y_x)	– ·50	·40	·15	1·36	2·30	$\overline{y}=\cdot 68$ $\sigma_y^2=2\cdot 65$

—Arbitrary mean from which true means and standard deviations are calculated.

$$\eta^2=\frac{S\{n_x(y_x-\overline{y})^2\}}{N\sigma_y^2}=\cdot 3577,$$

$$\therefore\ \eta=\cdot 59.$$

APPENDIX III. *Marks and Ranks**.

Form	Name	Age	Rank	Total Marks	Rank	Geom.	Rank	Arith.	Rank	Alg.	Rank	A	Rank	B	Rank	C	Rank	D	Rank	E	Rank	F	Rank	G	Rank	H	Rank	I	Rank
VII. *1*	R. A. A.	16·3	23½	145	28½	55	30	49	32	41	24	40	8	30	37	44	41½	15	50	72	19½	51	58½	17	24	53	11	70	13½
	A. M. J. B.	15·5	44½	220	9	48	39½	93	6	79	4	18	60	30	37	44	41½	9	65	77	15½	88	11	40	3½	62	4½	86	5
	W. T. C.	14·11	55	212	10½	85	8	65	15	62	10	40	8	27	59	92	4	42	11½	80	12	70	30½	25	13½	70	3	87	4
	W. K. C.	14·9	59½	253	5	132	1	59	20	62	10	48	2	40	6	127	1	78	2	60	33½	84	14	24	15	53	11	88	3
5	C. K. D.	15·8	36½	256	3	125	2	94	5	37	29	50	1	30	37	120	2	79	1	101	5	91	9	37	7½	41	24½	45	26
	H. A. E. J.	17·3	10½	168	19½	68	17	52	29	48	18	20	47	40	6	60	21½	30	28	65	25½	87	13	20	19	35	33	57	20
	L. P. F.	16·0	29	178	15	93	5	53	26½	32	35½	30	26	40	6	80	7	60	3	58	36½	80	16½	5	55½	30	40½	33	39
	H. B. H.	14·3	73	159	23	58	26	60	18	41	24	30	26	30	37	60	21½	40	15½	56	40½	93	8	20	19	42	22½	27	47
	A. G. L.	18·6	2	242	6½	81	9	87	9	74	5	30	26	28	58	70	10½	30	28	73	17½	76	20½	40	3½	46	16½	73	11
10	S. S. M.	15·4	46	242	6½	80	10½	96	4	66	8	38	15½	39	12½	69	13	30	28	104	2½	108	6	35	9½	58	7½	75	10
	R. H. P.	15·8	36½	187	14	80	10½	80	10	27	43	38	15½	40	6	86	5	37	20	85	10	75	22½	37	7½	33	35½	45	26
	D. W. R. R.	15·7	39½	212	10½	64	19	91	7	57	12	40	8	40	6	63	16	18	46	86	9	105	7	40	3½	53	11	69	15
	A. S.	16·4	20	284	1	111	3	78	11	95	2	39	14	30	37	95	3	58	4	88	8	115	5	40	3½	74	2	100	1
	E. J. S.	14·11	55	154	26	54	34	59	20	41	24	20	47	30	37	60	21½	30	28	77	15½	65	41½	12	28½	32	37½	46	23½
15	A. C. T.	17·10	4½	176	17½	74	13	46	39	56	13	30	26	30	37	50	34	30	28	67	24	71	26	16	26	55	9	77	9
U. VI.	S. M. A.	15·3	48½	102	52	27	68½	35	54½	40	26½	20	47	30	37	35	49	0	75	43	62½	48	60½	10	40½	44	19	56	21½
	O. D. A.	14·4	70	199	13	56	27	90	8	53	14½	30	26	30	37	50	34	48	7½	93	6	121	2	17	24	58	7½	72	12
	C. E. B.	14·0	77	266	2	55	30	112	2	99	1	29	32½	22	62	60	21½	20	39	103	4	117	3	30	11	79	1	98	2
	H. A. D.	15·8	36½	145	28½	59	25	35	54½	51	16	28	34½	40	6	65	14½	45	10	44	60½	43	65	10	40½	46	16½	83	8
20	C. J. F.	16·5	17½	209	12	89	6	71	13	49	17	35	17	35	16½	80	7	48	7½	89	7	116	4	35	9½	50	15	46	23½
	W. F.	15·9	33	136	34	33	58½	41	45½	62	10	12	65	20	67½	32	51	20	39	50	50	66	39	0	72	44	19	84	6½
	M. A. H.	16·11	13½	117	38	39	48	25	69½	53	14½	26	36	20	67½	44	41½	17	48	32	76	40	66½	0	72	43	21	56	21½
	F. R. L.	14·3	73	158	24	96	4	27	67	35	32½	40	8	25	60	80	7	42	11½	50	50	74	24½	27	12	52	13½	66	16
	G. C. M.	16·11	13½	142	30½	60	23	46	39	36	30½	30	26	30	37	54	28	20	39	55	43	70	30½	10	40½	37	28	43	28
25	J. C. T. R.	15·9	33	176	17½	60	23	43	42½	73	6	34	18	30	37	62	17	35	21½	65	25½	68	37½	10	40½	62	4½	84	6½
	J. T. S.	16·5	17½	141	32	72	14½	48	34½	21	49½	20	47	30	37	70	10½	50	5½	64	28	65	41½	15	27	20	55	34	37
	C. M. S.	15·3	48½	68	67	17	78½	40	48	11	62½	0	82	10	81	2	81	10	59	70	22	74	24½	20	19	8	75	20	56
	R. M. S.	15·8	36½	254	4	88	7	123	1	43	21½	40	8	30	37	65	14½	40	15½	118	1	128	1	40	3½	35	33	28	46
	H. S.	14·8	62	142	30½	40	46½	57	23	45	20	20	47	30	37	50	34	10	59	57	39	70	30½	0	72	36	30	70	13½
30	H. A. T.	16·4	20	86	57	17	78½	33	58	36	30½	10	69½	30	37	20	71	0	75	53	47	38	70½	0	72	28	43½	36	34
	G. P. W.	16·2	26	241	8	60	23	99	3	82	3	40	8	40	6	70	10½	30	28	104	2½	88	11	10	40½	60	6	60	17
	N. S. W.	14·1	75	164	21	66	18	58	22	40	26½	40	8	30	37	55	26½	40	15½	72	19½	65	41½	20	19	40	26	58	19
	W. J. W.	16·3	23½	90	55	33	58½	34	56½	23	47½	1	80	20	67½	30	57½	20	39	49	53	44	64	10	40½	22	51	32	41
L. VI.	P. B.	14·0	77	61	73	16	80	38	51½	7	67½	6	77	20	67½	10	78½	0	75	54	45½	69	35½	10	40½	22	51	10	69
35	J. P. C.	16·0	29	77	63	53	37	11	82	13	58½	20	47	20	67½	30	57½	40	15½	33	75	45	63	2	59	21	53½	31	43
	F. S. H. F.	15·2	51½	76	64½	35	53½	30	64	11	62½	20	47	30	37	40	45	10	59	34	74	65	41½	0	72	10	70½	18	59½
	S. W. I.	17·11	3	115	40	35	53½	48	34½	32	35½	20	47	40	6	30	57½	0	75	58	36½	68	37½	9	52	30	40½	32	41
	G. L.	16·6	16	104	51	44	43½	31	61½	29	39	33	19½	30	37	40	45	30	28	43	62½	69	35½	0	72	42	22½	42	29
	E. P. M.	17·4	9	132	35	41	45	63	16	28	41½	30	26	30	37	17	74	19	44	73	17½	75	22½	10	40½	44	19	18	59½
40	T. C. M.	17·3	10½	105	49½	55	30	25	69½	25	44½	30	26	30	37	50	34	10	59	48	54	53	56	5	55½	29	42	25	50
	L. P. B. M.	14·10	57½	155	25	35	53½	77	12	43	21½	20	47	20	67½	20	71	10	59	78	13½	77	19	10	40½	41	24½	45	26
	G. M.	15·6	42	162	22	72	14½	51	30	39	28	30	26	30	37	60	21½	20	39	56	40½	58	49	10	40½	38	27	59	18

	C. F. P.	17·10	4½	80	61	28	65½	21	72	31	37	24	37	30	37	29	64	2	66	36	69½	28	76	0	72	36	30	41	30
	H. L. P.	16·3	23½	65	70½	35	53½	28	66	2	78½	33	19½	32	18	33	50	10	59	44	60½	51	58½	0	72	6	77	4	75½
45	S. L. R. S.	18·10	1	66	69	34	56	14	80	18	53	20	47	29	56½	30	57½	0	75	18	82	26	78	0	72	26	45½	17	63
	T. L. S.	14·10	57½	110	44	28	65½	47	37	35	32½	9	74	20	67½	20	71	0	75	60	33½	56	53½	10	40½	31	39	36	34
	E. G. T.	15·9	33	105	49½	38	49½	46	39	21	49½	18	60	20	67½	20	71	16	49	70	22	70	30½	10	40½	22	51	39	31½
	S. H. T.	16·4	20	120	37	44	43½	48	34½	28	41½	30	26	30	37	48	39	18	46	40	64½	38	70½	10	40½	33	35½	30	44
	R. W.	15·6	42	115	40	55	30	40	48	20	51	18	60	19	75	50	34	50	5½	58	36½	61	45	8	53	28	43½	29	45
50	W. T. W.	17·5	8	153	27	77	12	43	42½	33	34	23	38	30	37	52	29	35	21½	46	57½	80	16½	0	72	25	48	23	52
U. V.^{A}	W. C. A.	14·7	64½	65	70½	27	68½	26	68	12	60½	19	56½	30	37	22	68	40	15½	47	55½	58	49	0	72	21	53½	20	56
	K. C. M. A.	14·8	62	83	59	47	41	31	61½	5	72	20	47	30	37	50	34	30	28	35	72½	38	70½	0	72	14	64½	12	68
	D. I. B.	16·1	27	110	44	53	37	34	56½	23	47½	40	8	38	14½	50	34	30	28	50	50	53	56	0	72	25	45	14	66
	A. C. H.	15·7	39½	168	19½	53	37	69	14	46	19	20	47	30	37	50	34	20	39	54	45½	57	52	40	3½	36	30	34	37
55	R. J. J.	15·2	51½	108	46½	40	46½	53	26½	15	57	28	34½	30	37	38	47½	10	59	58	36½	88	11	10	40½	16	59	17	63
	H. M. L.	14·0	77	108	46½	37	51	62	17	9	64	30	26	35	16½	38	47½	14	51	81	11	79	18	20	19	11	67½	5	73
	E. J. L.	13·11	80	76	64½	28	65½	19	76	29	39	13	63½	30	37	30	57½	0	75	24	80	23	80	10	40½	35	33	39	31½
	M. M.	16·7	15	91	54	23	75	44	41	24	46	20	47	19	75	12	76	0	75	61	32	53	56	5	55½	26	45½	32	41
	F. C. M.	14·6	67	177	16	55	30	53	26½	69	7	20	47	23	61	60	21½	30	28	55	43	63	44	20	19	52	13½	36	34
60	R. F. N.	15·11	31	89	56	38	49½	48	34½	3	76½	20	47	20	67½	30	57½	10	59	55	43	47	62	10	40½	6	77	3	78
	C. T. N.	15·3	48½	111	42	54	34	32	59	25	44½	40	8	40	6	60	21½	40	15½	47	55½	56	53½	0	72	25	48	21	53½
	J. G. P.	17·2	12	78	62	18	76½	31	61½	29	39	0	82	10	81	10	78½	13	52	40	64½	58	49	5	55½	32	37½	34	37
	N. P. V. P.	15·5	44½	107	48	48	39½	42	44	17	55	19	56½	30	37	40	45	20	39	62	31	82	15	25	13½	15	61½	26	48½
	F. L. W.	14·9	59½	115	40	71	16	38	51½	6	70	40	8	40	6	55	26½	46	9	50	50	70	30½	2	59	15	61½	4	75½
65	H. E. W.	17·9	6	130	36	62	20	50	31	18	53	29	32½	38	14½	60	21½	20	39	63	30	70	30½	17	24	15	61½	26	48½
U. V.^{B}	L. J. B.	13·4	83	55	75	32	62	16	79	7	67½	18	60	30	37	28	65	0	75	36	69½	40	66½	0	72	17	57½	8	71
	H. B.	13·11	80	50	78	25	72	17	78	8	65	10	69½	30	37	30	57½	0	75	24	80	8	82½	10	40½	14	64½	20	56
	A. L. C.	15·3	48½	48	79	26	70½	20	74	2	78½	10	69½	20	67½	20	71	10	59	36	69½	35	74	2	59	10	70½	4	75½
	H. W. D.	14·8	62	138	33	61	21	59	20	18	53	40	8	39	12½	70	10½	38	19	78	13½	70	30½	20	19	17	57½	18	59½
70	A. C. F.	14·3	73	82	60	46	42	24	71	12	60½	20	47	40	6	50	34	10	59	28	77	28	76	0	72	13	66	13	67
	L. C. G.	16·0	29	45	80½	12	82	29	65	4	74½	2	79	19	75	0	82½	12	53	35	72½	38	70½	0	72	9	73½	9	70
	I. H. G.	15·6	42	72	66	33	58½	36	53	3	76½	20	47	29	56½	30	57½	0	75	36	69½	38	70½	0	72	6	77	7	72
	R. O. G.	14·7	64½	67	68	24	73½	39	50	4	74½	10	69½	30	37	30	57½	0	75	45	59	70	30½	0	72	10	70½	17	63
	V. G. T. H.	14·4	70	33	82	15	81	18	77	0	81½	10	69½	10	81	10	78½	10	59	37	67	48	60½	0	72	0	81½	0	81
75	J. T. H.	14·4	70	63	72	32	62	31	61½	0	81½	3	78	19	75	13	75	18	46	50	50	25	79	12	28½	0	81½	0	81
	B. L.	16·3	23½	53	76½	28	65½	20	74	5	72	13	63½	30	37	30	57½	0	75	27	78	18	81	10	40½	5	79	4	75½
	A. F. L.	15·1	53	30	83	24	73½	6	83	0	81½	10	69½	30	37	30	57½	0	75	10	83	8	82½	0	72	0	81½	0	81
	F. G. B. L.	14·6	67	92	53	33	58½	54	24	5	72	8	75½	19	75	30	57½	20	39	70	22	58	49	10	40½	11	67½	21	53½
	A. M.	14·11	55	45	80½	32	62	13	81	0	81½	10	69½	18	78	25	66½	26	34	24	80	28	76	0	72	0	81½	0	81
80	A. H. R.	14·6	67	53	76½	26	70½	20	74	7	67½	18	60	30	37	25	66½	0	75	39	66	38	70½	10	40½	9	73½	18	59½
	H. C. W.	17·7	7	56	74	8	83	41	45½	7	67½	0	82	10	81	0	82½	0	75	46	57½	76	20½	0	72	10	70½	0	81
	F. B. H. W.	13·9	82	110	44	54	34	40	48	16	56	10	69½	30	37	44	41½	30	28	64	28	58	49	10	40½	18	56	24	51
83	D. H. B.	13·11	80	84	58	18	76½	53	26½	13	58½	8	75½	10	81	10	78½	0	75	64	28	60	46	10	40½	15	61½	16	65

Explanation of letter-headings. A. Marks for memory of definitions and general principles (e.g. principle of superposition) in Geometry. B. Memory of constructions (Geom.). C. Memory of preceding propositions and power of applying them (Geom.). D. Recognition of necessity of generality in proof, and power of recognising general relations in a particular case (Geom.). E. Accuracy in Arithmetic. F. General memory of rules and power of applying them (Arith.). G. Power of doing sums in percentage and proportion (Arith.). H. Accuracy in Algebra. I. General memory of rules and power of applying them (Algebra).

* Note the seemingly great bias of marks for even figures, especially for tens, in columns A, B, C, and D. This is due to the fact that in Geometry the part-capacities marked were such that in most cases they either gained the maximum mark, viz. 10, or failed to obtain any marks at all. See Appendix I, C.

APPENDIX III. *Marks and Ranks**.

Form	Name	Age	Rank	Total Marks	Rank	Geom.	Rank	Arith.	Rank	Alg	Rank	A	Rank	B	Rank	C	Rank	D	Rank	E	Rank	F	Rank	G	Rank	H	Rank	I	Rank
VII 1	R. A. A.	16·3	23½	145	28½	55	30	49	32	41	24	40	8	30	37	44	41½	15	50	72	19½	51	58½	17	24	53	11	70	14½
	A. M. J. B.	15·5	44½	220	9	48	39½	93	6	79	4	18	60	30	37	44	41½	9	65	77	15½	88	11	10	3½	62	4½	86	5
	W. T. C.	14·11	55	212	10½	85	8	65	15	62	10	40	8	27	59	92	4	42	11½	80	12	70	30½	25	13½	70	3	87	4
	W. K. C.	14·9	59½	253	5	132	1	59	20	62	10	48	2	40	6	127	1	78	2	60	33½	84	14	24	15	53	11	88	3
5	C. K. D.	15·8	31½	256	3	125	2	94	5	37	29	50	1	30	37	120	2	79	1	101	5	91	9	37	7½	41	24½	45	26
	H. A. E. J.	17·3	10½	168	19½	68	17	52	29	48	18	20	47	40	6	60	21½	30	28	65	25½	87	13	20	19	45	33	57	20
	L. P. F.	16·0	29	178	15	93	5	53	26½	32	35½	30	26	40	6	80	7	60	3	58	36½	80	16½	5	55½	30	40½	33	39
	H. B. H.	14·3	73	159	23	58	26	60	18	41	24	30	26	30	37	60	21½	40	15½	56	10½	93	8	20	19	42	22½	27	17
	A. G. L.	18·6	2	242	6½	81	9	87	9	71	5	30	26	28	58	70	10½	30	28	73	17½	76	20½	40	3½	46	16½	73	11
10	S. S. M.	15·4	46	242	6½	80	10½	96	4	66	8	38	15½	39	12½	69	13	30	28	104	2½	108	6	35	9½	58	7½	75	10
	R. H. P.	15·8	36½	187	14	80	10½	80	10	27	13	38	15½	40	6	86	5	37	20	85	10	75	22½	47	7½	33	35½	45	26
	D. W. R. B.	15·7	39½	212	10½	64	19	91	7	57	12	40	8	40	6	63	16	18	46	86	9	105	7	40	3½	53	11	69	15
	A. S.	16·4	20	284	1	111	3	78	11	95	2	39	14	30	37	95	3	58	1	88	8	115	5	40	3½	71	2	100	1
	E. J. S.	14·11	55	154	26	54	34	59	20	41	24	20	17	30	37	60	21½	30	28	77	15½	65	41½	12	28½	32	37½	16	24½
15	A. C. T.	17·10	4½	176	17½	74	13	16	39	56	13	30	26	30	37	50	34	30	28	67	24	71	26	16	26	55	9	77	9
U. VI.	S. M. A.	15·3	48½	102	52	27	68½	35	54½	40	26½	20	17	30	37	35	49	0	75	43	62½	48	60½	10	40½	44	19	56	21½
	O. D. A.	14·4	70	199	13	56	27	90	8	53	14½	30	26	30	37	50	34	18	7½	93	6	121	2	17	24	58	7½	72	12
	C. E. B.	14·0	77	266	2	55	30	112	2	99	1	29	32½	22	62	60	21½	20	39	103	4	117	3	30	11	79	1	98	2
	H. A. D.	15·8	36½	145	28½	59	25	35	54½	51	16	28	34½	40	6	65	14½	45	10	44	60½	43	65	10	40½	46	16½	83	8
20	C. J. F.	16·5	17½	209	12	89	6	71	13	49	17	35	17	35	16½	80	7	18	7½	89	7	116	4	35	9½	50	15	46	23½
	W. F.	15·9	33	136	34	33	58½	41	45½	62	10	12	65	20	67½	32	51	20	39	50	50	66	39	0	72	44	19	84	6½
	M. A. H.	16·11	13½	117	38	39	48	25	69½	53	14½	26	36	20	67½	44	41½	17	48	32	76	40	66½	0	72	43	21	56	21½
	F. R. L.	14·3	73	158	24	96	4	27	67	35	32½	40	8	25	60	80	7	42	11½	50	50	74	24½	27	12	52	13½	66	16
	G. C. M.	16·11	13½	142	30½	60	23	46	39	36	30½	30	26	30	37	51	28	20	39	55	43	70	30½	10	40½	37	28	43	28
25	J. C. T. R.	15·9	33	176	17½	60	23	43	42½	73	6	34	18	30	37	62	17	35	21½	65	25½	68	37½	10	40½	62	4½	84	6½
	L. T. S.	16·5	17½	141	32	72	14½	48	34½	21	49½	20	47	30	37	70	10½	50	5½	64	28	65	41½	15	27	20	55	34	47
	C. M. S.	15·3	48½	68	67	17	78½	40	48	11	62½	0	82	10	81	2	81	10	59	70	22	74	21½	20	19	8	75	20	56
	R. M. S.	15·8	36½	254	4	88	7	123	1	43	21½	40	8	30	37	65	14½	40	15½	118	1	128	1	40	3½	35	33	28	46
	H. S.	14·8	62	142	30½	40	46½	57	23	15	20	20	47	30	37	50	34	10	59	57	39	70	30½	0	72	36	30	70	13½
30	H. A. T.	16·4	20	86	57	17	78½	33	58	36	30½	10	69½	30	37	20	71	0	75	53	47	38	70½	0	72	28	13½	36	34
	G. P. W.	16·2	26	241	8	60	23	99	3	82	3	10	8	40	6	70	10½	30	28	104	2½	88	11	10	40½	60	6	60	17
	N. S. W.	14·1	75	164	21	66	18	58	22	40	26½	40	8	30	37	55	26½	40	15½	72	19½	65	41½	20	19	40	26	58	19
	W. J. W.	16·3	23½	90	55	33	58½	34	56½	23	17½	1	80	20	67½	40	57½	20	39	49	53	44	64	10	40½	22	51	32	41
L. VI.	P. B.	14·0	77	61	73	16	80	38	51½	7	67½	6	77	20	67½	10	78½	0	75	54	45½	69	35½	10	40½	22	51	10	69
35	J. P. C.	16·0	29	77	63	53	37	11	82	13	58½	20	47	20	67½	30	57½	40	15½	33	75	45	63	2	59	21	53½	31	43
	F. S. H. F.	15·2	51½	76	64½	35	53½	30	64	11	62½	20	47	30	37	40	45	10	59	34	71	65	41½	0	72	10	70½	18	59½
	S. W. I.	17·11	3	115	10	35	53½	48	34½	32	35½	20	47	40	6	30	57½	0	75	58	36½	68	37½	9	52	30	10½	32	41
	G. L.	16·6	16	104	51	44	43½	31	61½	29	39	33	19½	30	37	40	45	30	28	43	62½	69	35½	0	72	42	22½	42	29
	E. P. M.	17·4	9	132	35	41	45	63	16	28	41½	30	26	30	37	17	74	19	44	73	17½	75	22½	10	40½	44	19	18	59½
40	T. C. M.	17·3	10½	105	49½	55	30	25	69½	25	44½	30	26	30	37	50	34	10	59	48	54	53	56	5	55½	29	42	25	50
	L. P. B. M.	14·10	57½	155	25	35	53½	77	12	43	21½	20	47	20	67½	20	71	10	59	78	13½	77	19	10	40½	11	24½	45	26
	G. M.	15·6	42	162	22	72	14½	51	30	39	28	30	26	30	37	60	21½	20	39	56	40	58	49	10	40½	38	27	59	18
	C. F. I.	17·10	4½	80	61	28	65½	21	72	31	31	24	37	[illegible]	[illegible]	27	64	2	66	36	69½	28	76	0	72	36	30	41	30
	H. L. P.	16·3	23½	65	70½	35	53½	28	66	2	76½	33	19½	32	18	33	50	10	59	44	60½	51	58½	0	72	6	77	4	75½
45	S. L. R. S.	18·10	1	66	69	34	56	14	80	18	53	20	17	29	56½	30	57½	0	75	18	82	26	78	0	72	26	15½	17	63
	T. L. S.	14·10	57½	110	44	28	65½	47	37	35	32½	9	74	20	67½	20	71	0	75	60	33½	56	53½	10	40½	31	39	36	34
	E. G. T.	15·9	33	105	49½	38	49½	46	39	21	49½	18	60	20	67½	20	71	16	49	70	22	70	30½	10	40½	22	51	39	31½
	S. H. T.	16·4	20	120	37	44	43½	48	34½	28	41½	30	26	30	37	48	39	18	46	40	64½	38	70½	10	40½	33	35½	30	44
	R. W.	15·6	12	115	40	55	30	10	48	20	51	18	60	19	75	50	34	50	5½	58	46½	61	45	8	53	28	43½	29	15
50	W. T. W.	17·5	8	153	27	77	12	43	42½	33	34	23	38	30	37	52	29	35	21½	46	57½	80	16½	0	72	25	48	23	52
U. V^A	W. C. A.	14·7	64½	65	70½	27	68½	26	68	12	60½	19	56½	30	37	22	68	40	15½	47	55½	58	49	0	72	21	53½	20	56
	K. C. M. A.	14·8	62	83	59	47	41	31	61½	5	72	20	47	30	37	50	34	30	28	35	72½	38	70½	0	72	14	64½	12	68
	D. I. B.	16·1	27	110	44	55	37	34	56½	23	47½	40	8	38	14½	50	34	30	28	50	50	53	56	0	72	25	45	14	66
	A. C. H.	15·7	39½	168	19½	53	37	69	14	46	19	20	47	30	37	50	34	20	39	54	45½	57	52	40	3½	36	30	34	37
55	R. J. J.	15·2	51½	108	46½	10	46½	53	26½	15	57	28	34½	30	37	38	47½	10	59	58	36½	88	11	10	40½	16	59	17	63
	H. M. L.	14·0	77	108	46½	37	51	62	17	9	64	30	26	35	16½	38	47½	14	51	81	11	79	18	20	19	11	67½	5	73
	E. J. L.	13·11	80	76	64½	28	65½	19	76	29	39	13	63½	30	37	30	57½	0	75	24	80	23	80	10	40½	35	33	39	31½
	M. M.	16·7	15	91	54	23	75	44	41	24	46	20	47	19	75	12	76	0	75	61	32	53	56	5	55½	26	45½	32	41
	F. C. M.	14·6	67	177	16	75	30	53	26½	69	7	20	47	23	61	60	21½	30	28	55	13	63	44	20	19	52	13½	36	34
60	R. F. N.	15·11	31	89	56	38	49½	48	34½	3	76½	20	47	20	67½	30	57½	10	59	55	43	47	62	10	40½	6	77	3	78
	C. T. N.	15·3	48½	111	42	54	34	32	59	25	44½	40	8	40	6	60	21½	40	15½	47	55½	56	53½	0	72	25	48	21	53½
	J. G. P.	17·2	12	78	62	18	76½	31	61½	29	39	0	82	10	81	10	78½	13	52	40	64½	58	49	5	55½	32	37½	34	37
	N. P. A. P.	15·5	11½	107	48	48	39½	42	44	17	55	19	56½	30	37	40	45	20	39	62	31	82	15	25	13½	15	61½	26	48½
	F. L. W.	14·9	59½	116	40	71	16	38	51½	6	70	40	8	40	6	55	26½	46	9	50	50	70	30½	2	59	15	61½	4	75½
65	H. E. W.	17·9	6	130	36	62	20	50	31	18	53	29	32½	38	14½	60	21½	20	39	63	30	70	30½	17	24	15	61½	26	48½
U. V^B	L. J. B.	13·4	83	55	75	32	62	16	79	7	67½	18	60	30	37	28	65	0	75	36	69½	40	66½	0	72	17	57½	8	71
	H. B.	13·11	80	50	78	25	72	17	78	8	65	10	69½	30	37	30	57½	0	75	24	80	8	82½	10	40½	14	64½	20	56
	A. L. C.	15·3	48½	48	79	26	70½	20	74	2	78½	10	69½	20	67½	20	71	10	59	36	69½	35	74	2	59	10	70½	4	75½
	H. W. D.	14·8	62	138	33	61	21	59	20	18	53	40	8	39	12½	70	10½	38	19	78	13½	70	30½	20	19	17	57½	18	59½
70	A. C. F.	14·3	73	82	60	46	42	24	71	12	60½	20	47	40	6	50	34	10	59	28	77	28	76	0	72	13	66	13	67
	L. C. G.	16·0	29	45	80½	12	82	29	65	4	74½	2	79	19	75	0	82½	12	53	35	72½	38	70½	0	72	9	73½	9	70
	I. H. G.	15·6	42	72	66	33	58½	36	53	3	76½	20	47	29	56½	30	57½	0	75	36	69½	38	70½	0	72	6	77	7	72
	R. O. G.	14·7	64½	67	68	24	73½	39	50	4	74½	10	69½	30	37	30	57½	0	75	45	59	70	30½	0	72	10	70½	17	63
	V. G. T. H.	14·4	70	33	82	15	81	18	77	0	81½	10	69½	10	81	10	78½	10	59	37	67	48	60½	0	72	0	81½	0	81
75	J. P. H.	14·4	70	63	72	32	62	31	61½	0	81½	3	78	19	75	13	75	18	46	50	50	25	79	12	28½	0	81½	0	81
	B. L.	16·3	23½	53	76½	28	65½	20	74	5	72	13	63½	30	37	30	57½	0	75	27	78	18	81	10	40½	5	79	4	75½
	A. F. L.	15·1	53	30	83	24	73½	6	83	0	81½	10	69½	30	37	30	57½	0	75	10	83	8	82½	0	72	0	81½	0	81
	F. G. B. L.	14·6	67	92	53	33	58½	54	24	5	72	8	75½	19	75	30	57½	20	39	70	22	58	49	10	40½	11	67½	21	53½
	A. M.	14·11	55	45	80½	32	62	13	81	0	81½	10	69½	18	78	25	66½	26	34	24	80	28	76	0	72	0	81½	0	81
80	A. H. R.	14·6	67	53	76½	26	70½	20	74	7	67½	18	60	30	37	25	66½	0	75	39	66	38	70½	10	40½	9	73½	18	59½
	H. C. W.	17·7	7	56	74	8	83	41	45½	7	67½	0	82	10	81	0	82½	0	75	46	57½	76	20½	0	72	10	70½	0	81
	F. B. H. W.	13·9	82	110	44	54	34	40	48	16	56	10	69½	30	37	44	41½	30	28	64	28	56	49	10	40½	18	56	24	51
83	D. H. B.	13·11	80	84	58	18	76½	53	26½	13	58½	8	75½	10	81	10	78½	0	75	64	28	60	46	10	40½	15	61½	16	65

Explanation of letter-headings. A. Marks for memory of definitions and general principles (e.g. principle of superposition) in Geometry. B. Memory of constructions (Geom.). C. Memory of preceding propositions and power of applying them (Geom.). D. Recognition of necessity of generality in proof, and power of recognising general relations in a particular case (Geom.). E. Accuracy in Arithmetic. F. General memory of rules and power of applying them (Arith.). G. Power of doing sums in percentage and proportion (Arith.). H. Accuracy in Algebra. I. General memory of rules and power of applying them (Algebra).

* Note the seemingly great bias of marks for even figures, especially for tens, in columns A, B, C, and D. This is due to the fact that in Geometry the part-capacities marked were such that in most cases they either gained the maximum mark, viz. 10, or failed to obtain any marks at all. See Appendix I, C.

DARWINISM, BIOMETRY AND SOME RECENT BIOLOGY. I.

BY KARL PEARSON, F.R.S.

(1) THE principle of the "survival of the fitter" as the basis of progressive evolution depends upon the co-existence of three factors, (i) the destruction, or elimination as far as reproduction is concerned, of those less fitted to their environment, (ii) the inheritance of the somatic characters on which the fitness depends, and (iii) the absence of any differential fertility. These essential principles of natural selection are frequently overlooked not only by sociologists but by biologists, and whenever they are disregarded, there an inroad of a serious nature has been made into the Darwinian theory of evolution. It is possible to illustrate this point in a variety of ways. Thus Nature in the struggle for existence in wild life, regards and can regard only somatic characters. A somatic character which is not inherited in a greater or lesser degree has no value in the Darwinian theory of evolution. Again, a genetic constitution which does not manifest itself in somatic characters, i.e. is not directly or indirectly correlated with some one or other somatic character, cannot form the subject of Darwinian evolution. To what extent are patent qualities inherited? This must always be the fundamental problem of any Darwinian interpretation of living forms. And—above all in populations where selective mating is small—this problem must and will remain in the first place a statistical problem. The knowledge of an universal physiological law of heredity would immensely aid artificial breeding; it would essentially aid civilized man in the generations to come, when his knowledge dictates and controls his emotional needs*. But from the pure standpoint of Darwinian evolution, the fundamental problem remains the intensity of the transmission of somatic characters in the

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* There is not the least doubt that good physique generally and that mental capacity in many cases, can in the present state of human development have sexual attraction; deformity and imbecility produce sexual repulsion. With greater and more widely spread knowledge of genetics, the gametic constitution of an individual as roughly manifested in the average somatic characters of other members of the stock may well come to be a potent emotion-controlling factor of a practically unconscious kind. Even to-day many human beings would, as far as matters of sex are concerned, experience revulsion rather than attraction from an individual whose relatives were physically, mentally or socially defective. There is no doubt that this emotional state will be emphasised the more we are trained from childhood to give the proper relative weights to environment and heredity.

general population of any species. Those who desire to test the efficiency of Darwinian theory must examine nature in the first place from this standpoint.

(2) Again, recent work on evolution seems to have very largely disregarded the direct possibility of testing in wild life, or under artificial conditions, the destruction of the less fit. The correlations between death-rate and environment, and between somatic characters and death-rate, are essentially problems which can be treated by biometric methods, and which are vital to further progress in our study of evolution. It is true that human vital statistics present an immense material for the relationship of death-rate to environment; but they have not hitherto provided the much needed measures of the somatic characters of the selected. What is much needed is a widening of the breeder's outlook,—that he should not only find what is inherited, but that the age at death and the fertility of his stud should be recorded with special relation to the somatic characters of its members, and to marked differences in environment. Light, warmth, food are all in a high degree within his power of differentiation, and his greatest difficulty, environmental growth changes, ought to be surmountable by the use of standard populations. For the purpose of evolutionary study, once the intensity of heredity has been determined, the physiological process of heredity, important as it is for other branches of science, is secondary; the next important step is the correlation of death-rate with somatic characters. I venture to think that more experimental work than has been recently attempted might well be undertaken in this direction.

(3) Lastly I would refer to the third important factor of evolution—the absence of a differential fertility. This point, it seems to me, we again and again overlook. It is extremely hard to believe that fertility could by any mechanism come to be highly correlated with the peculiar type of character fittest to survive in a given environment. The only easy way to suppose it lies in the case of those species in which length of life is at once a measure of fitness and a measure of fertility. We cannot apply this consideration, however, to the case of species which breed only once. If on the other hand fertility were inherited, and were at the same time correlated with other characters, it is difficult to believe that a selective death-rate could perform its functions, except in the case where the maximum of fertility coincided with the optimum of the selected characters. Personally, the truth that natural selection presupposes the practical absence of genetic selection, has only gradually been forced upon me by the discovery of case after case in which there was little or no demonstrable inheritance of fertility. I began with the conception that fertility would be found to be a markedly hereditary character; I expected to find genetic selection masking or even reversing natural selection. Then, as very small or even insensible values came out for man, horse, swine and mice, I have been forced to the conclusion that the smallness of the hereditary factor in fertility is an essential feature of Darwinian evolution. I should be surprised to find a large inheritance of fertility now, just as I should be surprised not to find a large inheritance of any somatic character. And on the

evidence we have at present, I should not expect to find a correlation between fertility and a somatic character; if it does exist in any case, I should anticipate very rapid changes going on in that species.

(4) One of the most recent investigations in this direction is that of Raymond Pearl on the inheritance of fecundity in Barred Plymouth Rocks. His paper on the number of eggs of this race is an extremely interesting one. It is true that he only gives the crude correlations and does not give the data which would be necessary for their correction. Thus he had 31 mothers* and 180 daughters, and some of his "arrays" were probably due to a *single* mother bird, but this cannot be ascertained from his Tables. In working at fertility in man I found exceedingly small values, until correction was made for weighting of the more fertile mother, and then the values appeared to be small but appreciable. If we put aside, however, the small number of mothers used by Pearl and this question of correction, there cannot be the least doubt that Pearl's data show a low, if not zero, intensity of heredity in fertility, thus confirming in poultry what we already know of many mammals, i.e. the absence of inheritance in the case of fertility which seems a necessary foundation of the idea of Darwinian evolution.

Pearl does not take this zero correlation as confirmation of existing results for fertility. He considers it in some way a confirmation of the pure line theory of Johannsen. We are sorry that we cannot in the least agree with him on this point. For if his view were correct, every high correlation would tell against such a theory, which it certainly does not; and in the case of mammals, where the fertility correlations are zero and the body characters highly correlated between parents and offspring, one correlation would refute, another confirm the pure line theory in the same species. Nor again can data taken from measurements of a character highly influenced by environment be in the least conclusive, if they are handled in the manner recently adopted by Jennings, who selected a few individuals with large values of a character and compared their offspring with the general population. There is no security whatever that the individuals selected did not have excess of character owing to environmental and not to hereditary conditions. Such sweeping and dogmatic statements as those made by Jennings that: "The 'standard deviation' and 'coefficient of variation' express in a pure race mere temporary conditions of no consequence in heredity" merely beg the question of what a pure race may be, or else show a sad ignorance of what work has been done on this very point.

The only manner in which it appears to me that an answer can be found to the question which Pearl and Jennings take as now settled—that is to say, the problem of whether or no selection within the individuals descended from a single unit can or cannot produce an effect—is to correlate ancestry with offspring in species in which self-fertilisation or parthenogenesis takes place. If this be done, then the relation of offspring to parent ought to be identical with the relation of offspring to grandparent. Warren and Johannsen himself[*] are the only investigators who, so far as I am aware, have hitherto provided material in sufficient mass for approximately answering such a question, and what do we find:

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* Nothing short of 500 mothers would really give a convincing result, I think, where the actual correlation is so extremely small. See Pearl, R. and Surface, F. M.: "Data on the Inheritance of Fecundity obtained from the Records of Egg Production of the Daughters of '200-Egg Hens'." *Maine Agricultural Experiment Station, Bulletin*, No. 166, 1909.

Johannsen: *Phaseolus vulgaris.*

	Correlation	Regression
Offspring and parent	·35 ± ·01	·27
Offspring and grandparent	·24 ± ·01	·11
Warren: *Daphnia.*		
Offspring and parent	·47 ± ·05	·62
Offspring and grandparent	·27 ± ·12	·50
Warren: *Hyalopterus.*		
Offspring and parent	·40 ± ·03	·47
Offspring and grandparent	·24 ± ·04	·25

In every one of these cases, although, as the probable errors show, not based on sufficient numbers to be absolutely final, the parental is higher than the grandparental correlation. The ratio of the two is almost exactly the 3/5 found for the same ratio in mammals with bisexual generation. Pearl and Jennings put such results—and many others of the Biometric School—on one side, without even reference; on what bases? The former on the foundation that fertility is not inherited in hens—an interesting result but one that might be predicted from what we know of mice and swine. The latter because he has failed under the method adopted by him to discover inheritance in *Paramecium*. From that he argues not only to the whole range of possible heredity in Paramecium but apparently in every living form, whatever its mode of reproduction. The fundamental condition of a crucial experiment of this character must be to show (1) on a sample of sufficient size that the character is inherited at all, and (2) that if it be inherited the relationship between offspring and grandparent is as intense as between offspring and parent. Whether the pure line theory be true or not, it cannot be demonstrated either by showing that a fertility correlation is zero, or by the mere *ipse dixit* of Jennings that "'variations' are mere temporary fluctuations, without effect in heredity, so that their relation to evolution is *nil*."

No biometrician asserts that *every* variation is of hereditary value; on the contrary he asserts the very opposite whenever he tables a correlation less than perfect in parthenogenetic reproduction. What he does assert is that on the average such variations form in a measurable degree the mark of a certain associated gametic character. He would not have thought it worth while to try and determine the correlation of gametic and somatic characters[†] had he believed every individual variation capable of direct inheritance. Nor would his results be in the least upset if—and some theorists in heredity seem coming to that—every individual were in himself a "pure line." He might mildly wonder how all these pure lines came into existence in a single local race on any philosophical scheme of evolution; but it would not affect his treatment of inheritance in populations. Why then should he criticise the work of Johannsen, Hanel or Jennings on "pure lines"? Simply because the experiments made by these biologists are in his opinion insufficient to provide a basis for the sweeping inferences drawn from them, and further because they contradict other biological work, which he thinks just as important and valid. The assumption made by the pure linist is that in reproduction from a single individual the same gametic constitution is transferred to every offspring. The biometrician sees no sufficient justification of such a view in our present state of knowledge; two seeds from the same self-fertilised plant may differ in gametic constitution, and this differentiation may appear in the correlated somatic characters. Two Aphides from the same mother, according to Warren, do show such differentiation. Does or does not differentiation follow the division in *Paramecia*? We do not know, because no heredity of the "pure lines" has yet been shown to exist for the character selected by Jennings in the only way that would satisfy a statistician, i.e. a correlation table of the pure line parents with their offspring. On the other hand, having convinced himself that size of litter, for example, is not inherited to any appreciable extent in mice, but that colour and area of colouring are, he is not likely to accept the conclusion of Pearl, based on the non-inheritance of fertility in hens, that "the chief if not the entire function of selection in breeding is to *isolate* pure strains from a mixed population. It is found in actual experience impossible to bring about by selection improvement beyond a point already existing in the pure (isolated) strain at the beginning" (*loc. cit.* p. 79). It is difficult to understand where in the pure strain at the beginning there existed the chief characteristics of any of our domestic animals of to-day. Such statements appear too sweeping in the light of any evidence at present available; they discard the Darwinian spirit of slow and patient inquiry, at the same time that they destroy the Darwinian theory of selective evolution*.

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[*] Elise Hanel's results for *Hydra grisea* will be discussed at length below; the character of the selections made prevents full comparison here.

[†] See *Biometrika*, Vol. VI., pp. 90, 91.

That selection in a *particular direction* cannot go beyond certain limits is of course a commonplace of breeders, indeed it was illustrated at a very early stage by one of the first of biometricians, Sir Francis Galton himself, in his discussion on organic stability. The selection of any one character changes all correlated characters, and a condition is rapidly reached at which the organism is either incompatible with its environment, or possibly its germinal determinants become unstable and assume a new form of equilibrium.

(5) While Jennings himself, as I have said, provides us with no data upon which we can test (i) whether the character he is dealing with is really inherited in *Paramecia*, (ii) whether in an *unselected* population the degree of resemblance of parent to offspring is no greater than that of grandparent—while in fact both Pearl and he appear to confuse non-heredity of a character with pure line heredity

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* Pearl, R. and Surface, F. M.: "Is there a cumulative effect of Selection?" *Zeitschrift für induktive Abstammungs- und Vererbungslehre*, Bd. II., S. 257, 1909, answer a very big question from the same egg data.

—yet he has given his approval and sanction on two separate occasions[*] to a memoir by Elise Hanel[†] on asexual propagation in *Hydra grisea.* We have thus a very fair test of what his judgment on a statistical proof of "pure line" heredity is worth.

I propose to discuss at some length Hanel's paper, because it seems to me that no definite conclusions as to "pure lines" can be obtained by investigations of this type, until the fundamental features of the research are modified and the observations treated with an adequate theoretical knowledge of statistics. Hanel begins with a very careful investigation of the growth and environmental changes in the character selected—the number of tentacles in *Hydra grisea.* There is general agreement with Parke's results that the number of tentacles changes with age, size, food and place of culture. Differences in these factors can produce very considerable differences in individuals, and differences in the averages of differentially treated groups, which can amount to as much as 0·5 to 0·8 of a tentacle. These are precisely of the order of the average hereditary differences. Thus:

PARENT		OFFSPRING	
Number of Individuals	Number of Tentacles	Number of Individuals	Average of Tentacles
9	6	364	6·943
9	7	310	7·296
4	8	166	7·344
4	9	125	7·383

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[*] "Heredity, Variation and Evolution in Protozoa" II., *Proc. Amer. Phil. Soc.* Vol. XLVII. 1908, p. 521, Jennings definitely asserts that Johannsen in 1903 showed that in beans and barley selection within the pure line had no effect. He then proceeds to state that Hanel has found the same state of affairs—no selection within the pure line will produce an effect—in Hydra. In a second very dogmatic paper "Heredity and Variation in the Simplest Organisms," *The American Naturalist*, Vol. XLIII. 1909, p. 332, Jennings writes: "Work with 'pure lines'—where no intercrossing of races or individuals occurs—is possible with few organisms, and little of it has been done. In the few investigations carried on in this way, the same conditions have been found that we have set forth above for Paramecium. They were first shown by Johannsen to hold for beans and barley, and later by Elise Hanel for Hydra. The fact that there exist diverse races, tending to breed true, has of course been shown for many species, but in most cases it is difficult to maintain pure lines, and thus to *absolutely* demonstrate the relations above set forth, as has been done for beans, barley, Hydra and Paramecium." The italics are mine. He also cites Barber "On Heredity in certain Micro-Organisms," *The Kansas University Science Bulletin*, IV. 1907, pp. 1—48. Now several comments are needed here. Jennings directly overlooks the researches of the Biometric school, which was first to work in the field of pure lines, and actually found (see Warren's papers, *R. S. Proc.* Vol. 65, 1899; *Biometrika*, Vol. I. p. 128, 1901) that variations within the line *were* inherited. The same result appears to flow from Johannsen's and Hanel's own experiments, but the characters they have chosen are so influenced by environment that they have not observed the fact. It does not flow from Jennings' or Barber's experiments because they have not demonstrated that heredity exists at all for the characters selected. In fact Jennings repeatedly (and Hanel also) illustrates how markedly environment dominates the characters taken as a measure of heredity. Jennings then assumes that by selecting a character, the heredity of which he has never demonstrated, he can reach a general and "absolute demonstration" of the truth of the theory of pure lines!

[†] "Vererbung bei ungeschlechtlicher Fortpflanzung von Hydra grisea," *Jenaische Zeitschrift*, Bd. XLIII. 1908, pp. 322—372.

It will be at once recognised that the differences here are rather less than many of the environmental differences, and there is no security whatever that these 26 foundation stock Hydra are really represented by differentiated hereditary numbers of tentacles. Yet this table as it stands embraces Hanel's proof that number of tentacles is an hereditary character in the "pure line." What evidence is there that any one of the numbers of tentacles attached to those 26 parents is truly constitutional and not environmental? In every generation that follows Hanel selects from each line two or three individuals to be parents of the following generation. In selecting a few isolated individuals in each generation, where non-hereditary influences are so influential, we may break the effect of heredity at each step, and since such influences are equally effective with heredity, the chances are that we shall do so once in every *two* selections. Only by taking large numbers of the high and large numbers of the low, would it have been possible to average out the effect of environmental changes. When the line is carried through two or three selected individuals only, there is no certainty that the non-hereditary variations will average out. Of course this statement proceeds on the assumption that heredity is small in this case compared with other factors. Table I gives Hanel's data for the 26 original parents and their immediate offspring*. The statistical constants obtained are†:

TABLE I.

Number of Tentacles of Stem Parent and Offspring.

Number of Tentacles of Stem Parent.

Number of Tentacles of Offspring.	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	Totals
5	1	—	—	—	—	—	—	—	1
6	24	81	54	25	12	1	—	2	199
7	70	139	136	75	36	8	—	8	472
8	23	44	93	50	22	1	—	14	247
9	4	4	23	14	8	6	—	—	59
10	—	—	4	1	2	3	—	1	11
11	—	—	—	1	—	—	—	—	1
Totals	122	268	310	166	80	19	—	25	990

	Parents (26)	Offspring (990)	Correlation	Regression
Mean	6·996	7·201	·230	·138
Standard Deviation	1·435	·862	± ·011	± ·013

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* A protest must here be raised against the manner in which this author's data are tabled. No proper description is given of the tables, and it is quite impossible to pick out the pedigree of any individual beyond the parent. Further, instead of the frequencies of offspring with each number of tentacles being given, the total number of offspring is stated and the percentages with each number of tentacles. A laborious process had therefore to be undertaken to replace Hanel percentages by the original frequencies. It was then found that many of this author's percentages must be wrongly calculated, for their totals came to 111, 93, etc., instead of 100, while in other cases it was impossible to determine where one or more individuals should be placed on the basis of the percentages given. It is impossible to make anything of a return in which 13 individuals are said to occur in percentages of 15, 31, 40 and 8! On this account 21 families had to be dropped, out of 245 for which data are provided.

† I have to thank most heartily Miss Ethel M. Elderton for the whole of the arithmetical reductions of Hanel's data.

Now these results bring out several interesting points. It is doubtful whether the probable error of the correlation ought to be calculated on 990 cases and not rather on the number of parents, 26. Further, these parents are far from a random sample; the S.D. as compared with that of the offspring shows that the parents have been selected with markedly high and low numbers of tentacles. The regression has thus more meaning than the correlation*. We see then that tentacles in Hydra are really inherited, but with a value ·138, only about a third to a fourth of the intensity of heredity in many other characters in other species—compare Warren's values in the case of *Daphnia* and *Hyalopterus*. Let us see exactly what this means: it signifies that if we were to take a species in which heredity was as marked as in most characters in man, or as marked as in Daphnia, we should have to mix it with two to three times as many pairs in which there was no resemblance between the character in order to weaken down the measure of hereditary resemblance to that manifested by Hanel's *Hydra*. In other words, a mixture in which one-third only exhibited normal hereditary values of the character and two-thirds characters having no hereditary significance, would produce the same intensity of heredity as is manifested in this material†. What chance, we may ask, has any selection of two or three individuals at most out of each family of 20 to 40, got of exhibiting continuous selective influence?

To test the generality of the above results a Table was now formed (Table II) giving the correlation between all available parents and their offspring. We find:

	Parents (224)	Offspring (6436)	Correlation	Regression
Mean	7·234	7·275	·182	·135
Standard Deviation	1·225	·908	± ·008	± ·006

Thus we clearly see when we pass from the 26 foundation stock to the available 224 parents, that the material has not been so markedly selected, but parents are

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* The regression would not be changed by the selection of parents, and would represent the correlation in a stable population.

† The low value of the correlation between parent and offspring is undoubtedly due in chief part to the variation with growth in the number of tentacles. This point has been demonstrated by Albrecht Hase in an interesting memoir ("Ueber die deutschen Süsswasser-Polypen Hydra," *Archiv für Rassen- und Gesellschafts-Biologie*, Jg. VI. S. 721). He shows that the average number of tentacles increases in *Hydra grisea* from the first day of separation as follows:

1st Day	20th Day	50th Day	End of Culture
6·14	6·71	8·42	9·14

It is clear that the hereditary factor must be obscured by such growth changes. But I do not think Hase is correct in his conclusion that the number of tentacles has *no* hereditary value; my analysis shows that the hereditary value is there, but much obscured. Hase holds that Hanel's experiments have shown complete regression in the line, because she is dealing with a character which has no "Erblichkeitswert," and that she has only thus obtained results in accordance with Johannsen's. I am entirely in agreement with him, that a first condition of any experimenting on "pure line" material, is to show that the character chosen has "Erblichkeitswert." This both Pearl and Jennings have failed to do. But Hanel's data do exhibit "Erblichkeitswert," if only in a very attenuated degree. But to the same degree they show, what Hanel and Jennings imagine they do not show, that regression does take place within the pure line, and that properly instituted selection would produce effect within the limits of the pure line. There is a similar attenuated "Erblichkeitswert" in Johannsen's own material which shows in a similar manner to Hanel a partial regression within the pure line.

still somewhat selected material as compared with the offspring. Taking, however, the regression coefficient as less influenced by such selection, we see that the intensity of heredity is essentially the same for all parents as for the foundation stock parents, i.e. ·135 and ·138.

TABLE II.

Number of Tentacles of Parent and Offspring.

Number of Tentacles of Parent.

Number of Tentacles of Offspring.	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	Totals
5	1	—	—	—	—	—	—	—	1
6	38	436	440	290	81	20	5	8	1318
7	86	862	875	724	254	81	5	20	2907
8	33	403	501	493	184	67	4	20	1705
9	4	75	114	134	62	25	—	11	425
10	—	—	10	13	13	13	—	1	50
11	—	—	2	12	9	7	—	—	30
Totals	162	1776	1942	1666	603	213	14	60	6436

We have now to consider the next stage, which presents considerable difficulty, for Hanel has not preserved all the offspring in order to reach a random sample of the second generation, but preserved two or three from each group only. We took however all the "grandchildren" of the original foundation stock with the following result:

TABLE III.

Number of Tentacles of Stem Grandparent and Offspring.

Number of Tentacles of Stem Grandparent.

Number of Tentacles of Offspring.	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	Totals
6	15	149	157	47	28	14	—	18	428
7	61	243	249	149	92	10	—	26	830
8	44	116	121	124	66	6	—	22	499
9	14	32	29	22	15	2	—	1	115
10	—	—	4	2	1	1	—	—	8
11	—	—	4	—	—	—	—	—	4
Totals	134	540	564	344	202	33	—	67	1884

	Grandparents (26)	Offspring (1884)	Correlation
Mean	7·199	7·181	·030
Standard Deviation	1·288	·884	± ·016

Now on the pure line theory the resemblance of the offspring to the parent ought to be no greater than it is to the grandparent. It is here more than *four times as great.* Another very good test of the "pure line," it occurred to me, might further be used to illustrate this point. If the theory of the pure line hold for *Hydra,* the relation of the offspring to the whole fraternity of which the parent is a member should be as close as to the parent itself—either equally represents the character of the pure line. This method, as far as I can see, might be very profitably used to test pure line theories, because the parent will generally have been reared in the same environment as its co-fraternity*. Table IV has then been formed giving the relation between offspring and each member of the parents' co-fraternity. We have :

TABLE IV.

Number of Tentacles of Member of Parents' Fraternity† and of Offspring of Parents.

Number of Tentacles of Parents' Brethren.

Number of Tentacles of Offspring.	*5*	6	*7*	*8*	9	*10*	*11*	Totals
6	4	3824	7681	3143	384	70	8	15114
7	15	6370	17869	7980	1587	218	25	34064
8	9	3322	10258	5919	1102	175	22	21807
9	6	733	2514	1235	338	52	8	3886
10	—	36	167	93	35	6	—	337
11	—	5	91	4	11	2	—	154
Totals	34	14290	38580	18415	3457	523	63	75362

	"Aunt or Uncle"	"Nephew or Niece" (75362)	Correlation	Regression
Mean	7·170	7·213	·091	·095
Standard Deviation	·816	·851	± ·002	± ·003

It is clear that the correlation between parent and offspring is *double* what it is between the parents' co-fraternity and the offspring, a result wholly inexplicable on the theory of the pure line.

We can follow this idea further and remark that the degree of resemblance between every member of the pure line ought to be exactly the *same.* There ought to be no distinction at all between the correlation of individuals with their ascendants, their descendants and their collaterals in all grades. Now in dealing with man and with mammals generally, the biometric school has found the resemblance of brethren to be slightly greater than that of parent and offspring, parental correlations running from ·4 to ·5 and fraternal from ·5 to ·6. For greyhounds, Barrington, Lee and Pearson found the ratio of parental to fraternal correlations to be ·466 to ·529, for the horse ·522 to ·633, for man ·460 to ·519; and Barrington and Pearson for shorthorns ·40 to ·53. Again Warren found for mean results for Hyalopterus and Daphnia ·500 to ·656. It will accordingly be clear that the ratio of fraternal to parental correlation lies between 1·1 and 1·3. If we suppose the weakening of heredity to be as great in the filial as the parental generation, we should expect fraternal resemblance in Hydra to lie between 1·1 × ·136 and 1·3 × ·136, or say between ·15 and ·18. Our results are:

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* I do not see further why it should not be applied to test "pure lines" in the case of bisexual reproduction, where it is met at once by the fact that the relationship between uncle or aunt and nephew or niece has only about one *half* the intensity of that between parent and child; yet the parent and the uncle or aunt are both products of the same pair of "pure lines." If it be said, which I am ready to admit, that the gametes of one individual are not all alike, that on bisexual reproduction the zygote will not always be the same, and that the somatic characters will be related to these selected gametic characters, the obvious reply is that there is no evidence at present that, either in budding or in parthenogenetic reproduction, there is always a transfer of precisely the same type of cells as basis for the new individual.

† Parent excluded, i.e. "Uncle and Nephew."

TABLE V.

Number of Tentacles of Pairs of "Brethren" (Siblings).

Number of Tentacles of First Sibling.

Number of Tentacles of Second Sibling.	*5*	*6*	*7*	*8*	*9*	*10*	*11*	Totals
5	—	5	23	11	3	—	—	42
6	5	1878	3599	1426	231	28	7	7174
7	23	3599	8968	4473	941	114	20	18138
8	11	1426	4473	2880	723	107	12	9632
9	3	231	941	723	254	63	7	2222
10	—	28	114	107	63	10	1	323
11	—	7	20	12	7	1	—	47
Totals	42	7174	18138	9632	2222	323	47	37578

(37,578) Members of same Co-fraternity*		Correlation
Mean	7·212	·167 ± ·003
Standard Deviation ...	·861	

The correlation of "brethren" is thus significantly greater than the corrected correlation, ·135, of parent and offspring, the actual ratio being 1·23. It differs also significantly from the relation between "uncle" and "nephew," the ratio in that case being 1·76, which compares closely with the value for man ·50 to ·30, i.e. about 1·67. It appears impossible therefore, from the three tests made (i.e. comparison of the parental correlation with the correlations of individuals with their grandparents, with their "uncles or aunts" and with their brethren) not to admit that the individual has a special relation to each grade of its line, and is *not* equally related to all members of its line.

Of the results we have reached that for the foundation stock and their grandchildren (·03) is, perhaps, the least satisfactory; it is so low. But the source of this is fairly obvious. Out of each array of offspring of the 26 original Hydra, one, two or three, in one case only, four, individuals were selected to carry on the line. These individuals were not selected at random, nor were they taken proportionally to the frequency distribution of the tentacles in the offspring. The result is that the grandchildren as they stand are not random samples of the grandchildren which would be due to the original individual. They are arbitrary and not properly representative groups. Thus the correlation of the foundation stock of 26 with their 1452 great-grandchildren is ·059, greater than the value, ·030, of the same individuals' correlation with their grandchildren. It may readily take any small value, because the grandchildren and great-grandchildren are not typical of the stock of the foundation individuals, there having been either one or two selections of two or three individuals of a quite arbitrary character in between. While therefore the correlations for these more distant generations are lower than in the case of parent and offspring, they are quite erratic, and I see no obvious method of allowing theoretically for these arbitrary intermediate selections of parents. It is safest to base the fundamental criticism on the result that the correlation of brethren is sensibly higher, and of "aunts and uncles" very sensibly lower than that of parents and offspring, a result absolutely opposed to the theory of the pure line.

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* Pairs of "brethren" in the first generation.

(6) But if those who lightly accept without criticism such results as those of Hanel have not the statistical training necessary to measure degree of resemblance, they might at least have examined Hanel's own tables carefully, and compared the average number of tentacles in the offspring of the parents possessing the highest and lowest numbers of tentacles which occur in each pure line. They would have found the following results:

"Stammthier"	Line	Lowest Number of Parental Tentacles	Number of Offspring	Mean	Highest Number of Parental Tentacles	Number of Offspring	Mean
6	1	6	83	6·82	7	65	7·04
6	2^a	6	66	6·84	8	13	7·46
5	2^b	5	30	6·70	8	29	7·34
8	3	6	143	7·42	8	199	7·43
7	4	7	93	7·60	9	64	7·62
6	5	6	202	6·69	8	77	7·15
9	6	6	183	7·06	12	35	7·69
7	7	6	32	7·03	10	31	7·66
7	8	7	100	6·67	8	23	6·78
8	9	6	22	7·32	8	101	7·41
8	10	6	39	7·39	8	116	7·22
6	11	5	39	6·90	7	85	7·11
6	12	6	154	6·97	8	83	7·05
7	13	6	89	6·35	8	50	6·36
7	14	6	24	6·50	8	18	6·61
8	15	7	21	6·93	9	56	7·73
12	16	8	84	7·08	12	25	7·64
10	17	8	31	8·23	10	116	8·45
7	18	6	119	7·34	9	49	7·35
5	19	5	49	7·06	9	26	7·30
7	20	6	95	7·47	9	18	7·44
6	21	6	103	7·49	8	39	8·11
7	22	7	109	7·67	10	23	7·12
5	23	5	44	7·22	9	85	7·40
7	25	7	98	8·05	10	60	7·63
9	26	7	94	7·17	9	78	7·25

The above Table has been prepared from Hanel's means, because these are what Jennings had before him. Many of these means are, I have found, erroneous to a greater or less extent. Further, in a number of cases Hanel did not preserve as parents individuals with the highest and lowest number of tentacles occurring in the "pure line." But what do we perceive even in the data thus presented? *That although the character is very slightly inherited and largely the product of other factors than heredity, yet if we reproduce from the individuals in each pure line with* (a) *the lowest and* (b) *the highest number of tentacles available, the produce of the latter class has in* 22 *out of the* 26 *"pure lines" more tentacles than the produce of the former class.*

In other words, so far from Hanel's observations demonstrating as that author and Jennings suppose that:

(1) "In einer Population von Hydra ist die Selektion wirksam, innerhalb der reinen Linien ist sie ganz ohne Einfluss.

(2) Im Falle 1 ist die Regression eine teilweise, im 2 Falle ist sie vollständig,"

they actually show that regression exists within the "pure line," and that as far as Hydra is concerned the idly termed "fluctuating variations" have a real selective value. The "erfreuliche Uebereinstimmung mit Johannsen's schönen Resultaten," which Hanel says the experiments indicate, is real, but far different in its bearing from what that author imagines. The *Phaseolus* experiments of 1903 and the *Hydra* experiments of 1908 both alike indicate that the offspring are closer to the immediate parent than to other members of the same "pure line." Yet they have both alike been accepted without criticism by distinguished biologists who, suffering from "neo-chytophilia," seem peculiarly anxious to remove the only philosophical basis of Darwinism as a memorial of its fiftieth anniversary.

(7) Let us recognise fully what the "pure line" theory leads to in the case of any self-reproducing organism. It assumes that the cells of an individual destined for the production of new organisms are all alike in character and that variety in the somatic characters of such new organisms, when resulting from a single individual, is not due to any differentiation in the groups of the cells from which the fraternal individuals arise. Every self-reproducing species must have originated with an indefinite number of pure lines—differentiation could not have been produced by selection—or else, differentiation must have been produced at some epoch by what has been termed a fit of mutation. Either solution, the initial indefinite differentiation of a species, or the epochs of mutation, appears to me to destroy Darwinism as a consistent logical system. What then? Simply, that if Darwinism be proved to be in error it must be dropped. But the proofs which are being accepted to-day, that it is in error, are no proofs at all. They are hurried conclusions drawn from observations, which, when treated by any adequate logic, demonstrate—even on ill-suited material—the very reverse of what

they are assumed to prove. I fully appreciate the careful experimental work done by both Jennings and Hanel; I am not pledged to any theory of life, or of inheritance; it is the method and logic of their reasoning which concerns me in the first place. These are essentially inadequate and do not meet the points really at issue. That the degree of resemblance decreases in a geometrical progression, i.e. that offspring are more like their parents than their grandparents or great-grandparents, is not for me a theory, it is a deduction from the observed facts; it becomes a theory, which flows from an hypothesis, if you accept, say Mendelism[*]. Those facts would not be changed, if the theory of pure lines were demonstrated for either *Paramecium* or *Hydra*. But the establishment of that conclusion would affect biometricians in the indirect manner that any fundamental subversal of Darwinian principles destroys the only philosophical theory of life at present available, and so lessens the interest many feel in devoting time and energy to the measurement of life.

(8) As I have taken Hanel to illustrate recent biological work on the inheritance of variation, I will consider in this section another biological memoir, one on fertility.

I have hitherto made no reference to a recent interesting paper[†] by Captain R. E. Lloyd, I.M.S. He states that he was led by some observations of mine on the fertility of Shirley poppies to question whether fertility was related to weight in rats. He gives two correlation tables which I reproduce below, but he makes no statistical reduction of these tables, but draws apparently from the mere examination of them the conclusion that gigantic and dwarf rats are just as fertile as common rats of average size. "There is clear evidence that the largest and smallest rats are *quite as fertile as those of average size*" (p. 264).

Now the first impression I formed from Lloyd's tables before they were reduced was that they showed quite sensible correlation between weight of rat and number of young. And this on reduction proved to be the case, the dwarf rats have fewer offspring than the average rat, and the large rats have more offspring than the

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[*] A curiously ignorant account of the biometric treatment of heredity has recently been given by W. Weinberg: "Ueber Vererbungsgesetze beim Menschen," *Zeitschrift für induktive Abstammungs- und Vererbungslehre*, Bd. I. 1909, S. 377 *et seq*. He does not appear while writing his paper to have known the difference between the correlation and regression coefficients; and only when penning the last paragraph did a vague inkling of the difference come to him—through the writings of Johannsen! He overlooks entirely the allowances for assortative mating in biometric work, and accuses the biometric school of neglecting environment, when at any rate in the case of man it is the only group, which by statistical method and by direct investigation has endeavoured to allow for it. Weinberg has stated that the correlation between parents and offspring observed by biometricians in the case of the phthisical diathesis is due to a mixture of classes having different tuberculosis death-rates; and this regardless of the fact that (i) the data of the biometricians were purposely drawn from very uniform classes, and (ii) the correlation of husband and wife for the same material was shown to be much smaller than that of blood relations and in some cases zero. It hardly seems needful to reply to criticisms of this character.

[†] "The Relation between Fertility and Normality in Rats," *Records of the Indian Museum*, Vol. III. Part III. pp. 261—5.

TABLE VI.

Poona Rats.

Number of Offspring.

Weight in grs. of Mother.	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	Totals
50	—	—	—	1	3	1	—	—	—	—	—	—	5
60	—	—	—	1	6	2	—	—	—	—	—	—	9
70	—	—	2	10	17	12	4	—	1	—	—	—	46
80	1	1	11	8	18	10	9	3	2	—	—	—	63
90	2	5	7	18	30	28	12	5	1	—	—	—	108
100	3	5	10	25	37	35	21	7	2	1	—	—	146
110	1	4	12	19	38	37	29	6	2	—	—	—	148
120	2	6	9	21	36	26	30	14	6	—	1	—	151
130	4	4	9	12	35	29	17	17	6	1	1	1	136
140	1	4	6	9	12	27	15	6	2	1	—	—	83
150	—	3	—	2	13	11	6	6	2	—	—	—	43
160	—	2	—	1	11	11	9	3	4	—	—	—	41
170	1	—	1	1	2	4	2	2	1	—	1	—	15
180	—	—	1	1	—	2	2	2	—	—	—	—	8
190	—	—	—	—	—	—	—	—	—	1	—	—	1
Totals	15	34	68	129	258	235	156	71	29	4	3	1	1003

TABLE VII.

Belgaum Rats.

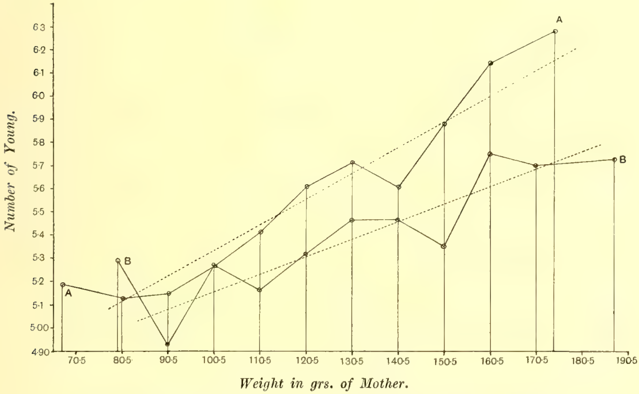
Number of Offspring.

Weight in grs. of Mother.	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	Totals
70	—	—	—	—	1	1	—	—	—	—	2
80	—	—	—	10	7	5	3	—	—	—	25
90	—	1	3	13	27	11	2	1	—	—	58
100	2	1	1	18	43	23	14	3	—	—	105
110	—	2	7	23	56	25	15	2	1	—	131
120	2	2	7	23	54	40	23	1	3	—	155
130	1	—	5	18	46	30	16	4	4	—	124
140	2	3	7	15	39	44	18	6	3	—	137
150	—	1	5	20	41	38	10	3	1	1	120
160	—	2	2	4	20	18	7	7	2	—	62
170	—	1	1	4	14	12	9	2	1	—	44
180	—	—	—	3	9	5	4	4	—	—	25
190	—	—	—	1	2	4	1	1	—	—	9
200	—	—	1	2	1	—	2	—	—	—	6
210	—	—	—	—	—	—	3	—	—	—	3
Totals	7	13	39	154	360	256	127	34	15	1	1006

average rat. If w stands for weight, f for number of young, then we have the following results:

	1003 Poona Rats	1006 Belgaum Rats
Mean Weight $\bar{w}$	$=$113·92	128·84
Mean Number of Young $\bar{f}=$	5·49	5·38
$\sigma_w=$	24·86	25·35
$\sigma_f=$	1·68	1·29
$r_{wf}=$	·1670	·1498
$\eta\ =$	·1733	·1634

As the above results indicate, the regression curves of fertility on weight are almost linear, and the average number of offspring increase at an almost uniform rate from the dwarf to the giant rats. This is well illustrated on the accompanying diagram. It will be seen at once that Captain Lloyd's view is not supported by his own data. There is a very sensible relationship between the weight of the rat and the number of its offspring. But it is not a relation in which the type rat is the most fertile. In other words the case seems at first sight precisely one in which the effect of reproductive selection would cause, without regard to natural selection, a progressive increase in the size of rats. But is this inference really justified? It seems so very important to determine whether any such cases of genetic selection actually occur that we may be pardoned for hoping that further information may be forthcoming. It is conceivable that in certain stages of pregnancy even the size of the litter might have some effect on the weight, but this is hardly likely to produce *all* the difference noted. Another important point



Regression Curves of Number of Young on Mother's Weight in Rats.
AA Poona Rats. BB Belgaum Rats.

is the question of age. Were the bigger rats on the average the older, and do the older rats have on the average larger litters? If anything of this kind takes place in rats, we should not be comparing the fertility of large and small rats, but the fertility of rats in different stages of development, which is a very diverse point. I am inclined to think that the real relation between number of young and weight of rat, although overlooked by Captain Lloyd, actually has this simple origin. It is suggested, but not demonstrated by the relation which is manifest in mice between size of litter and number of litter. Thus in 700 litters of mice recorded by the late Professor Weldon, I find:

	Number	Average size
First litter	307	5·46
Second "	254	5·57
Third "	139	5·76

It will be seen that the size of the litter increased uniformly with the increasing age of the mouse, and there is little doubt that the weight of the mouse also increased with age. Only a direct experiment in which age and number of litter were taken into account could settle the point. In Captain Lloyd's experimental weighing, there was no record of these characters. I have small doubt that the very sensible correlation between weight and number of young shown by his data is only a secondary result of the relations between age and weight, and again between age and number of young. It is not evidence of a real correlation between fertility and a somatic character, and thus has no bearing on genetic selection, and does not oppose the general principle that fertility is not highly correlated with inherited characters.

(9) The reader may ask: Why have I considered in the same paper such diverse work as that of Jennings, Hanel and Lloyd? The reason lies in this: We have been told recently in an *ex cathedra* fashion by a distinguished biologist* that:

"Of the so-called investigations of heredity pursued by extensions of Galton's non-analytical method and promoted by Professor Pearson and the English Biometrical School it is now scarcely necessary to speak.......A preliminary acquaintance with the natural history of heredity and variation was sufficient to throw doubt on the foundations of these elaborate researches. To those who hereafter may study this episode in the history of biological science, it will appear inexplicable that work so unsound in construction should have been respectfully received by the scientific world."

It seems therefore, however regrettable, needful to re-emphasise the point from which this Journal started more than eight years ago. Biology requires the help of a more exact logic than it appears to possess, above all of an adequate statistical theory to enable it to interpret its observations and test its own theories. I have endeavoured to illustrate this in the present paper, by indicating how a want of statistical insight has resulted in at least two cases where experiments have been interpreted in a sense, which is in no way justified by the observations themselves. Regression within the pure line exists in Hydra, and fertility is associated with weight in rats. These are far from the only cases in which principles bearing on evolution have been propounded on the basis of researches, which prove in fact their negation. The biometrician does not desire to waste the energies that may be given to better work in personal recriminations with anonymous writers, nor in controversy with biologists of repute; but it is desirable to assert occasionally that biometry is very far from moribund, and that biology stands to-day even more in need of its methods than it did eight years ago; for it has now developed theories of such complexity, that without the aid of the highest mathematical analysis, it is wholly unable to state whether its theories are accurate or not. And this many Mendelians admit to-day, and all will have to admit in the very near future. From small fragments our Frankenstein has created his soulless monster; he will need all the mathematical aid he can enlist, if his creation is to be brought under control, and not to end in destroying him. But I propose to deal further with some recent Mendelian developments on another occasion.

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* W. Bateson: *Mendel's Principles of Heredity*, Cambridge, 1909, pp. 6—7. Luckily the doctrine of infallibility is not yet accepted in the world of science.

ADDITIONAL TABLES AND DIAGRAM FOR THE DETERMINATION OF THE ERRORS OF TYPE OF FREQUENCY DISTRIBUTIONS.

SECOND PAPER.

BY A. RHIND, B.Sc.

IN *Biometrika*, Vol. VII. page 131, a diagram was given to determine the type to which an observed frequency distribution belongs for given values of β_1 and β_2.

The probable errors of β_1 and β_2 define a region within which the most probable values lie.

Since, however, β_1 and β_2 are highly correlated this area is not a rectangle but an ellipse; a contour of the surface

$$z = \frac{N}{2\pi\sigma_{\beta_1}\sigma_{\beta_2}\sqrt{1 - R^2_{\beta_1\beta_2}}} e^{-\frac{1}{2}\frac{1}{1 - R^2_{\beta_1\beta_2}}\left\{\frac{x^2}{\sigma^2_{\beta_1}} - \frac{2R_{\beta_1\beta_2}xy}{\sigma_{\beta_1}\sigma_{\beta_2}} + \frac{y^2}{\sigma^2_{\beta_2}}\right\}},$$

or, referred to principal axes,

$$z = \frac{N}{2\pi\Sigma_1\Sigma_2} e^{-\frac{1}{2}\left\{\frac{X^2}{\Sigma_1^2} + \frac{Y^2}{\Sigma_2^2}\right\}} \qquad\qquad \text{(A)}.$$

The semi-axes of an elliptic cylinder, with section similar to a contour of (A), and containing half the frequency, are $1{\cdot}177\Sigma_1$ and $1{\cdot}177\Sigma_2$.

These are inclined to the axis of β_2 at angles θ given by

$$\tan 2\theta = \frac{2R_{\beta_1\beta_2}/\sigma_{\beta_1}\sigma_{\beta_2}}{\dfrac{1}{\sigma^2_{\beta_1}} - \dfrac{1}{\sigma^2_{\beta_2}}} = R_{\beta_1\beta_2}\tan 2\phi, \text{ where } \tan\phi = \sigma_{\beta_1}/\sigma_{\beta_2}.$$

The directions were calculated from this formula and Diagram C (p. 389), constructed in such manner that the tangent and normal to one of the broken-line curves at any point are the major and minor axes respectively. The *major* axis is inclined to the axis of β_2 at angles given by Table IX.

A distinction between the directions of the major and minor axes is readily made when we note that a contour of (A) is inscribed in a rectangle of sides $2\sigma_{\beta_1}$ and $2\sigma_{\beta_2}$, and that the major axis of such an ellipse is inclined to the greater side at an angle less than $\pi/4$.

For if $ax^2 + 2hxy + by^2 = 1$ be an ellipse inscribed in a rectangle sides $2q$ and $2p$ having the greater side $2q$ parallel to the axis of x, and centre as origin, then

$$p^2 = \frac{a}{ab - h^2}, \quad q^2 = \frac{b}{ab - h}, \quad \therefore b > a.$$

Again
$$a\cos^2\theta + h\sin 2\theta + b\sin^2\theta = \frac{1}{\gamma^2},$$

$$(b - a)\sin 2\theta + 2h\cos 2\theta = -\frac{2}{\gamma^2}\frac{dr}{d\theta}.$$

For maximum or minimum r

$$\tan 2\theta_1 = \frac{-2h}{b - a},$$

$$2(b - a)\cos 2\theta_1 - 4h\sin 2\theta_1 = -\frac{2}{\gamma_1^3}\frac{d^2r}{d\theta^2} \text{ at a maximum or minimum,}$$

$$\therefore \cos 2\theta_1 \frac{(b - a)^2 + 4h^2}{(b - a)} = -\frac{2}{\gamma_1^2}\frac{d^2r}{d\theta^2}.$$

Hence $\frac{d^2r}{d\theta^2}$ is negative when $\cos 2\theta$ is positive,

$$\therefore \theta \text{ for a maximum is } < \pi/4.$$

If
$$\sin\gamma = \frac{2\sqrt{1 - R^2_{\beta_1\beta_2}}\,\sigma_{\beta_1}\sigma_{\beta_2}}{\sigma^2_{\beta_1} + \sigma^2_{\beta_2}},$$

then
$$\Sigma_1 = \sqrt{\sigma^2_{\beta_1} + \sigma^2_{\beta_2}}\,\sin\tfrac{1}{2}\gamma,$$

$$\Sigma_2 = \sqrt{\sigma^2_{\beta_1} + \sigma^2_{\beta_2}}\,\cos\tfrac{1}{2}\gamma.$$

From these formulae Σ_1 and Σ_2 were calculated and the values of $1{\cdot}177\Sigma_1$ and $1{\cdot}177\Sigma_2$ tabulated in VII. and VIII. respectively. The number must of course be multiplied by the factor $\cdot 67449/\sqrt{N}$.

Since the population outside an ellipse of semiaxes $\kappa\Sigma_1$ and $\kappa\Sigma_2$ is $Ne^{-\frac{1}{2}\kappa^2}$, it follows that, if an ellipse of twice the above linear dimensions be drawn, the probability that a point lies outside it is 1/16.

If an ellipse be drawn with semiaxes $\kappa\Sigma_1$ and $\kappa\Sigma_2$, the probability P that a point lies outside the ellipse is $P = e^{-\frac{1}{2}\kappa^2}$. Let

$$\kappa\Sigma_1 = 1{\cdot}177\sqrt{N}\Sigma_1 \times \frac{\cdot 67449}{\sqrt{\frac{N}{q}}}.$$

Then when $q = 10$, $\kappa^2 = 6{\cdot}3006$ and $p = \frac{1}{23}$ approximately,

" " $q = 12$, $\kappa^2 = 7{\cdot}5609$ " $p = \frac{1}{44}$ " .

Hence to find the semiaxes of ellipses containing 22/23 or 43/44 of the whole frequency; take the numbers from Tables VII. and VIII. and multiply by the factor from Gibson's Tables (*Biometrika*, Vol. IV. p. 385) corresponding to populations of $N/10$ and $N/12$ respectively.

To facilitate the construction of the ellipse, a series of ellipses has been drawn on Diagram C of different dimensions and varying in ratio of Σ_1/Σ_2 from ·1 to ·3.

In practice the values of the axes should be taken from Tables VII. and VIII. and multiplied by the appropriate factor. Then their ratio gives Σ_1/Σ_2. By means of a piece of tracing paper applied to the corresponding series of ellipses, the required curve can be approximately drawn and transferred to the diagram. It will then be possible to determine whether the frequency really lies with a definite probability within the chosen type, as indicated by the β_1, β_2.

The series of tables has been extended by the inclusion of Table X. which gives the probable error of

$$\kappa_2 = \frac{\beta_1(\beta_2+3)^2}{4(4\beta_2-3\beta_1)(2\beta_2-3\beta_1-6)}.$$

The S.D. was determined thus:

$$\begin{aligned}\frac{1}{\kappa_2}\delta\kappa_2 &= \frac{2}{\beta_2+3}\delta\beta_2 + \frac{1}{\beta_1}\delta\beta_1 - \frac{4\delta\beta_2 - 3\delta\beta_1}{4\beta_2-3\beta_1} - \frac{2\delta\beta_2 - 3\delta\beta_1}{(2\beta_2-3\beta_1-6)} \\ &= \left[\frac{2}{(\beta_2+3)} - \frac{4}{(4\beta_2-3\beta_1)} - \frac{2}{(2\beta_2-3\beta_1-6)}\right]\delta\beta_2 \\ &\qquad + \left[\frac{1}{\beta_1} + \frac{3}{(4\beta_2-3\beta_1)} + \frac{3}{(2\beta_2-3\beta_1-6)}\right]\delta\beta_1,\end{aligned}$$

whence

$$\sigma_{\kappa_2} = c^2\left\{A^2\sigma^2_{\beta_1} + 2ABR_{\beta_1\beta_2}\sigma_{\beta_1}\sigma_{\beta_2} + B^2\sigma^2_{\beta_2}\right\}^{\frac{1}{2}},$$

where

$$\begin{aligned}c &= \frac{(\beta_2+3)}{2(4\beta_2-3\beta_1)(2\beta_2-3\beta_1-6)}, \\ A &= 8\beta_2(\beta_2-3) - 9\beta_1^2, \\ B &= 18\beta_1\frac{4+\beta_1(\beta_1+5)-\beta_2(\beta_1+4)}{(\beta_2+3)}.\end{aligned}$$

Erratum. In the printing of my former paper a slip of sign escaped my notice. On p. 129, *Biometrika*, Vol. VII., l. 7, for $\alpha = (2\beta_2 - 3\beta_1 - 6)/(\beta_2 - 3)$ read $\alpha = (2\beta_2 - 3\beta_1 - 6)/(\beta_2 + 3)$, which was, of course, the value used in the calculation of the tables and diagrams.

DIAGRAM C.

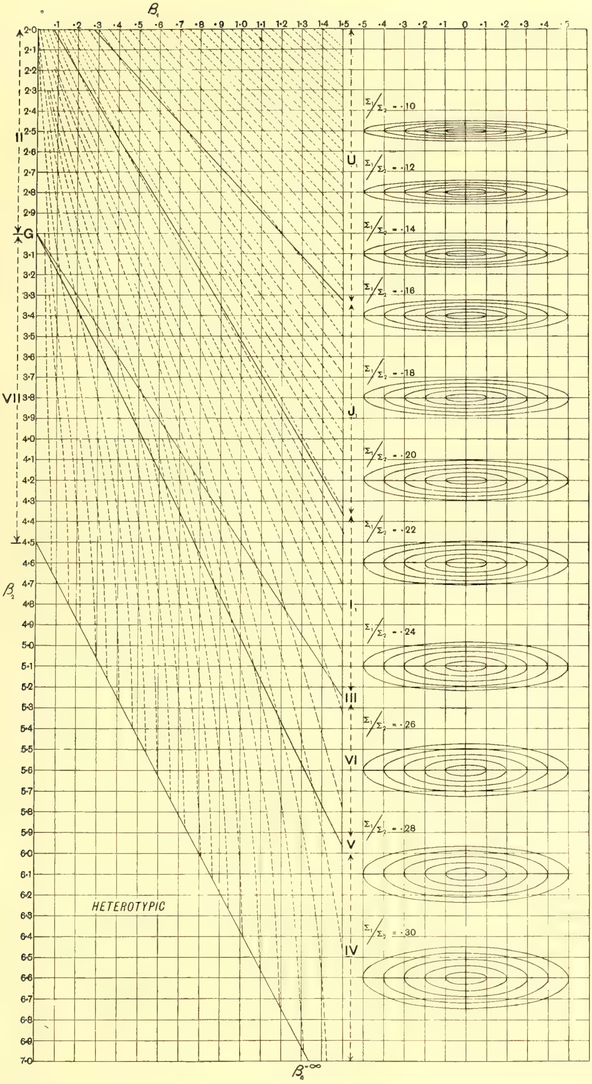
β_1
·1 ·2 ·3 ·4 ·5 ·6 ·7 ·8 ·9 1·0 1·1 1·2 1·3 1·4 1·5
·5 ·4 ·3 ·2 ·1 0 ·1 ·2 ·3 ·4 ·5
2·0 2·1 2·2 2·3 2·4 2·5 2·6 2·7 2·8 2·9
G
3·1 3·2 3·3 3·4 3·5 3·6 3·7 3·8 3·9 4·0 4·1 4·2 4·3 4·4 4·5 4·6 4·7 4·8 4·9 5·0 5·1 5·2 5·3 5·4 5·5 5·6 5·7 5·8 5·9 6·0 6·1 6·2 6·3 6·4 6·5 6·6 6·7 6·8 6·9 7·0
II
VII
β_2
U_1
J_1
I_1
III
VI
V
IV
HETEROTYPIC
$\Sigma_1/\Sigma_2 = -·10$
$\Sigma_1/\Sigma_2 = -·12$
$\Sigma_1/\Sigma_2 = -·14$
$\Sigma_1/\Sigma_2 = -·16$
$\Sigma_1/\Sigma_2 = -·18$
$\Sigma_1/\Sigma_2 = -·20$
$\Sigma_1/\Sigma_2 = -·22$
$\Sigma_1/\Sigma_2 = -·24$
$\Sigma_1/\Sigma_2 = -·26$
$\Sigma_1/\Sigma_2 = -·28$
$\Sigma_1/\Sigma_2 = -·30$
$\beta_4 = \infty$

TABLE VII.

Values of $1{\cdot}77\sqrt{N}\Sigma_1$ *for given values of* β_1, β_2 *(Semi-Minor Axis of Probability Ellipse).*

β_2 \ β_1	*0*	*·05*	*·1*	*·15*	*·2*	*·25*	*·3*	*·35*	*·4*	*·45*	*·5*	*·55*	*·6*	*·65*	*·7*	*·75*
2·0	0·0	0·5	0·7	0·8	0·8	0·8	0·8	0·8	0·7	0·7	0·7	0·6	0·6	0·5	0·5	0·4
2·1	0·0	0·5	0·7	0·8	0·9	0·9	0·8	0·8	0·8	0·8	0·7	0·7	0·7	0·6	0·6	0·5
2·2	0·0	0·5	0·8	0·9	0·9	0·9	0·9	0·9	0·9	0·8	0·8	0·8	0·7	0·7	0·7	0·6
2·3	0·0	0·5	0·8	0·9	1·0	1·0	1·0	1·0	1·0	0·9	0·9	0·9	0·8	0·8	0·8	0·7
2·4	0·0	0·6	0·9	1·0	1·0	1·0	1·0	1·0	1·0	1·0	1·0	0·9	0·9	0·9	0·9	0·8
2·5	0·0	0·6	0·9	1·0	1·1	1·1	1·1	1·1	1·1	1·1	1·0	1·0	1·0	1·0	1·0	0·9
2·6	0·0	0·6	1·0	1·1	1·1	1·2	1·2	1·2	1·1	1·1	1·1	1·1	1·1	1·0	1·0	1·0
2·7	0·0	0·7	1·0	1·2	1·2	1·3	1·3	1·3	1·2	1·2	1·2	1·2	1·1	1·1	1·1	1·1
2·8	0·0	0·7	1·1	1·3	1·3	1·3	1·3	1·3	1·3	1·3	1·3	1·3	1·2	1·2	1·2	1·2
2·9	0·0	0·8	1·2	1·3	1·3	1·4	1·4	1·4	1·4	1·4	1·4	1·4	1·3	1·3	1·3	1·3
3·0	0·0	0·8	1·2	1·4	1·4	1·5	1·5	1·5	1·5	1·5	1·5	1·5	1·4	1·4	1·4	1·4
3·1	0·0	0·9	1·3	1·5	1·5	1·6	1·6	1·6	1·6	1·6	1·6	1·6	1·6	1·5	1·5	1·5
3·2	0·0	1·0	1·4	1·6	1·6	1·7	1·7	1·7	1·7	1·7	1·7	1·7	1·7	1·6	1·6	1·6
3·3	·00	1·1	1·6	1·8	1·8	1·9	1·9	1·9	1·8	1·8	1·8	1·8	1·8	1·7	1·7	1·7
3·4	·00	1·2	1·7	1·9	1·9	2·0	2·0	2·0	2·0	2·0	1·9	1·9	1·9	1·9	1·9	1·8
3·5	·00	1·4	1·8	2·0	2·1	2·1	2·1	2·1	2·1	2·1	2·1	2·1	2·1	2·0	2·0	1·9
3·6	·00	1·5	2·0	2·1	2·2	2·3	2·3	2·3	2·3	2·3	2·3	2·2	2·2	2·1	2·1	2·1
3·7	·00	1·6	2·1	2·3	2·4	2·5	2·5	2·5	2·5	2·5	2·4	2·4	2·4	2·3	2·3	2·3
3·8	·00	1·8	2·3	2·5	2·6	2·7	2·7	2·7	2·7	2·7	2·6	2·6	2·6	2·5	2·5	2·4
3·9	·00	2·0	2·5	2·7	2·9	3·0	3·0	3·0	3·0	2·9	2·9	2·8	2·8	2·7	2·7	2·6
4·0	·00	2·2	2·8	3·0	3·2	3·3	3·3	3·3	3·3	3·2	3·1	3·1	3·0	2·9	2·9	2·8
4·1	—	—	3·0	3·3	3·5	3·6	3·6	3·6	3·6	3·5	3·4	3·4	3·3	3·2	3·2	3·1
4·2	—	—	3·3	3·6	3·8	3·9	4·0	4·0	3·9	3·8	3·8	3·7	3·6	3·5	3·5	3·4
4·3	—	—	3·6	4·0	4·2	4·3	4·4	4·4	4·3	4·2	4·1	4·0	3·9	3·8	3·8	3·7
4·4	—	—	4·0	4·4	4·7	4·8	4·8	4·8	4·7	4·6	4·5	4·4	4·3	4·2	4·1	4·0
4·5	—	—	4·4	4·9	5·2	5·3	5·3	5·3	5·2	5·1	5·0	4·9	4·7	4·6	4·5	4·4
4·6	—	—	—	—	—	—	5·9	5·9	5·8	5·7	5·6	5·4	5·2	5·1	5·0	4·8
4·7	—	—	—	—	—	—	6·7	6·7	6·6	6·4	6·2	6·0	5·8	5·7	5·6	5·3
4·8	—	—	—	—	—	—	7·6	7·6	7·4	7·2	6·9	6·6	6·4	6·3	6·2	5·9
4·9	—	—	—	—	—	—	8·5	8·5	8·4	8·1	7·8	7·5	7·2	7·0	6·8	6·5
5·0	—	—	—	—	—	—	11	10	9·7	9·3	8·9	8·5	8·1	7·7	7·4	7·1
5·1	—	—	—	—	—	—	—	—	—	—	—	—	9·3	8·6	8·1	7·7
5·2	—	—	—	—	—	—	—	—	—	—	—	—	11	9·5	8·9	8·6
5·3	—	—	—	—	—	—	—	—	—	—	—	—	12	11	10	9·6
5·4	—	—	—	—	—	—	—	—	—	—	—	—	14	13	12	11
5·5	—	—	—	—	—	—	—	—	—	—	—	—	16	15	14	13
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE VII.—(*continued*).

Values of $1{\cdot}77\sqrt{N}\,\Sigma_1$ *for given values of* β_1, β_2 *(Semi-Minor Axis of Probability Ellipse).*

β_1

·8	*·85*	*·9*	*·95*	*1·0*	*1·05*	*1·1*	*1·15*	*1·2*	*1·25*	*1·3*	*1·35*	*1·4*	*1·45*	*1·5*	β_2
0·4	0·3	0·3	0·2	0·1	0·1	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	*2·0*
0·5	0·5	0·4	0·3	0·2	0·2	0·2	0·1	0·1	0·1	0·0	0·0	0·0	0·0	0·0	*2·1*
0·6	0·6	0·5	0·5	0·4	0·4	0·3	0·3	0·3	0·2	0·1	0·0	0·0	0·0	0·0	*2·2*
0·7	0·7	0·6	0·6	0·5	0·5	0·4	0·4	0·4	0·3	0·2	0·1	0·1	0·0	0·0	*2·3*
0·8	0·8	0·7	0·7	0·6	0·6	0·5	0·5	0·4	0·4	0·3	0·2	0·2	0·1	0·0	*2·4*
0·9	0·8	0·8	0·7	0·7	0·7	0·6	0·6	0·5	0·5	0·4	0·3	0·3	0·2	0·1	*2·5*
1·0	0·9	0·9	0·8	0·8	0·8	0·7	0·7	0·7	0·6	0·6	0·5	0·4	0·3	0·2	*2·6*
1·1	1·0	1·0	0·9	0·9	0·9	0·8	0·8	0·8	0·7	0·7	0·6	0·5	0·4	0·3	*2·7*
1·1	1·1	1·1	1·0	1·0	1·0	0·9	0·9	0·9	0·8	0·8	0·7	0·6	0·5	0·5	*2·8*
1·2	1·2	1·2	1·1	1·1	1·0	1·0	1·0	1·0	1·0	0·9	0·8	0·7	0·7	0·7	*2·9*
1·3	1·3	1·3	1·2	1·2	1·2	1·1	1·1	1·1	1·0	1·0	0·9	0·9	0·8	0·8	*3·0*
1·4	1·4	1·4	1·3	1·3	1·3	1·2	1·2	1·2	1·1	1·1	1·0	1·0	0·9	0·9	*3·1*
1·5	1·5	1·5	1·4	1·4	1·4	1·3	1·3	1·2	1·2	1·2	1·1	1·1	1·0	1·0	*3·2*
1·7	1·6	1·6	1·5	1·4	1·4	1·3	1·3	1·3	1·2	1·2	1·2	1·1	1·1	1·0	*3·3*
1·8	1·7	1·7	1·6	1·6	1·6	1·5	1·5	1·4	1·4	1·3	1·3	1·2	1·2	1·1	*3·4*
1·9	1·9	1·8	1·8	1·7	1·7	1·6	1·6	1·5	1·5	1·4	1·4	1·3	1·3	1·2	*3·5*
2·0	2·0	2·0	1·9	1·8	1·8	1·7	1·7	1·6	1·6	1·5	1·5	1·4	1·4	1·4	*3·6*
2·2	2·2	2·1	2·1	2·0	2·0	1·9	1·8	1·7	1·7	1·6	1·6	1·6	1·5	1·5	*3·7*
2·4	2·3	2·2	2·2	2·1	2·1	2·0	2·0	1·9	1·9	1·8	1·7	1·7	1·6	1·6	*3·8*
2·6	2·5	2·4	2·4	2·3	2·3	2·2	2·1	2·0	2·0	1·9	1·9	1·8	1·7	1·7	*3·9*
2·8	2·7	2·6	2·6	2·5	2·5	2·4	2·3	2·2	2·2	2·1	2·1	2·0	2·0	1·9	*4·0*
3·0	2·9	2·8	2·8	2·7	2·7	2·6	2·5	2·4	2·4	2·3	2·3	2·2	2·2	2·1	*4·1*
3·2	3·2	3·1	3·0	2·9	2·9	2·8	2·7	2·6	2·6	2·5	2·4	2·3	2·3	2·2	*4·2*
3·5	3·5	3·4	3·3	3·2	3·1	3·0	2·9	2·8	2·8	2·7	2·6	2·5	2·4	2·4	*4·3*
3·9	3·8	3·7	3·6	3·5	3·4	3·3	3·1	3·0	3·0	2·9	2·8	2·7	2·7	2·6	*4·4*
4·2	4·1	4·0	3·9	3·7	3·6	3·5	3·4	3·3	3·2	3·1	3·1	3·0	3·0	2·9	*4·5*
4·6	4·5	4·4	4·2	4·1	3·9	3·8	3·7	3·5	3·4	3·3	3·3	3·2	3·2	3·1	*4·6*
5·1	4·9	4·8	4·6	4·4	4·3	4·2	4·0	3·8	3·7	3·6	3·5	3·4	3·4	3·3	*4·7*
5·6	5·4	5·2	5·0	4·8	4·7	4·6	4·4	4·2	4·0	3·9	3·8	3·7	3·6	3·5	*4·8*
6·2	6·0	5·7	5·5	5·3	5·1	5·0	4·8	4·6	4·4	4·2	4·1	4·0	3·9	3·8	*4·9*
6·8	6·6	6·3	6·1	5·8	5·6	5·4	5·2	5·0	4·8	4·6	4·4	4·3	4·2	4·1	*5·0*
7·4	7·2	7·0	6·7	6·4	6·2	5·9	5·7	5·4	5·1	4·9	4·7	4·6	4·4	4·3	*5·1*
8·3	8·0	7·7	7·4	7·1	6·8	6·5	6·2	5·8	5·5	5·2	5·0	4·9	4·7	4·5	*5·2*
9·2	8·9	8·6	8·2	7·8	7·5	7·1	6·7	6·3	5·9	5·6	5·4	5·2	5·0	4·7	*5·3*
10	10	9·6	9·1	8·6	8·2	7·8	7·3	6·9	6·5	6·1	5·8	5·6	5·3	5·0	*5·4*
12	11	11	10	9·5	9·0	8·5	8·0	7·5	7·1	6·7	6·4	6·1	5·8	5·5	*5·5*
—	—	12	11	10	10	9·5	8·9	8·4	7·9	7·4	7·0	6·6	6·3	6·0	*5·6*
—	—	14	13	12	11	10	10	9·4	8·8	8·3	7·8	7·4	7·0	6·7	*5·7*
—	—	16	14	13	12	12	11	10	10	9·4	8·8	8·3	7·8	7·4	*5·8*
—	—	18	16	15	14	14	13	12	11	10	10	9·5	8·9	8·4	*5·9*
—	—	22	20	18	16	15	14	13	12	12	11	11	10	9·5	*6·0*
—	—	—	—	—	—	18	16	15	14	13	13	12	12	11	*6·1*
—	—	—	—	—	—	21	19	17	16	15	14	13	13	12	*6·2*
—	—	—	—	—	—	24	21	19	18	17	16	15	14	13	*6·3*
—	—	—	—	—	—	27	24	22	20	19	18	17	16	15	*6·4*
—	—	—	—	—	—	32	28	25	23	21	20	19	18	16	*6·5*
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	*6·6*
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	*6·7*
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	*6·8*
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	*6·9*
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	*7·0*

TABLE VIII.

Values of $1{\cdot}77\sqrt{N}\,\Sigma_2$ *for values of* β_1, β_2 (*Semi-Major Axis of Probability Ellipse*).

β_1

β_2	*0*	*·05*	*·1*	*·15*	*·2*	*·25*	*·3*	*·35*	*·4*	*·45*	*·5*	*·55*	*·6*	*·65*	*·7*	*·75*
2·0	1·6	1·9	2·2	2·5	2·8	3·1	3·4	3·7	4·1	4·4	4·7	5·0	5·4	5·7	6·1	6·4
2·1	1·7	2·1	2·4	2·7	2·9	3·2	3·5	3·8	4·1	4·4	4·6	4·9	5·3	5·6	6·0	6·3
2·2	1·9	2·3	2·6	2·9	3·1	3·4	3·7	3·9	4·1	4·4	4·6	4·9	5·2	5·5	5·9	6·2
2·3	2·1	2·5	2·8	3·1	3·3	3·5	3·8	4·0	4·2	4·4	4·6	4·9	5·2	5·5	5·8	6·1
2·4	2·4	2·8	3·1	3·3	3·5	3·7	3·9	4·1	4·3	4·5	4·7	4·9	5·1	5·4	5·7	6·0
2·5	2·8	3·3	3·5	3·7	3·8	4·0	4·1	4·3	4·5	4·7	4·8	5·0	5·2	5·4	5·7	5·9
2·6	3·3	3·8	4·0	4·2	4·3	4·5	4·6	4·7	4·8	4·9	5·0	5·1	5·3	5·5	5·7	6·0
2·7	3·8	4·4	4·6	4·7	4·9	5·0	5·1	5·2	5·3	5·4	5·4	5·4	5·5	5·6	5·8	6·1
2·8	4·4	5·0	5·3	5·4	5·6	5·7	5·8	5·9	5·9	5·9	5·8	5·8	5·8	5·9	6·0	6·2
2·9	5·1	5·7	6·0	6·2	6·3	6·4	6·5	6·6	6·5	6·5	6·4	6·3	6·3	6·3	6·4	6·5
3·0	5·8	6·5	6·9	7·1	7·2	7·3	7·3	7·3	7·2	7·1	7·0	6·9	6·8	6·8	6·8	6·8
3·1	6·7	7·5	8·0	8·3	8·4	8·4	8·4	8·3	8·2	8·0	7·9	7·8	7·7	7·6	7·5	7·4
3·2	7·8	8·7	9·2	9·6	9·8	9·8	9·7	9·5	9·4	9·2	9·0	8·8	8·6	8·4	8·3	8·1
3·3	9·2	10	11	11	11	11	11	11	11	11	10	10	9·6	9·4	9·2	8·9
3·4	11	11	12	12	13	13	13	12	12	12	11	11	11	11	10	10
3·5	13	13	14	14	15	15	14	14	14	14	13	12	12	12	11	11
3·6	15	16	17	17	17	17	16	16	16	16	15	14	14	14	13	13
3·7	18	19	20	20	20	20	19	19	18	18	17	17	16	16	15	14
3·8	21	23	24	24	24	23	22	22	21	21	20	19	18	18	17	16
3·9	25	27	28	28	28	27	26	25	24	24	23	22	21	20	19	18
4·0	29	32	34	33	32	31	30	29	28	27	26	24	23	22	21	20
4·1	—	—	43	41	39	37	36	34	32	31	30	28	26	25	24	23
4·2	—	—	55	51	48	45	42	40	38	36	34	32	30	29	27	26
4·3	—	—	69	62	57	53	50	48	45	43	40	37	35	33	31	29
4·4	—	—	87	76	69	64	60	56	53	50	46	43	40	38	36	34
4·5	—	—	113	98	86	78	71	66	62	57	53	49	46	43	41	39
4·6	—	—	—	—	—	—	90	79	73	67	61	57	53	50	48	45
4·7	—	—	—	—	—	—	110	99	90	80	72	67	62	58	55	52
4·8	—	—	—	—	—	—	140	124	110	97	86	79	73	68	64	60
4·9	—	—	—	—	—	—	200	160	135	119	105	95	86	79	74	69
5·0	—	—	—	—	—	—	384	234	182	153	130	115	102	92	85	79
5·1	—	—	—	—	—	—	—	—	—	—	—	—	124	110	101	93
5·2	—	—	—	—	—	—	—	—	—	—	—	—	148	134	120	109
5·3	—	—	—	—	—	—	—	—	—	—	—	—	188	166	146	129
5·4	—	—	—	—	—	—	—	—	—	—	—	—	245	203	177	154
5·5	—	—	—	—	—	—	—	—	—	—	—	—	405	297	230	194
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE VIII.—(*continued*).

Values of $1{\cdot}77\sqrt{N}\,\Sigma_2$ *for values of* β_1, β_2 *(Semi-Major Axis of Probability Ellipse).*

β_1

·8	*·85*	*·9*	*·95*	*1·0*	*1·05*	*1·1*	*1·15*	*1·2*	*1·25*	*1·3*	*1·35*	*1·4*	*1·45*	*1·5*	β_2
6·8	7·2	7·6	7·9	8·3	8·7	9·1	9·5	9·9	10	11	11	11	12	12	*2·0*
6·7	7·0	7·4	7·8	8·1	8·5	8·9	9·2	9·6	10	11	11	11	11	12	*2·1*
6·5	6·8	7·2	7·6	8·0	8·3	8·6	9·0	9·4	9·8	10	10	11	11	12	*2·2*
6·4	6·7	7·1	7·4	7·8	8·1	8·5	8·8	9·2	9·6	9·9	10	11	11	12	*2·3*
6·3	6·6	6·9	7·3	7·6	8·0	8·3	8·7	9·0	9·4	9·8	10	10	11	11	*2·4*
6·2	6·5	6·8	7·1	7·5	7·8	8·2	8·5	8·9	9·2	9·6	10	10	11	11	*2·5*
6·2	6·5	6·8	7·1	7·4	7·7	8·0	8·3	8·7	9·0	9·4	9·9	10	11	11	*2·6*
6·3	6·5	6·8	7·0	7·3	7·6	7·9	8·2	8·5	8·8	9·2	9·7	10	10	11	*2·7*
6·4	6·6	6·8	7·0	7·2	7·5	7·7	8·0	8·3	8·7	9·1	9·5	9·8	10	10	*2·8*
6·6	6·7	6·8	7·0	7·2	7·4	7·6	7·9	8·2	8·6	8·9	9·3	9·6	10	10	*2·9*
6·9	7·0	7·1	7·2	7·3	7·5	7·7	7·9	8·2	8·5	8·8	9·1	9·4	9·7	10	*3·0*
7·3	7·3	7·3	7·4	7·5	7·6	7·8	8·0	8·2	8·5	8·7	9·0	9·3	9·6	9·9	*3·1*
7·9	7·9	7·8	7·8	7·8	7·9	8·1	8·2	8·3	8·5	8·7	8·9	9·2	9·5	9·8	*3·2*
8·7	8·5	8·4	8·3	8·3	8·3	8·3	8·4	8·5	8·7	8·9	9·1	9·3	9·5	9·7	*3·3*
9·5	9·3	9·1	9·0	8·8	8·8	8·8	8·8	8·9	9·0	9·1	9·2	9·4	9·5	9·7	*3·4*
11	10	10	9·8	9·6	9·5	9·4	9·3	9·3	9·3	9·3	9·4	9·5	9·6	9·8	*3·5*
12	11	11	11	10	10	10	10	9·9	9·9	9·8	9·7	9·7	9·8	9·9	*3·6*
13	12	12	12	11	11	11	11	11	10	10	10	10	10	10	*3·7*
15	14	14	13	13	13	12	12	12	11	11	10	10	10	11	*3·8*
17	16	16	15	14	14	14	13	13	12	12	11	11	11	11	*3·9*
19	18	18	17	16	16	15	14	14	13	13	12	12	12	12	*4·0*
22	21	20	19	18	17	17	16	15	15	14	14	13	13	13	*4·1*
24	23	22	21	20	19	18	18	17	16	15	15	14	14	14	*4·2*
28	26	25	23	22	21	20	20	19	18	17	16	15	15	15	*4·3*
32	30	29	27	25	24	23	22	21	20	19	18	17	17	17	*4·4*
37	35	33	31	29	27	26	25	24	23	22	21	20	19	18	*4·5*
43	40	37	35	33	31	29	28	27	25	24	23	22	21	20	*4·6*
49	45	42	39	37	35	33	32	30	28	27	26	24	23	22	*4·7*
56	52	48	45	42	40	38	36	34	32	31	29	27	26	25	*4·8*
64	59	55	51	48	46	42	40	38	36	35	33	31	29	28	*4·9*
73	68	63	59	55	52	49	46	43	41	39	37	35	33	32	*5·0*
85	78	72	67	63	59	55	52	49	46	43	41	39	37	36	*5·1*
99	90	82	77	72	67	63	58	55	52	49	46	43	41	40	*5·2*
114	104	95	88	82	76	71	66	62	58	55	51	48	46	44	*5·3*
136	123	112	102	95	87	80	75	70	65	61	57	54	51	49	*5·4*
167	147	132	119	108	100	92	85	79	74	69	64	60	57	54	*5·5*
—	—	160	141	126	115	105	96	89	83	78	73	68	64	60	*5·6*
—	—	206	169	148	132	120	110	102	95	88	82	76	72	67	*5·7*
—	—	258	206	175	150	136	126	116	108	100	93	87	81	75	*5·8*
—	—	318	255	215	190	168	150	136	125	115	107	99	92	85	*5·9*
—	—	446	332	273	228	200	178	161	147	134	123	113	104	98	*6·0*
—	—	—	—	—	—	264	215	190	171	157	144	130	120	112	*6·1*
—	—	—	—	—	—	345	268	230	207	184	167	150	138	127	*6·2*
—	—	—	—	—	—	480	364	294	250	215	194	174	159	144	*6·3*
—	—	—	—	—	—	680	477	370	299	252	224	201	181	165	*6·4*
—	—	—	—	—	—	1047	680	456	368	312	268	237	212	191	*6·5*
—	—	—	—	—	—	—	—	—	—	—	—	280	248	223	*6·6*
—	—	—	—	—	—	—	—	—	—	—	—	338	297	266	*6·7*
—	—	—	—	—	—	—	—	—	—	—	—	412	362	320	*6·8*
—	—	—	—	—	—	—	—	—	—	—	—	525	446	390	*6·9*
—	—	—	—	—	—	—	—	—	—	—	—	809	584	491	*7·0*

TABLE IX.

Angle between Major-Axis and Axis of β_2 (Probability Ellipse) measured in degrees.

β_1

β_2	*0*	*·05*	*·1*	*·15*	*·2*	*·25*	*·3*	*·35*	*·4*	*·45*	*·5*	*·55*	*·6*	*·65*	*·7*	*·75*
2·0	0	12	23	28	31	33	35	36	37	38	39	40	41	41	42	42
2·1	0	11	21	25	28	30	32	34	35	37	38	39	40	40	41	41
2·2	0	10	19	23	26	28	30	32	33	35	36	38	39	39	40	40
2·3	0	10	18	22	25	27	28	30	32	34	35	37	38	38	39	39
2·4	0	9	17	20	23	25	26	29	31	33	34	35	36	37	38	38
2·5	0	8	15	18	21	23	25	27	29	31	33	34	35	36	37	37
2·6	0	7	14	17	20	22	24	26	28	30	31	33	34	35	35	36
2·7	0	7	13	16	19	21	23	25	26	28	29	31	32	33	34	35
2·8	0	6	12	15	17	19	21	23	25	27	28	30	31	32	33	33
2·9	0	6	11	14	16	18	20	22	23	25	26	28	29	30	31	32
3·0	0	5	10	13	15	17	19	21	22	24	25	27	28	29	30	31
3·1	0	5	9	12	14	16	18	20	21	23	24	26	27	28	29	30
3·2	0	5	9	12	14	16	17	19	20	21	22	24	25	26	27	28
3·3	0	4	8	11	13	15	16	18	19	20	21	22	24	25	26	27
3·4	0	4	8	10	12	14	15	17	18	19	20	21	22	23	24	25
3·5	0	3	7	9	11	13	14	15	17	18	19	20	21	22	23	24
3·6	0	3	6	8	10	12	13	14	15	16	18	19	20	21	22	23
3·7	0	3	5	7	9	11	12	13	14	15	16	17	18	19	20	21
3·8	0	3	5	7	9	10	11	12	13	14	15	16	17	18	19	20
3·9	0	2	4	6	8	9	10	11	12	13	14	15	16	17	18	19
4·0	0	2	4	6	7	8	9	10	11	12	13	14	15	16	17	18
4·1	—	—	3	5	6	7	8	9	10	11	12	13	14	15	16	17
4·2	—	—	3	4	5	6	7	8	9	10	11	12	13	14	15	16
4·3	—	—	2	4	5	5	6	7	9	10	10	11	12	13	14	15
4·4	—	—	2	3	4	5	6	7	8	9	9	10	11	12	13	14
4·5	—	—	1	2	3	4	5	6	7	8	8	9	10	11	12	13
4·6	—	—	—	—	—	—	4	5	6	7	7	8	9	10	11	12
4·7	—	—	—	—	—	—	3	4	5	6	6	7	8	9	10	11
4·8	—	—	—	—	—	—	3	4	4	5	5	6	7	8	9	10
4·9	—	—	—	—	—	—	2	3	3	4	5	6	6	7	8	9
5·0	—	—	—	—	—	—	1	2	2	3	4	5	6	6	7	8
5·1	—	—	—	—	—	—	—	—	—	—	—	—	5	5	6	7
5·2	—	—	—	—	—	—	—	—	—	—	—	—	4	5	6	7
5·3	—	—	—	—	—	—	—	—	—	—	—	—	3	4	5	5
5·4	—	—	—	—	—	—	—	—	—	—	—	—	2	3	4	5
5·5	—	—	—	—	—	—	—	—	—	—	—	—	1	2	3	4
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE IX.—(*continued*).

Angle between Major-Axis and Axis of β_2 (Probability Ellipse) measured in degrees.

β_1

·8	·85	·9	·95	1·0	1·05	1·1	1·15	1·2	1·25	1·3	1·35	1·4	1·45	1·5	β_2
43	43	44	44	45	45	46	46	46	46	47	47	48	48	49	2·0
42	42	43	44	44	44	45	45	45	46	46	47	47	48	48	2·1
41	41	42	42	43	43	44	44	45	45	46	46	46	47	47	2·2
40	40	41	41	42	42	43	43	44	44	45	45	46	46	47	2·3
39	39	40	40	41	41	42	42	43	43	44	44	45	45	46	2·4
38	38	39	39	40	40	41	41	42	42	43	43	44	44	45	2·5
37	37	38	39	39	39	40	40	41	41	42	42	43	43	44	2·6
36	36	37	38	38	39	39	40	41	41	42	42	43	43	44	2·7
34	35	36	36	37	38	38	39	40	40	41	41	42	42	43	2·8
33	34	35	35	36	37	38	38	39	39	40	40	41	41	42	2·9
32	33	34	34	35	36	37	37	38	38	39	39	40	40	41	3·0
30	31	32	33	34	35	35	36	37	38	38	39	39	40	40	3·1
29	30	31	32	33	34	34	35	36	37	38	38	39	39	39	3·2
28	29	30	31	32	32	33	34	35	36	37	37	38	38	38	3·3
26	27	28	29	30	31	32	33	34	35	36	36	37	38	38	3·4
25	26	27	28	29	30	31	32	33	34	35	35	36	36	37	3·5
24	25	26	27	28	29	30	31	32	33	34	34	35	35	36	3·6
22	23	24	25	26	27	28	29	31	32	33	33	34	34	35	3·7
21	22	23	24	25	26	27	28	29	30	31	32	33	33	34	3·8
20	21	22	23	24	25	26	27	28	29	30	31	32	32	33	3·9
19	20	21	22	23	24	25	26	27	28	29	30	31	31	32	4·0
18	19	20	21	22	23	24	25	26	27	28	29	30	30	31	4·1
17	18	19	19	20	21	22	23	25	26	27	28	29	30	30	4·2
16	17	18	19	19	20	21	22	23	24	25	26	27	28	29	4·3
15	16	17	18	18	19	20	21	22	23	24	25	26	27	28	4·4
14	15	16	16	17	18	19	20	21	22	23	24	25	25	26	4·5
13	14	15	15	16	17	18	19	20	21	22	23	24	25	25	4·6
12	13	14	14	15	16	17	18	19	20	21	22	23	24	24	4·7
11	12	13	14	15	15	16	17	18	19	20	21	22	23	23	4·8
10	11	12	13	14	14	15	16	17	18	19	20	21	21	22	4·9
9	10	11	12	13	13	14	15	16	17	18	19	20	20	21	5·0
8	9	10	11	12	13	13	15	16	16	17	18	19	20	20	5·1
7	8	9	10	11	12	13	14	15	15	16	17	18	19	19	5·2
6	7	8	9	10	11	12	13	14	14	15	16	17	18	18	5·3
6	7	8	8	9	10	11	12	13	13	14	15	16	17	17	5·4
5	6	7	7	8	9	10	11	12	12	13	14	15	16	17	5·5
—	—	6	7	7	8	9	10	11	12	13	13	14	15	16	5·6
—	—	5	6	6	7	8	9	10	11	12	12	13	14	15	5·7
—	—	4	5	5	6	7	8	9	10	11	11	12	13	14	5·8
—	—	3	4	5	5	6	7	8	9	10	10	11	12	13	5·9
—	—	2	3	4	4	5	6	7	8	9	9	10	11	12	6·0
—	—	—	—	—	—	—	—	6	7	8	9	9	10	11	6·1
—	—	—	—	—	—	—	—	5	6	7	8	8	9	10	6·2
—	—	—	—	—	—	—	—	4	5	6	6	7	8	9	6·3
—	—	—	—	—	—	—	—	3	4	5	6	6	7	8	6·4
—	—	—	—	—	—	—	—	2	3	4	5	6	6	7	6·5
—	—	—	—	—	—	—	—	—	—	—	—	5	6	7	6·6
—	—	—	—	—	—	—	—	—	—	—	—	4	5	6	6·7
—	—	—	—	—	—	—	—	—	—	—	—	3	4	5	6·8
—	—	—	—	—	—	—	—	—	—	—	—	2	3	4	6·9
—	—	—	—	—	—	—	—	—	—	—	—	1	3	4	7·0

TABLE X.

Probable Error of Criterion κ_2. Values of $\sqrt{N}\Sigma_{\kappa_2}$ for values of β_1, β_2.

β_1

β_2	·00	·05	·10	·15	·20	·25	·30	·35	·40	·45	·50	·55	·60	·65	·70
2·0	·000	·242	·332	·399	·454	·498	·545	·582	·631	·671	·716	·758	·806	·854	·899
2·1	·000	·271	·367	·430	·483	·521	·557	·600	·639	·678	·717	·760	·800	·843	·890
2·2	·000	·310	·415	·480	·527	·565	·597	·626	·656	·691	·728	·767	·809	·845	·890
2·3	·000	·355	·477	·550	·596	·635	·660	·678	·700	·725	·753	·790	·826	·858	·895
2·4	·000	·417	·560	·642	·691	·722	·748	·759	·770	·787	·806	·830	·857	·884	·914
2·5	·000	·500	·697	·771	·816	·841	·855	·860	·867	·873	·881	·892	·909	·928	·947
2·6	·000	·660	·840	·946	1·01	1·03	1·04	1·02	1·01	1·00	1·00	1·00	1·00	1·00	1·00
2·7	·000	1·04	1·30	1·35	1·34	1·33	1·30	1·27	1·24	1·19	1·15	1·12	1·10	1·09	1·08
2·8	·000	1·83	1·97	1·98	1·93	1·84	1·74	1·64	1·55	1·47	1·40	1·33	1·27	1·23	1·20
2·9	·000	3·51	3·71	3·42	3·00	2·66	2·42	2·22	2·05	1·89	1·73	1·61	1·51	1·45	1·39
3·0	—	18·8	9·89	6·94	5·30	4·30	3·62	3·10	2·73	2·43	2·18	2·00	1·84	1·71	1·61
3·1	·000	—	62·0	20·5	8·47	6·36	5·22	4·42	3·78	3·34	2·96	2·66	2·41	2·20	2·01
3·2	·000	7·82	91·7	—	49·7	20·2	12·2	8·48	6·44	5·09	4·28	3·71	3·21	2·86	2·58
3·3	·000	2·99	11·7	70·2	—	142	32·4	15·4	10·8	8·56	6·88	5·62	4·66	3·98	3·46
3·4	·000	1·82	4·80	13·8	55·5	—	344	182	28·9	16·8	11·8	8·69	6·88	5·60	4·76
3·5	·000	1·43	2·89	6·43	15·8	50·6	380	—	127	46·5	25·1	16·1	11·5	8·55	6·91
3·6	·000	1·17	2·18	4·08	8·00	17·2	46·5	215	—	277	76·4	35·4	22·4	15·8	11·5
3·7	·000	1·04	1·79	3·08	5·18	9·36	24·6	44·3	155	—	767	133	54·6	28·5	19·2
3·8	·000	·979	1·54	2·54	4·09	6·33	14·3	26·2	44·0	126	855	—	230	80·0	40·8
3·9	·000	·920	1·41	2·20	3·32	4·91	8·84	13·1	20·5	38·2	105	448	—	—	125
4·0	·000	·869	1·35	2·00	2·83	4·08	5·98	9·08	13·7	22·3	41·4	98·0	296	—	—
4·1	—	—	1·30	1·94	2·60	3·61	5·08	7·25	10·4	15·8	24·4	40·5	78·7	216	—
4·2	—	—	1·33	1·94	2·58	3·43	4·54	6·09	8·49	11·8	16·8	25·3	39·4	67·2	169
4·3	—	—	1·44	2·01	2·63	3·37	4·20	5·41	7·19	9·52	12·8	17·8	26·2	45·2	72·2
4·4	—	—	1·58	2·15	2·74	3·39	4·20	5·18	6·58	8·56	10·9	14·6	20·2	29·0	40·4
4·5	—	—	1·81	2·32	2·94	3·59	4·36	5·29	6·45	7·76	9·85	12·4	16·1	21·1	29·9
4·6	—	—	—	—	—	—	4·90	5·63	6·60	7·84	9·20	11·1	13·6	17·2	24·2
4·7	—	—	—	—	—	—	5·87	6·46	7·11	7·97	8·96	10·2	12·2	15·3	20·0
4·8	—	—	—	—	—	—	7·43	7·61	7·94	8·41	9·16	10·1	12·0	14·3	17·5
4·9	—	—	—	—	—	—	10·1	9·45	9·08	9·36	9·80	10·6	12·0	13·7	15·9
5·0	—	—	—	—	—	—	15·1	11·3	10·4	10·4	10·8	11·5	12·4	13·6	15·4
5·1	—	—	—	—	—	—	—	—	—	—	—	—	13·5	14·0	15·3
5·2	—	—	—	—	—	—	—	—	—	—	—	—	15·3	15·5	16·0
5·3	—	—	—	—	—	—	—	—	—	—	—	—	18·3	17·8	17·7
5·4	—	—	—	—	—	—	—	—	—	—	—	—	22·8	20·6	20·0
5·5	—	—	—	—	—	—	—	—	—	—	—	—	30·6	26·0	23·3
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE X.—(*continued*).

Probable Error of Criterion κ_2. *Values of* $\sqrt{N}\Sigma_{\kappa_2}$ *for values of* β_1, β_2.

β_1: *·75*	*·80*	*·85*	*·90*	*·95*	*1·00*	*1·05*	*1·10*	*1·15*	*1·20*	*1·25*	*1·30*	*1·35*	*1·40*	*1·45*	*1·50*	β_2
·949	1·00	1·06	1·12	1·18	1·25	1·31	1·39	1·46	1·56	1·64	1·75	1·86	1·97	2·11	2·25	*2·0*
·937	·992	1·05	1·10	1·16	1·22	1·28	1·35	1·42	1·49	1·58	1·67	1·77	1·88	2·00	2·12	*2·1*
·936	·987	1·04	1·09	1·14	1·19	1·25	1·31	1·37	1·43	1·51	1·59	1·69	1·79	1·89	1·99	*2·2*
·939	·982	1·04	1·08	1·12	1·17	1·22	1·27	1·32	1·38	1·45	1·53	1·62	1·70	1·79	1·87	*2·3*
·950	·990	1·03	1·07	1·11	1·15	1·19	1·24	1·29	1·34	1·40	1·47	1·55	1·62	1·69	1·77	*2·4*
·972	·998	1·03	1·07	1·10	1·14	1·18	1·22	1·27	1·32	1·37	1·43	1·49	1·55	1·61	1·68	*2·5*
1·01	1·03	1·06	1·09	1·12	1·15	1·18	1·21	1·26	1·31	1·36	1·41	1·46	1·51	1·57	1·63	*2·6*
1·07	1·08	1·09	1·11	1·14	1·17	1·19	1·21	1·25	1·30	1·35	1·39	1·43	1·48	1·53	1·59	*2·7*
1·17	1·15	1·16	1·17	1·18	1·19	1·21	1·23	1·27	1·31	1·35	1·39	1·43	1·47	1·52	1·58	*2·8*
1·33	1·28	1·26	1·26	1·25	1·25	1·25	1·27	1·29	1·32	1·35	1·39	1·43	1·47	1·52	1·57	*2·9*
1·53	1·47	1·42	1·38	1·36	1·34	1·34	1·34	1·35	1·36	1·38	1·40	1·43	1·46	1·51	1·56	*3·0*
1·85	1·72	1·62	1·56	1·52	1·49	1·46	1·44	1·43	1·43	1·43	1·44	1·45	1·47	1·51	1·57	*3·1*
2·34	2·13	1·96	1·83	1·75	1·68	1·62	1·57	1·54	1·52	1·51	1·50	1·50	1·51	1·54	1·58	*3·2*
3·03	2·69	2·44	2·23	2·06	1·94	1·85	1·77	1·72	1·67	1·63	1·60	1·59	1·58	1·59	1·61	*3·3*
4·08	3·50	3·06	2·74	2·49	2·29	2·14	2·02	1·92	1·84	1·78	1·73	1·70	1·68	1·68	1·69	*3·4*
5·72	4·75	4·12	3·56	3·15	2·83	2·59	2·38	2·22	2·09	1·99	1·91	1·85	1·81	1·79	1·79	*3·5*
8·85	6·90	5·71	4·79	4·15	3·68	3·22	2·90	2·67	2·49	2·31	2·20	2·09	2·00	1·96	1·92	*3·6*
14·2	10·4	8·22	6·69	5·70	4·81	4·17	3·67	3·31	3·05	2·81	2·62	2·45	2·29	2·19	2·10	*3·7*
24·9	18·1	13·8	10·1	8·16	6·71	5·76	4·92	4·28	3·84	3·48	3·16	2·91	2·68	2·50	2·36	*3·8*
65·5	36·5	21·7	15·8	12·2	10·0	8·17	6·75	5·75	5·00	4·43	3·93	3·52	3·18	2·91	2·72	*3·9*
242	86·6	48·5	30·0	20·9	15·4	12·0	9·61	7·86	6·60	5·67	4·90	4·34	3·88	3·47	3·19	*4·0*
—	374	127	62·9	38·3	26·0	18·3	14·4	11·5	9·15	7·53	6·45	5·66	4·98	4·38	3·80	*4·1*
—	—	—	200	91·0	51·6	32·7	23·4	17·4	13·4	10·6	8·70	7·50	6·48	5·52	4·61	*4·2*
144	478	—	—	314	112	70·0	42·9	29·1	21·3	16·1	12·7	10·3	8·55	7·24	6·25	*4·3*
62·8	135	—	—	—	580	192	93·9	55·8	37·1	26·5	19·8	15·4	12·1	9·74	8·26	*4·4*
42·8	68·3	119	280	—	—	—	286	126	72·8	46·4	32·2	23·9	18·3	14·6	11·8	*4·5*
32·3	44·7	62·7	99·6	240	742	—	—	—	181	91·0	58·9	41·0	30·0	22·3	16·9	*4·6*
26·2	33·8	46·1	68·0	105	240	532	—	—	—	260	128	76·7	50·4	36·0	26·6	*4·7*
21·6	27·1	35·0	47·3	66·8	104	182	413	—	—	—	403	172	99·3	63·0	44·8	*4·8*
18·9	22·6	26·9	33·7	44·0	61·5	84·2	115	337	—	—	—	—	249	140	80·8	*4·9*
17·8	20·7	24·6	30·1	37·8	48·9	66·0	97·0	157	286	—	—	—	—	—	172	*5·0*
17·2	20·0	23·0	27·1	32·5	40·6	51·5	69·2	99·8	147	253	559	—	—	—	—	*5·1*
17·3	19·3	21·7	24·8	29·5	35·4	43·1	54·7	70·6	94·6	138	216	—	—	—	—	*5·2*
18·0	19·7	21·4	23·9	27·1	31·5	37·0	44·2	54·5	69·3	93·2	132	205	380	—	—	*5·3*
19·6	20·5	21·6	23·3	25·6	28·6	32·8	38·0	44·8	54·0	68·6	94·0	130	185	—	—	*5·4*
22·3	22·0	22·5	23·6	25·2	27·5	30·5	34·3	39·4	45·4	54·7	67·3	86·5	116	169	275	*5·5*
—	—	—	24·5	25·8	27·6	29·8	32·2	35·6	39·6	44·8	51·4	63·2	83·0	118	168	*5·6*
—	—	—	26·8	27·8	28·8	30·0	31·5	33·8	36·5	39·8	44·4	53·1	67·0	85·2	116	*5·7*
—	—	—	30·7	30·8	31·0	31·3	32·0	33·4	35·3	38·2	42·2	48·4	56·6	69·9	87·9	*5·8*
—	—	—	38·4	36·0	34·4	33·5	33·1	34·1	36·0	38·7	42·2	47·0	53·1	61·9	74·8	*5·9*
—	—	—	50·4	42·5	38·3	36·8	36·4	36·5	37·9	40·0	43·0	46·6	51·3	57·8	66·2	*6·0*
—	—	—	—	—	—	—	41·0	41·0	41·6	42·7	44·6	47·2	50·3	55·0	61·5	*6·1*
—	—	—	—	—	—	—	48·2	47·0	46·1	46·0	46·7	48·0	50·0	53·5	58·5	*6·2*
—	—	—	—	—	—	—	57·3	54·0	51·6	49·8	49·1	49·4	50·0	52·5	56·8	*6·3*
—	—	—	—	—	—	—	79·4	66·6	58·6	54·2	51·8	51·2	50·8	52·5	55·5	*6·4*
—	—	—	—	—	—	—	128	84·0	66·9	59·9	55·9	53·9	53·1	53·6	55·4	*6·5*
—	—	—	—	—	—	—	—	—	—	—	—	—	57·4	57·0	56·7	*6·6*
—	—	—	—	—	—	—	—	—	—	—	—	—	63·5	61·5	59·2	*6·7*
—	—	—	—	—	—	—	—	—	—	—	—	—	73·6	67·8	63·5	*6·8*
—	—	—	—	—	—	—	—	—	—	—	—	—	97·2	77·7	70·5	*6·9*
—	—	—	—	—	—	—	—	—	—	—	—	—	134	96·0	82·0	*7·0*

MISCELLANEA.

I. Dr Galloway's "Canary Breeding."

BY C. B. DAVENPORT.

IN a paper published in *Biometrika*, Parts I and II of Vol. VII, Dr Galloway has criticised a "recent memoir" by me, "especially the material used" and the "use of terms." Criticism is always to be sought for by scientific workers and is usually beneficial to the progress of science; and, just as criticism is good, a rejoinder is often helpful to afford the original author, in the light of that criticism, opportunity to state any necessary change of conclusions or to make clear any difficulty which the critic has encountered. Accordingly I feel it to be my duty, although a not altogether agreeable one, to take the time and space for a rejoinder.

The principal points of criticism that Dr Galloway makes are (1) that I should "have selected the original stock with much greater care" (p. 2); (2) that my "sole criterion of a perfect crest...seems to be absence of a bald occiput" (p. 2); (3) that "yellow does not mean merely absence of dark feathering, as in Davenport's paper, but a particular quality of feather which may occur in a green, variegated or clear bird"; (4) that my statement that hybrids with the yellow canary "frequently show more or less of the canary yellow" is to be contradicted; and (5) that in my green × "yellow" matings, my "green" is not green but variegated and my "yellows" are also variegated. Besides these criticisms there are some points of difference in experimental results, namely; (*a*) in the offspring of "variegated" × clear green, or variegated (pp. 3, 4, 26), (*b*) in the mating of two crests, and (*c*) in the absence of homozygous crests. Finally, I shall be glad if I can assist the author and science by a few criticisms of some new points in his paper.

Of the five criticisms it will be observed that four deal with definitions and one with the material used by me. The latter point may be considered first. The criticism on my original stock is not directed towards any concealment on my part, for the data of origin, so far as known, are given and one or more paintings of original stock published. It is rather directed towards the unfitness of races that are bred for song to give evidence concerning inheritance of colour and towards the fact that my birds were not "pure-bred." I am somewhat embarrassed in replying to this criticism since the author does not define what he means by pure-bred. I know what notion I associate with "pure breeding," namely, continual self-fertilization, such as occurs in sweet peas and many other plants, but obviously that kind of pure breeding is unknown among canaries. Some fanciers, I know, mean by pure breeding mating inside the same race or variety; but this definition is hopelessly vague since there is no agreement as to the limits of relationship or similarity compatible with these terms. A few fanciers mean by "pure-bred" birds, birds having a characteristic *a* all of whose ancestors for several generations have the same characteristic *a*. For such, the offspring of Crested Norwich × Crested

Manchester Coppy would be "pure-bred." This meaning of "pure-bred" is at least clear; but its utility may be doubted; nor does such "pure breeding" insure uniformity of progeny, since even if the crested parents have had only crested ancestors for many generations they may still throw plainheads. As a matter of fact, all philosophical breeders of experience realize that the fetish of "pure breeding" is a will-of-the-wisp, and for the most part a hopelessly obscure, and unsatisfactory conception. Practically the experienced breeder need not greatly care (within limits) about the ancestry of his birds. A little experience in breeding will give him greater assurance as to the character of the germ plasm than the affidavits of the fancier. To return to my stock, they were live, healthy and mostly fecund birds,—*canary* birds, no doubt. That is about as far as it was necessary to go. If I offered the additional information that they might be classified as "Harz" or "Norwich," that might help a possible critic; but I laid little stress on the statement and, from my experience, think it unimportant. I would defy anybody to *classify* my canaries now, but nevertheless I know more about their germ plasm than most punctilious fanciers do about that of their pure breeds. And a knowledge of performance of germ plasm, not capacity for satisfying the conditions of the "Standard," is what I have worked for.

As for my terms, it is the privilege of an author to use them as he sees fit provided he defines them clearly. Not having the fortune to live at Aberdeen and to mingle with Scotch fanciers, I have, no doubt, defined some terms differently from my critic. That was not only harmless but necessary. I specifically defined "imperfect" crest as I was about to use it for the immediate purpose of the Table and four paragraphs of Text (p. 11) as "more or less bald on the occiput." I cannot find that my critic anywhere gives his definition of the term. He merely says (p. 2) "this bald spot accompanies most of the perfect crests of the present day." Our use of the word "perfect" in the two sentences differs—that is clear and sufficient.

Next, my critic objects to my sense of the word "yellow." I was not unaware of the use of "Yellow" by most English fanciers in a technical sense. Thus Blakston says (p. 94), "we speak of a Yellow Green or a Buff Green, a Yellow Cinnamon or a Buff Cinnamon, when it is patent that neither of these colours can be yellow or buff in reality; and the words, therefore, taken in their general application, are technicalities." Now I was not writing for English fanciers merely, I was writing for men of science without regard to nationality, and to have used "Yellow" in the fanciers' sense without a long explanation would have led to confusion. I preferred, in common with some other authors in English (and as clearly stated on page 14 of my book), to use *Jonque* and *Mealy* instead of "Yellow" and "Buff." That left me free to use yellow in what Blakston calls the *real* sense; the sense also in which the German fanciers, as exemplified by Karl Russ, use it when they speak of Hochgelbe and Strohgelbe. If I had intended to use yellow as a technical group-name I should certainly have printed it in capitals, as Blakston does, and as I print "Buff Cochin." As for the assertion that my definitions of "jonque" and "mealy" are inadequate, referring to colour merely, instead of including the form of the feathering and that of the whole body of the bird, I may say merely that not having been acquainted with my critic I could not have known his use of the terms; that my definitions were based on and are practically identical with those of Blakston's great English classic (p. 94). In so far as they do not include form of the body of the bird they are devoid of the probably fantastic associations of a special group of fanciers in the vicinity of Aberdeen. At least, I find in my hybrids no necessary correlation of jonque and mealy plumage colour with the form of the body of the bird.

As for the *emphatic* denial (p. 4) of my statement that hybrids of the yellow canary "frequently show more or less of the canary yellow," I need only ask any intelligent reader if the assertion is contradicted by my critic's statement that "the great bulk" of such hybrids are dark. "Frequently" implies less than half of them light; "the great bulk" more than half dark. I might willingly have added to my book the same words as my critic, except to avoid tautology. Finally, my critic's categorical statement (p. 32) that my "greens" and "yellows" are both variegated (in his sense) is as gratuitous as it is unscientific and false as a matter of fact.

We may next consider the differences between Dr Galloway's experimental results and mine.

(*a*) That, as stated on pages 3, 4, my critic should have got 15 unmottled to 5 mottled* offspring from non-crested birds, where 1 : 1 was expected is no argument against Mendelian results, since the numbers are insufficient. In my breeding, single families showed such deviation from expectation, but the totals of 32 : 30 revealed the true significance of the mating. As to my critic's divergent proportions with *crested* birds no conclusion can be drawn without a knowledge of his definition of "variegated." I suspect he includes in this category birds with pigmented crests ; I did not. There is an interesting correlation between crest and head pigment in my birds and so head pigment was disregarded in crested birds, and such as had no other pigment were called "clears" or "yellows."

(*b*) Dr Galloway finds that crest × *crest-bred plainhead* gives an equal number of crested and crestless young. As I showed in my paper this result follows when plainheads of *any* origin are mated with heterozygous *crests*. He finds that in the offspring of two crested parents, crests are to plainheads as 2 : 1. This is a very unanalytical grouping. The results are 1 : 0 or 3 : 1 according as parents are homozygous or heterozygous in crest.

(*c*) Dr Galloway finds no case of crest × plain that produces crests only. This is not strange, as at least twice as many heterozygous as homozygous crested are to be expected.

Passing now to a criticism of other points in Dr Galloway's paper one might first wish for a clearer table of matings and progeny, with individual numbers so that one could trace results in cases where the same bird is used several times in different matings. This information the appendix does not readily, if at all, afford. Secondly, one can only regret that, as so commonly the case with scientifically untrained writers, so much stress is laid on crude speculation, in this case as to the origin of domestic races of birds through a "cinnamon sport." In its application to poultry, certainly, the cherished theory is not appealing. And why is the commonest of Hervieux' canary colour types, translated literally "gray" instead of technically "green" (p. 13), said to be closely allied to the cinnamon canary (p. 18) ; and even given as a sort of synonym of the latter (p. 30) ?

At the end of this polemic it is a pleasure to speak of the valuable data in this paper of Dr Galloway's, the account of the inheritance in eye colour with its confirmation of the Bateson-Punnett theory and the conclusion that two Mealies cannot produce Jonques. With the general theory that the races of domestic animals have arisen by mutations the writer finds himself quite in accord with Dr Galloway. One can only wonder that, where the Mendelian theory is so often appreciated and supported, the author should occasionally fail to grasp it. And, best of all, it is a pleasure to see the increasing catholicity of the active editorship of *Biometrika* in accepting such a paper for publication†.

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* Mottled, as used by me, is *not* the equivalent of variegated as used by Blakston and, apparently, by Dr Galloway. It includes "light variegated," "heavily variegated" and "marked."

† [This statement of Dr Davenport's needs an editorial comment. No paper dealing with heredity from the Mendelian standpoint has ever been refused by this journal, although such papers would be declined if they were considered inadequate experimentally or theoretically. Biometric papers British, American, and foreign have been rejected on these grounds. K.P.]

II. Canary Breeding. A Rejoinder to C. B. Davenport.

By A. RUDOLF GALLOWAY.

In writing my "Canary Breeding," one of the chief objects I had in view, was to secure some sort of agreement between scientific men and fanciers ; for the attainment of such an end would undoubtedly result in an increase of knowledge, and would greatly benefit both parties.

Dr Davenport's frequent references to fanciers and their terms, and his acknowledgment of Blakston's *Great English Classic*, compel me to judge his statements from the fancier's point of view, as well as from that of the scientific man.

From the scientific standpoint only, his statements might be more consistent, though in all probability valueless ; from the fancier's point of view, which is pre-eminently that of *his* great authority Blakston, it is easy to show that his interpretation of terms is extremely faulty :—

In his Rejoinder, he makes the startling admission that he has in his tables classed dark crested birds as "clears or yellows." The very thought of this is enough to make Blakston turn in his grave, for the following is his (Blakston's) definition of a clear bird :—

"And here we draw the line" (at dark underflue), "and define 'Clear' to mean not having the remotest tinge of dark colour in quill, flue, or feather ; birds from which every trace of the green has been eliminated ; which have a clear, silky, snowy-white underflue and show by it that they have arrived at the goal" (Cassell's *Canaries and Cage Birds*, p. 100).

Such an interpretation of standard nomenclature is unwarranted, and indeed incomprehensible in one who regards Blakston with reverence and authority. It is needless to enter further into the meaning of the term "variegation," which Blakston so fully and clearly explains (pp. 95–101), for it is evident that Davenport's interpretation of it is his own conception (as indeed he admits), and consequently his results cannot be accepted by the initiated.

Another extract from the Rejoinder must be given in full :—

"As for the assertion that my definitions of 'jonque' and 'mealy' are inadequate, referring to colour merely, instead of including the form of the feathering and that of the whole body of the bird, I may say merely that not having been acquainted with my critic, I could not have known his use of the terms ; that my definitions were based on and are practically identical with those of Blakston's great English classic (p. 94). In so far as they do not include form of the body of the bird, they are devoid of the probably fantastic associations of a special group of fanciers in the vicinity of Aberdeen."

Nevertheless, if Dr Davenport will turn to p. 101 of the *Great English Classic* (of which, he will be glad to hear, a new edition will shortly be published—the original one having served its time and generation *) he will read as follows :—

"The feathering on the Buff birds is much denser than in the Jonques, the under-flue being very thick and long, as the fancier will be able to observe when he makes his first essay in washing. Altogether the Buff bird is *built on a larger and stouter scale*, and is, in every respect, the more *lusty* bird of the two."

It is hopeless for Dr Davenport to escape from his erroneous use of the word "yellow," for certainly Blakston never used it as he does, and if I am not greatly mistaken his appeal to German fanciers is equally at fault, for 'hochgelb' and 'weissgelb' mean undoubtedly jonque and mealy (yellow and buff). In support of this let me give a note received on Dec. 1st, 1908, from that well-known fancier and scientific writer C. L. W. Noorduijn* of Groningen :—

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* Cassell's *Canaries and Cage Birds* was published in London, Paris, *New York* and Melbourne, some *thirty* years ago.

"I mean by buff-yellow, light or whitish yellow (mealy) (weissgelb), in contra-distinction to clear or orange yellow (hochgelb)."

But even if we allow Dr Davenport to have his use of the term yellow, the mere fact that his matings take no account of the rudimentary distinction between the jonques and mealies, is amply sufficient to seriously detract from their value.

The following statements on Davenport's pages 6 and 7 must also be corrected :—

(1) That Hervieux in 1713 gives only 10 varieties (p. 7).

(2) That during the first half of the eighteenth century, the number of colour varieties was greatly increased, since Hervieux, in his edition of 1766, recorded 29 colour varieties (p. 7).

And (3) that the first edition of Hervieux was published in 1705 (p. 6).

The facts are as given in my "Canary Breeding" :—

(1) That there is no 1705 edition—the first one appearing in 1709.

(2) That in the 1709 edition, 28 varieties are mentioned.

This list also disproves the conjecture "that the frizzled characteristic is probably relatively recent, as it is relatively uncommon" for it occurs among the 1709 varieties.

It is perhaps superfluous to refer to the meaning of the word "frequently"; suffice it to say that the occurrence of 1 clear mule in 526 (see table on p. 4 of my "Canary Breeding") cannot be included under that category.

In the original draft of my paper, I found fault with Dr Davenport for translating the French word "gris" as green in his memoir (p. 7). This grumble was found wanting in the proof sheets, and I now refer to it only because in the Rejoinder the author finds fault with me for translating the same word "literally gray" instead of "technically green." The technicalities of science are worthy of reverence, but when they transform the colour grey into green, then there is something wrong, which should be corrected.

In reply to a query in the Rejoinder, I may state that grey and cinnamon sports are stated to be closely allied by me, because of my experience in keeping and breeding them—as may be seen throughout my paper (e.g. greenfinch family, p. 32).

With regard to the statement that I have not defined imperfect crest, it should not be necessary for me to refer to a whole paragraph on p. 2, where this is fully explained, nor to a special engraving on p. 24, which beautifully illustrates the modern crest of various degrees of perfection.

My "unanalytical grouping" of the progeny of double-crests (2 crests to 1 crest-bred plainhead) is due to the fact that I could not assume the existence of homozygous crests which I had not found.

Dr Davenport's tirade against pure-breeding is extraordinary as coming from the pen of an ultra-Mendelian, and from a follower of the great Blakston, who devotes the whole of Chapter XIII. to the discussion and praise of Pedigree breeding.

Let me again quote from the Rejoinder. "A few fanciers mean by 'pure-bred' birds, birds having a characteristic *a* all of whose ancestors for several generations have the same characteristic *a*. For such, the offspring of Crested Norwich × Crested Manchester Coppy* would be 'pure-bred.'" As the characteristic *a* must mean the crest and coppy (which however are very dissimilar) and as the rule in Crests and Coppies is to mate a crested bird with a crest-bred plainhead, a coppy with a coppy-bred plainhead, it is difficult to comprehend the sort of fancier to which reference is made.

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* Noorduijn discredits Davenport's Mottling Theory, criticizes adversely his inferences from Hervieux and Russ, and completes the similarity to my own position by showing that the frilled canary is not "probably relatively recent, as it is relatively uncommon"; he likewise disagrees with Davenport's crested experiments. (*Kritische Besprechungen und Referate. Archiv für Rassen- und Gesellschafts-Biologie*, Jahrg. 1909, S. 394.)

I must state, however, that our respective interpretations of the term "pure-bred" have nothing to do with the point at issue, which was my criticism of a result of Dr Davenport's—on the ground that pure-bred birds had *not* been used—purporting to overturn one of Russ', in which he (Russ) expressly stipulated that pure-bred birds were necessary. (See Davenport's quotation, p. 15.)

Concerning the "crude speculation" of my evolution theory, I sincerely hope that it will form the subject of serious discussion by competent authorities. I have many more facts in its support than could be produced in a paper which had to include such an amount of spade work.

With one statement in the Rejoinder, I am in complete agreement with Dr Davenport, viz. "I would defy anybody to classify my canaries now." He has succeeded in proving by his so-called scientific method, not only that a crestless bird is a perfect crest, but that it has a more perfect crest than three-fourths of the existing prize crested birds. By some ingenious allelomorphic scheme he might as easily prove that all bipeds were quadrupeds. From his false assumptions both in the matter of crest, and plumage-colour, it is quite impossible for him to arrive at any scientifically correct, or practically useful conclusion.

III. Inheritance in Canaries: A Study in Mendelism.

By DAVID HERON, M.A.

In 1908 there appeared a paper† on *Inheritance in Canaries*, by Chas. B. Davenport, Director of the Station for Experimental Evolution, Carnegie Institute of Washington, in which the author discusses the inheritance of form of crest and plumage-colour in canaries and concludes that those characters follow Mendelian rules.

These results are accepted without criticism by Bateson‡, and although Galloway§ has offered some criticism, it seems desirable that further attention should be called to the methods used in this paper.

It will not however be necessary to consider Davenport's discussion of the inheritance of plumage-colour; his treatment of the inheritance of form of crest on pages 8 to 13 and the general list of matings in Table E, p. 24, so far as it deals with crest-form, will be found quite sufficient for the present purpose, i.e. to measure the scientific weight of this paper.

Before considering his results a strong protest must be made against the very defective condition in which they have been presented. From the material we can only arrive at a lower limit to the number of blunders in those few pages (in all 114 lines of text and four Tables) by noting those cases in which the tables are self-contradictory, e.g., when a bird is said to be crested in one table and non-crested in another.

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* A redundant term!

† *Carnegie Institute of Washington Publication*, No. 95. Papers of the Station for Experimental Evolution, No. 10.

‡ *Mendel's Principles of Heredity*, pp. 37 and 43.

§ *Biometrika*, Vol. VII. p. 1 et seq.

In Table E, on p. 24, the full list of matings is given. As it stands, it is far from satisfactory, and the list of errors in the statement of form of crest which can be detected from a study of this table alone, is a large one. Only 66 matings in all are given.

In Experiments 501, 509, 616 and 723, a reference to Exp. 624 should be to Exp. 623. In Exp. 714, a reference to Exp. 625 should be to Exp. 624; Exp. 625 does not occur in this Table at all. In Exps. 716 and 725, a reference to Exp. 623 should be to Exp. 622. In Exp. 702, there is a mating with a bird which has no number at all. It is said to be a crested bird but there is no indication whether the crest is perfect or imperfect. Similarly no statement as to the presence or absence of baldness is made about Birds Nos. 50 (used twice), 269, and 240. Bird No. 110 used in Exp. 729, is not marked either for crest or baldness; it comes from Exp. 604 but it does not occur in the columns in which crest and baldness are indicated although it is marked "Yellow" in the colour columns. To balance matters, however, Bird No. 192 in this Experiment (604) is given in the columns for crest but is dropped in the columns for colour, while Bird No. 112 is entered twice in the colour columns. In Exp. 703, Bird No. 67 is said to come from Exp. 10; it really comes from Exp. 505. In Exp. 703 also, three birds are said to have died young when considering crest and four when dealing with colour. In Exp. 503, Bird No. 40 comes from Exp. 405, not from 505 as stated. In Exp. 709, Bird No. 143 comes from Exp. 609, not from 607 as stated. In Exp. 405, Birds Nos. 39 and 40 are said to have "crest present"; in Exps. 503 and 511, No. 40 is said to have "crest absent," while in Exp. 614, No. 39 is said to have "crest absent." As an example of an error of omission, there appears in Table II (Exp. 717), a mating between two birds which are not numbered; this mating does not occur in Table E at all.

Disregarding this serious list of blunders, however, we may now proceed to examine Tables I, II and III, and Davenport's deductions from them. Since those tables can only have been made up from the General List of Matings, Table E, we shall assume that Table E, in so far as it is not self-contradictory, represents the facts of the case*.

Dealing with presence or absence of crest, Davenport states (p. 8, 6 lines from bottom) that "crest is alternative in inheritance, for when crested and non-crested birds were paired, the offspring were either well crested or plain headed and there were no intergrades." Davenport here uses "non-crested" as equivalent to "plain headed." A few lines further on, however (p. 9, line 1), he says that Table I "gives an answer to the question whether non-crested heads are recessive to plain heads"; while in his conclusions on p. 23, he says that "crest is dominant over plain head." In the second of those three quotations "plain heads" ought to read "crested heads."

Turning now to Table I, we find again many blunders here.

For Exp. 514, read Exp. 509; for Exp. 613, read Exp. 614; for Exp. 614, read Exp. 615; for Exp. 615, read Exp. 616; for Exp. 623, read Exp. 622; for Exp. 624, read Exp. 623; for Exp. 711, read Exp. 712; for Bird No. 185 in Exp. 716, read Bird No. 186.

Exp. 619 gave 12 non-crested birds, not 11 as stated.

Further Bird No. 39 used in Exp. 614 (not Exp. 613 as stated) and said in Table I to have "crest absent," is said in Table E, Exp. 405, to have "crest present" and in Table E, Exp. 614, to have "crest absent." In Table I, however, the result of Exp. 405 is said to be two birds with "crest absent" and none with "crest present." In Table E, the result of Exp. 405 is said to be two crested birds and none without crest.

Similarly in Table E, Exp. 507 is said to give one bird with "crest present" (No. 71); in Table I however it is said to give one bird with "crest absent." This bird No. 71 is not used again, so further light cannot be thrown on the matter.

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* Davenport states (p. 9, line 1) that Table I is "extracted from the general table."

Again Exp. 726 in Table E, is said to be a mating of non-crested birds and thus ought to be included in Table I, while the father of Bird No. 142 used in Exp. 723 is crested, not non-crested as stated.

Davenport states that out of 102 offspring of non-crested parents, all were non-crested. It is true that in this conclusion he is supported by Galloway but the point to be insisted on here is that this conclusion cannot be reached from his own data since, according to Table E, Birds Nos. 39, 40, and 71, were crested birds and came from non-crested parents.

Turning now to Table II, we find again an extraordinary number of blunders.

For Exp. 710*a*, read Exp. 711; for Exp. 713, read 714; for Exp. 515, read 513; for Exp. 625, read 624; for Exp. 712, read 713.

Further, the results of Exps. 703, 604, 704 and 720, are not in accordance with the results of those experiments given in Table E.

In Exp. 703, instead of the proportion of non-crested to crested birds, 3 : 7, read 3 : 6; in Exp. 604, instead of 7 : 1, read 7 : 2; in Exp. 704, instead of 0 : 7, read 0 : 5; and in Exp. 720 instead of 0 : 1, read 0 : 2.

In Exp. 714 (not 713 as stated), Bird No. 240 is said to come from two crested parents. According to Table E, however, No. 240 is one of the "original stock of whose ancestry, consequently, nothing is known directly." Similarly in Exps. 503 and 511, Bird No. 3 is said to come from a mating of crest and non-crest, while on p. 11 and in Table E it is said to be "original stock."

Davenport's method of dealing with those tables is equally faulty.

In part 1 of Table II he deals with matings between crested birds. Now crested birds, on the assumption made by Davenport, that crest is dominant over absence of crest, may be either DD or DR, and if DD be mated with DD or DR all the offspring should be crested, while if DR be mated with DR, three-fourths of the offspring should be crested.

It is important therefore that DD's should be distinguished from DR's. If any of the offspring of a pair of crested birds are non-crested then both parents must be heterozygous; but if all the offspring are crested, it by no means follows that one or both of the parents is homozygous. So long as only crested birds are produced we cannot distinguish between DD's and DR's. All we can do is to appeal to the laws of probability and estimate the chance that all the offspring of a pair of crested birds, which are really heterozygous, shall be crested.

Now Davenport, as the author of a book on Statistical Methods, must be familiar with the various probabilities involved in those cases. He says that "at least two birds are homozygous in crest, No. 12 which has produced nine young all crested and No. 79 which has produced 11 young all crested." One is not surprised to find that according to Table E, No. 79 has produced only nine young all crested and this correction must first be applied. Further he says that No. 126 which has produced four young all crested is "possibly homozygous"; and that No. 9 has produced 16 crested birds and one non-crested.

Let us consider the probabilities involved in those cases. What is the chance that a pair of DR's, i.e. crested but heterozygous birds, will produce "n" young all crested? In the long run they will produce three crested birds to one non-crested bird, and thus the chances that n, $n-1$, $n-2$, etc. out of n will be crested are the successive terms of $(3+1)^n$ out of 4^n. Thus the chance that all out of nine shall be crested is $(\frac{3}{4})^9 = 1$ in 13.

Now he actually gets in his experiments the proportion 16 : 1. What is the probability of such a combination arising? It is

$$17 \cdot \frac{3^{16}}{4^{17}} = 1 \text{ in } 23.$$

Also the combination three birds without crest out of three arises. The chance that this will arise is still more remote—1 in 64.

Now Davenport rejects the combination 9 : 0 which occurs once in 13 times while he finds among his material combinations whose chance of occurring is far more remote, and yet retains them !

Homozygous and heterozygous birds in such cases can only be distinguished by increasing the number of experiments, but in cases of doubt crested birds should be mated with non-crested birds as then the chance that all of nine for example shall be crested is only 1 in $2^9 = 1$ in 512. To get the same probability when mating two crested birds, we must obtain 22 offspring. Davenport does not seem to recognise that whenever all the offspring are crested, whether there be only two or as many as nine, there is a possibility of one or both parents being homozygous.

Davenport's failure to test adequately the gametic constitution of the parents renders any appeal to his totals useless and a similar criticism applies to part 2 of the same table. All we can say is that in Table II, part 1, there are some matings which are DR × DR and others which may be DD × DD, DD × DR, or DR × DR, and that conclusions based on methods such as these are of no value.

His treatment of the inheritance of baldness however is even more faulty, and in Table III, one is confronted at the outset with a series of blunders and mis-statements which are absolutely unparalleled. In Table III, there are given the results of 26 matings, and in the following cases the statements made are contradicted either in the same table or elsewhere in the paper.

In Table A, Exp. 617, Bird No. 84 should be No. 82; and in Exp. 620, Bird No. 83 has a "perfect crest," not "imperfect" as stated. Exp. 620 should therefore be included in Table C and not in Table A. In Table B, α, Exp. 505, Bird No. 34 is said to have a perfect crest; in Exp. 513, C, β, it is said to have an imperfect crest, while in Table E, Exp. 403, it is said to have a perfect crest. In Table B, α, Exp. 505, four of the offspring are said to be non-crested; only two are given in Table E. In the same experiment (No. 505) Bird No. 7 is said to be without crest; in Table C, β, it is said to have a perfect crest, while in Tables I. and E it is said to be without crest. In Table B, α, Exp. 625 should be No. 624.

In Table B, β, among the offspring of Exp. 705, there are only three birds with perfect crest and not five as stated. In Exp. 714, in the same table, there is said to be one bird with imperfect crest and six in all. In Table E, for this experiment there are four birds with perfect crest, one with crest absent, and two which died young. No bird is marked as having an imperfect crest. According to his own figures instead of totals of 11 perfect crests and four imperfect crests there are nine perfect crests and three imperfect crests together with seven birds which are without crests; such birds are in the same table considered as perfect crested birds, i.e. they are not bald. There are thus 16 birds with perfect crests and three with imperfect crests.

Further, no imperfect crested birds are *known* to have occurred in the ancestry of Birds Nos. 157 and 240, and to state as he does that such is "probably" the case is to beg the whole question.

In Table C, α, Exp. 508, there should be one bird without crest. In Exp. 604, Bird No. 62 has a "perfect crest," not an "imperfect crest" as stated. This mating is therefore wrongly included in the Table. In Exps. 608 and 703, Bird No. 67 should have a "perfect crest," not an "imperfect crest" as stated. These matings should therefore be included in Table B and not in Table C. In Exp. 703, there are only two birds with perfect crest, not three as stated. In Exp. 704 there are only three birds with perfect crest, not five as stated. In Exp. 711 there are two birds with perfect crest and none with imperfect crest, not one of each as stated.

In Table C, *β*, in each of Exps. 513 and 621 (not 622 as stated) one bird without crest is dropped.

The climax is however reached in dealing with two matings between Birds No. 6 and 9.

In Table C, *a*, Exp. 510, this is said to be a mating of DR × RR; in Table C, *β*, Exp. 504, to be a mating of DD × RR. In the first case it is said to give four imperfect crested birds and none with perfect crest; in the second case it is said to give two perfect crested birds and one bird without crest. In the first case No. 6 is said to have an imperfect crest and No. 9 a perfect crest; in the second case No. 6 is said to have a perfect crest and No. 9 an imperfect crest. On p. 11, line 32, No. 9 is said to have a perfect crest and No. 6 an imperfect crest. The extraordinary versatility of these two birds is decidedly embarrassing and one is somewhat surprised to find that the tables in which such blunders occur are headed "Matings to Test Inheritance of Imperfect Crest." Davenport makes a similar blunder in dealing with two matings between Birds Nos. 7 and 34. It has already been pointed out that No. 7 has at one time no crest and at another time a perfect crest, and that No. 34 has at one time a perfect crest and at another time an imperfect crest. So also the mating 7 × 34 in Exp. 505 is said to be one of DD × DD or DD × DR, and in Exp. 513 the same mating is said to be one of DD × RR!

Again, Table B is divided into two sections *a* and *β*. Section *a* deals with matings DD × DD and DD × DR, while section *β* deals with the mating DR × DR. But the distinction is purely arbitrary; as has already been explained if a mating of birds with perfect crest results in one bird with imperfect crest, then the mating must be DR × DR but further we cannot go. The same criticism applies to his division of Table C into two parts.

Various conclusions from Table III are given on p. 13, e.g. "Table III shows that when two imperfect crested birds are mated, the offspring have imperfect crests (Table III, A). This indicates that absence of occipital feathering is recessive to its presence." He gives those matings in Table III, A, but omits Exp. 403 which is a mating between Birds Nos. 21 and 13. These birds are said on p. 11 to have imperfect crests. The result of Exp. 403 is, one bird with crest absent and one with crest present and perfect (No. 34). It has already been pointed out that this bird is stated twice to have a perfect crest and once to have an imperfect crest. The balance of probability being in favour of its having a perfect crest, the conclusions drawn are not justified and are flatly contradicted by Davenport's own material. Further, Exp. 620 should be deleted from Table A and placed in Table C, since Bird No. 83 has a "perfect crest" and not an "imperfect crest" as stated.

Again on p. 13 we find "when both parents have a perfect crest (being therefore DD or DR) all the offspring have the perfect crest (Table III, B, *a*)." This of course would only be the case for the matings DD × DD or DD × DR as stated in the Table, but the heading of Table III, B is "Neither parent has imperfect crest" and the table actually contains an incomplete list of matings in which at least one parent has a perfect crest and in which one parent has (in seven cases out of nine) no crest at all. These statements are all consistent with each other if non-crested birds are considered as having perfect crests, but Davenport excludes them in counting the offspring. It is quite justifiable for Davenport to group together birds with perfect crest and birds with no crest since neither according to him have the bald spot on the occiput, but, if he does so in considering the matings, he must also do so in counting the offspring, and this he fails to do. He actually leaves out of account altogether 39 birds out of 112.

It is impossible to tell what he intended to include in this table; he gives two cases of mating of perfect crest with perfect crest and seven cases of perfect crest with absence of crest. But if this is what he means, the list is far from complete. Assuming that his statements as to perfect crest, imperfect crest or absence of crest as given in Table E are correct, the following experiments must be added, Nos. 402, 502, 513, 604, 606, 608, 611, 701, 703, 709, and perhaps 514 and 702.

In those last two experiments it is not stated whether the crest is perfect or imperfect in the case of at least one of the parents, Birds Nos. 50, 269, and the mate of 269 which has no number.

Of this list, Nos. 513, 604, 608 and 703 are given in Tables III, C, α and β, but their inclusion is in flat contradiction to Table E.

Table III, C, α, is stated on p. 13 to consist of matings between perfectly feathered individuals that are probably hybrids between dominants and recessives, i.e. DR's and imperfectly feathered consorts. The heading of the Table is, however, "One Parent has Imperfect Crest." According to Table E, however, Bird No. 34 has a perfect crest and Exp. 513 should thus be included in Table B. As before, however, "absence of crest" is considered as equivalent to "perfect crest" in dealing with the parents, but birds without crest are rejected in counting the offspring.

In Table III, C the following should also be included: Exps. 511, 601, 620, 621 and perhaps 514, 613, 702 and 714, but absence of information in Table E leaves the matter in doubt.

The gametic scheme on p. 13 which assumes that in the crest there are two pairs of allelomorphs is not justified. It required the presence of bald plainheads, and out of Davenport's 154 non-crested birds none have been found to be bald. Galloway also says (*Biometrika*, Vol. VII. p. 25) that no bald plainheads have occurred in his experience.

The material used by Davenport is also unsatisfactory. Of the birds used in those experiments some are said to have been of the "Norwich" type, and in Plate I, Fig. 2, in Davenport's paper, is given an illustration of a crested Norwich canary. This is however so far removed from what is known as a Norwich crest that it seems desirable to consider the point further. I have examined carefully every Norwich canary exhibited at the 1910 exhibitions of the Scottish National Cage Bird Society at Edinburgh and of the London and Provincial Ornithological Society at the Crystal Palace, some hundreds in all, and saw no bird at all like that shown in Fig. 2. Such birds sometimes appear but they are rare and are certainly not representative of the class.

There are three well-marked types of Norwich canaries: the Norwich plainhead, the crest-bred plainhead, and the Norwich crest. These three birds are quite distinct and cannot be confused by anyone who has ever seen them. The crest-bred plainhead and the Norwich crest are usually bred together, but the Norwich plainhead is never bred with the other two. Now it is not clear whether Davenport's non-crested birds are plainheads or crest-bred plainheads or indeed whether he is aware of the distinction between these two types, but that they are quite distinct can be seen clearly from the illustrations given here. In Fig. 1, we have a typical Norwich plainhead. It is very neat in appearance and shows no trace of overhanging eyebrows and offers a strong contrast to Fig. 2 which is a crest-bred canary. In this bird the head feather is very dense, and when it is turned forward with the finger it reaches right over the tip of the beak. The illustration also shows the typically heavy eyebrows of this class.

In Fig. 3 is given an illustration of a Norwich crest. The crest is very regular and radiates from a well-defined centre; the eyes and beak are completely covered by the crest and it is clear that this bird has very little in common with the "Norwich crest" illustrated by Davenport.

It is obviously of the highest importance that the existence of these *three* distinct types should be recognised. In respect of crest the crest-bred plainhead occupies a distinctly intermediate position and cannot be classed either with the Norwich crest or with the Norwich plainhead.

It has thus been shown that every conclusion made by Davenport can be proved to be false from a study of his own material; that if a fact has to be stated twice the one statement is flatly opposed to the other and that blunder is heaped on blunder until patience is exhausted.

Yet such work is accepted as showing that Mendelian rules apply to Canaries! (See Bateson, *loc. cit.* pp. 37 and 43.)

Before the cautious man of science can be convinced of the truth of Mendelism, it will be necessary that in Mendelian writings a much higher standard of observational and statistical accuracy should be employed.

TYPES OF NORWICH CANARIES.

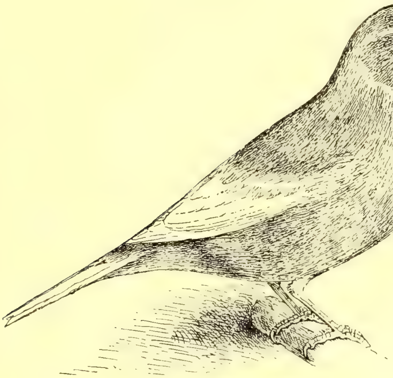

FIG. 1. Norwich Plainhead.

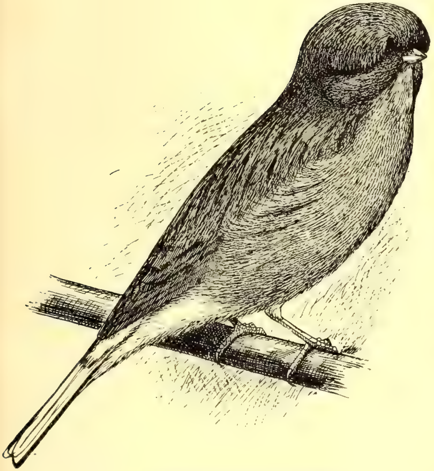

FIG. 2. Crest-bred Norwich Plainhead.

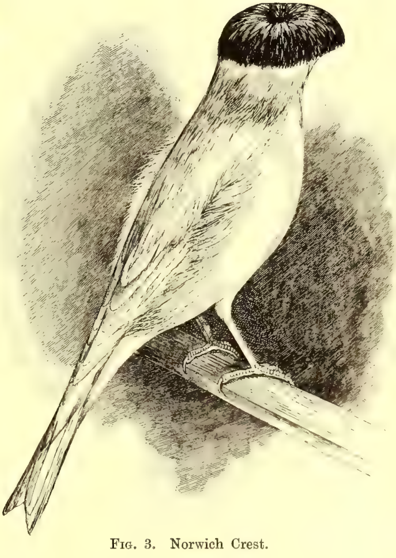

FIG. 3. Norwich Crest.

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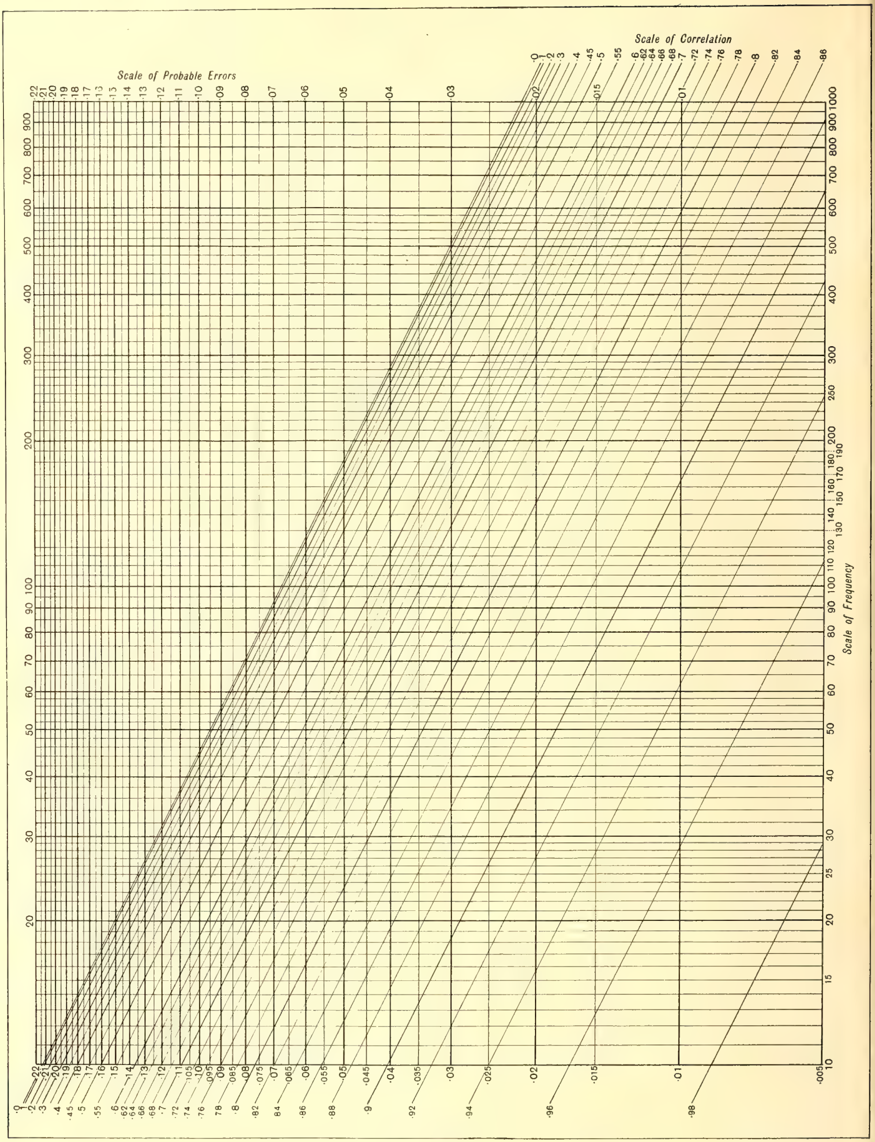

Abac for determining the Probable Errors of Correlation Coefficients.

BIOMETRIC NOTES.

I. An Abac for determining the Probable Errors of Correlation Coefficients.

By DAVID HERON, M.A.

The probable errors of correlation coefficients are so often required that the accompanying abac which enables them to be determined at once will save considerable labour. From this abac the probable errors can be read off correct to at least two decimal places; greater accuracy is seldom required.

The principle on which the abac is constructed is simple. If we take the well-known formula for the probable error of a correlation coefficient

$$E_r = \frac{\cdot 67449\,(1-r^2)}{\sqrt{n}},$$

where r is the correlation and n the number of observations, and express it in logarithmic form, we get

$$\log (E_r) = \bar{1}\cdot 8289755 + \log (1-r^2) - \tfrac{1}{2}\log n.$$

For any constant value of r there is thus a linear relationship between the logarithms of E_r and n. Hence by plotting E_r and n on logarithmic scales and ruling lines for a sufficient number of values of r, we can find E_r from r and n without difficulty.

The use of the abac can best be illustrated by an example. Let the correlation between two variables be ·55 and the number of observations 160; then to find the probable error of r, we must read along the perpendicular line from the number 160 on the scale of frequency until it crosses the diagonal line representing a correlation of ·55. The position of the point of intersection of these two lines is then read, by aid of the vertical lines, on the scale of probable errors and the value ·037 so obtained.

After the diagram had been drawn in pencil, the whole of the laborious work of ruling in the lines in ink and preparing the diagram for reproduction was undertaken by Miss H. G. Jones, and I have to thank her most heartily for her careful work.

II. On the Probable Error of a Partial Correlation Coefficient.

By DAVID HERON, M.A.

In a paper "On the Theory of Correlation for any Number of Variables, treated by a New System of Notation,"[*] G. Udny Yule has suggested that the probable error of a partial correlation coefficient, giving the correlation between two variables for a constant value of a third, is of the same form as the probable error of the direct correlation between any two variables, but his proof by reason of its very general nature is by no means easy to follow and it seems desirable to investigate the probable error of this coefficient on the lines laid down by Pearson and Filon in their classic paper on the probable errors of frequency constants*.

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[*] *Proc. R.S.*, Vol. 79, pp. 182 et seq.

Let ${}_3\rho_{12}$ be the correlation between variables 1 and 2 for a constant value of 3.

Then

$${}_3\rho_{12} = \frac{r_{12} - r_{13} r_{23}}{\sqrt{1 - r_{13}^2}\,\sqrt{1 - r_{23}^2}}.$$

Taking logarithmic differentials, we find,

$$\frac{d\,{}_3\rho_{12}}{{}_3\rho_{12}} = \frac{dr_{12}}{r_{12} - r_{13} r_{23}} + \frac{dr_{13}\,(r_{13} r_{12} - r_{23})}{(r_{12} - r_{13} r_{23})(1 - r_{13}^2)} + \frac{dr_{23}\,(r_{23} r_{12} - r_{13})}{(r_{12} - r_{13} r_{23})(1 - r_{23}^2)}.$$

Squaring, summing and dividing both sides by n, the total number of observations, and substituting for the standard deviation of the errors in r_{12}, r_{13}, r_{23}, and for the correlations of errors in those quantities the values given by Pearson and Filon in the paper already cited, we find after striking out the common factors $(1 - r_{13}^2)(1 - r_{23}^2)$

$$\begin{aligned}
&\frac{\Sigma^2{}_3\rho_{12}\,(r_{12} - r_{13} r_{23})^2 (1 - r_{13}^2)(1 - r_{23}^2)}{{}_3\rho_{12}^2} \,.\, n \\
&= (1 - r_{12}^2 - r_{13}^2 - r_{23}^2 + 2 r_{12} r_{13} r_{23}) \{(1 - r_{23}^2)(r_{23} - r_{12} r_{13}) r_{12} r_{13} \\
&\qquad + (1 - r_{13}^2)(r_{13} - r_{12} r_{23}) r_{12} r_{23} - (r_{13} - r_{12} r_{23})(r_{23} - r_{12} r_{13}) r_{13} r_{23}\} \\
&+ (1 - r_{13}^2)(1 - r_{23}^2) \{(1 - r_{12}^2) + (r_{13} - r_{12} r_{23})^2 + (r_{23} - r_{12} r_{13})^2 - 2 r_{23} (1 - r_{12}^2)(r_{23} - r_{12} r_{13}) \\
&\qquad - 2 (1 - r_{12}^2)(r_{13} - r_{12} r_{23}) r_{13} + 2 (r_{13} - r_{12} r_{23})(r_{23} - r_{12} r_{13}) r_{12}\}.
\end{aligned}$$

But the second part of the expression to the right of the sign of equality reduces to

$$(1 - r_{13}^2)(1 - r_{23}^2)(1 - r_{12}^2)(1 - r_{12}^2 - r_{13}^2 - r_{23}^2 + 2 r_{12} r_{13} r_{23}),$$

and the second factor of the first part reduces to

$$1 - r_{12}^2 - r_{13}^2 - r_{23}^2 + 2 r_{12} r_{13} r_{23} - (1 - r_{12}^2)(1 - r_{13}^2)(1 - r_{23}^2),$$

so that the whole expression on the right reduces at once to

$$(1 - r_{12}^2 - r_{13}^2 - r_{23}^2 + 2 r_{12} r_{13} r_{23})^2.$$

But

$${}_3\rho_{12}^2 = \frac{(r_{12} - r_{13} r_{23})^2}{(1 - r_{13}^2)(1 - r_{23}^2)},$$

so that

$$\Sigma_3\rho_{12} = \frac{1 - r_{12}^2 - r_{13}^2 - r_{23}^2 + 2 r_{12} r_{13} r_{23}}{\sqrt{n}\,(1 - r_{13}^2)(1 - r_{23}^2)}$$

$$= \frac{1 - {}_3\rho_{12}^2}{\sqrt{n}},$$

and the probable error of ${}_3\rho_{12}$ is

$$= \frac{\cdot 67449\,(1 - {}_3\rho_{12}^2)}{\sqrt{n}},$$

and is thus, as stated by Yule, of the same form as the probable error of the direct correlation between any two variables.

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[*] "On the Probable Errors of Frequency Constants and on the Influence of Random Selection on Variation and Correlation." By Karl Pearson, F.R.S., and L. N. G. Filon, B.A., *Phil. Trans. R.S.*, A, Vol. 191, pp. 229 et seq.

BOOK NOTES AND BIBLIOGRAPHY.

BOOK NOTES.

Interpolationsrechnung, by T. N. THIELE. Leipzig. B. G. Teubner, 1909, M. 10.

This interesting work which is divided into four chapters begins by evolving the general interpolation formula

$$X = A + (x-a)\left[\delta'(a, b) + (x-b)\{\delta''(a, \ldots, c) + (x-c)\{\delta'''(a, \ldots, d) + \ldots\}\}\right]$$

where $$\delta'(x, a) = \frac{X-A}{x-a} \text{ and } \delta''(x, \ldots, b) = \frac{\delta'(x, a) - \delta'(a, b)}{x-b}, \text{ etc.}$$

This formula does not depend as Newton's formula does on the tabulation of the function for even differences in the argument and in its practical application one differences the function in the usual way and then divides the differences so obtained by the corresponding difference in the argument. (Cp. De Morgan's *Differential and Integral Calculus*, Ch. XVIII., on Interpolation and Summation.) The remainder of the first chapter deals with Newton's Method and with series and the problems arising from them. The second chapter is devoted to symbolic work and although helpful and interesting in places it is rather overburdened with notation, and the third chapter gives auxiliary methods such as the graphic and explains the use of a "qualifizirte Differenz" which is based on expressions formed symbolically, $(E-a)f(x) = f(x+1) - af(x)$ for the first difference and $(E^2 - 2cbE + b^2)f(x) = f(x+2) - 2cbf(x+1) + b^2f(x)$ for the second difference and so on. These differences are applied in connection with exponential and periodic functions. The last chapter of the book deals with interpolation when there is more than one independent variable, but contains no new suggestions.

Interpolation is perhaps chiefly serviceable in its arithmetical applications and we can recommend the present work not only for its examples on this part of the subject but also for its interesting theoretical treatment.

W. P. E.

The Theory of the Construction of Tables of Mortality. A course of Lectures by G. F. HARDY, F.I.A., Delivered during the Session 1904—5. Published for the Institute of Actuaries by C. and E. Layton, 1909.

These Lectures which were intended mainly for Actuaries deal with graduation and the problems arising from it. They recount briefly the older methods of graduation such as the Graphic and Woolhouse's Difference Method but are chiefly concerned with the fitting of curves and with the practical use of Makeham's hypothesis for graduating a mortality Table.

It is a pity that owing to the omission of those parts of the subject with which his audience was familiar and to the fact that Mr Hardy has not had time to enlarge what were notes for a larger work the Lectures in their published form appear a little disjointed. This is unfortunate as it gives an impression of hurry which is enhanced by the fact that there is neither an index nor list of contents!

To biometricians Mr Hardy's treatment of frequency curves will not we think pass without a certain amount of criticism. On p. 39 the differential equation from which the Pearson-type curves are evolved is given as

$$\frac{1}{y}\cdot\frac{dy}{dx}=\frac{bx-x^2}{a-bx+cx^2},$$

which does not seem to lead to Type I, while the curves have been renumbered simply according to range which seems unsatisfactory, as, if we are to give up a numbering to which we are accustomed, it is well to adopt one which will not call for revision. This could have been done either according to the number of terms in the denominator of the right-hand side of the fundamental equation or according to the number of moments required, e.g. the curves requiring two moments numbered II a, II b, etc., and then further curves could be added if for any class of statistics a further term had to be included in the differential equation.

There are one or two other details with which we find it hard to agree; for example, on p. 46, Type III should we think have been used to fit the binomial series, while we find it hard to follow Mr Hardy's statement that Types III and IV are not specially useful to Actuaries, the reason given is hardly convincing, and we have found both curves in not a few cases from actuarial data.

To most people interested in graduation however the part of Mr Hardy's book which will be most attractive will certainly be his account of his own graduation of the select tables (i.e. tables giving the effect of medical selection) based on the new British Offices experience. Prior to this graduation there had we believe been no published graduation of a select table by any curve-fitting process, and though the two diagrams (pp. 74 and 75) do not betoken a very close fit it must be borne in mind that it is on the sum of the two curves that the agreement really depends, and little fault can be found with the resulting graduation. But apart from this one cannot expect a perfect method of graduation to be reached immediately, and Mr Hardy has probably done far more in actuarial circles for scientific graduation than most actuaries realise.

Our disagreement with some of the remarks made in Mr Hardy's book does not obscure but rather tends to accentuate in our minds the value of many of his actuarial suggestions, while we hope his work will stimulate other Actuaries to a study of general curve-fitting, so that the old methods which have been used so freely in the past may be displaced by those which Mr Hardy more than any other actuary has brought before the notice of his profession.

W. P. E.

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MOSQUITOES AND MALARIA.

A STUDY OF THE RELATION BETWEEN THE NUMBER OF MOSQUITOES IN A LOCALITY AND THE MALARIA RATE.

BY H. WAITE, M.A., B.Sc.

PROFESSOR ROSS, in his Report on the Prevention of Malaria in Mauritius*, has dealt briefly, from a mathematical point of view, with the influence of mosquitoes on malaria and he suggested in a letter to Professor K. Pearson that a further treatment from the more mathematical side would be of value. Such is the origin of the present investigation and it is to the above-mentioned Report that I am entirely indebted for the numerical data upon which this enquiry is based.

It has been proved conclusively that malaria is caused by the bite of a mosquito which has already bitten a previously infected person; and that the infection is probably conveyed only by the sub-family of mosquitoes called Anophelina. Hence, in order that the disease may spread in a neighbourhood, there must be present a source of infection in the shape of persons suffering from malaria, and a sufficient number of anophelines to act as carriers. Moreover, since recoveries are constantly taking place, it is clear that the increase or decrease in the number of cases will depend largely on the relative frequency of anophelines and of human beings.

Malaria Rate. The ratio of the number of persons affected with malaria to the total population of a district at a given time is called the *Malaria Rate* of the district at that time. In general, the rate is continually changing owing to (*a*) new infections, (*b*) recoveries, (*c*) emigration and immigration, (*d*) the birth and death rates, and (*e*) the extent to which cases are isolated, as well as owing to changes in the mosquito population.

As emigration and immigration vary considerably in different localities, and in the same locality at different times, their influence on the malaria rate cannot be satisfactorily dealt with except in particular cases where the necessary statistics are available; neither would results in general terms be of much practical use.

---

* *Report on the Prevention of Malaria in Mauritius*, by Ronald Ross, D.P.H., F.R.C.S., D.Sc., LL.D., F.R.S., C.B., published by Waterlow and Sons, London, 1908.

It is probable that the risk of infection may depend partly on the age and past history of an individual; for example, a person who has completely recovered from one or more attacks may become partially or entirely immune. So far, however, I have not found any statistics which deal with this phase of the question.

The combined effect of the birth and death rates will, as a rule, be small compared with the effects under (*a*) and (*b*); e.g., the birth rate for Mauritius in 1906 was 2·8 per thousand per month and the death rate 3·3; of the deaths, 1·2 per thousand were due to malaria. These figures do not differ appreciably from the average taken over a long period of years.

I propose, therefore, in the present paper, to confine my attention to the effects on the malaria rate of the first two of the above causes and to leave for a later consideration the minor contributory causes.

I shall also assume in this preliminary survey that both the human population and the anophelines are uniformly distributed over the area in question; that persons of all ages, whether they have previously suffered from malaria or not, are equally liable to infection; that the number of anophelines per unit of the population remains approximately constant during the period under review; and further that infected and non-infected persons are equally exposed to the risk of being bitten.

It is obvious that the results so obtained will require modification in practical application and ought to be taken into account when the theory is developed to a higher approximation; they will be, at the best, tentative and suggestive, but it is hoped that they will tend to throw light on the problems which arise when the specialist has to consider remedies for the disease.

Infection Rate. The number of new infections in a locality during any particular month has been called the *Monthly Infection Rate.* This depends chiefly on the number of infected anophelines in the neighbourhood, that is, the number which have bitten previously infected persons.

I shall in general follow the notation adopted by Professor Ross (*vide Report*, p. 31, *et seq.*).

Let p denote the average population of the locality during the enquiry, m the number of infected persons*, h the number of healthy persons at the beginning of the first month and im the average number of the infected persons whose blood contains enough of the malaria parasites to infect anophelines; evidently i is a fraction. Next, let a denote the average number of anophelines in the locality capable of carrying malaria and ba the average number of these which succeed in biting a single person during the month; here b must be a very small fraction, since probably many mosquitoes never succeed in biting men at all, and of those that do succeed, only a few will have bitten a particular individual. Hence the average number of anophelines which have bitten the suitably infected persons during the month will be *baim*.

---

* This differs from Professor Ross' use of m.

Now if any of these insects are, in their turn, to infect human beings, they must survive for at least a week or ten days, in order to give time for the parasites to mature within them; and by no means all of them will survive so long. Let s be the average proportion of anophelines which can survive for that period; then *sbaim* will be the number of infected mosquitoes which have survived long enough to infect men in their turn, where s is also a fraction. But not all of these will find opportunities to bite human beings again, though they have survived long enough to do so. Let f be the proportion which succeed in biting. Then *fsbaim* will be the average number of infecting anophelines which succeed in biting men; also, if each of these bite a separate person and only one person, the same expression will denote the average number of persons bitten by infecting mosquitoes during the month. Call this number n. The most probable distribution of the n bites between malarial and non-malarial persons will be in the ratio $m : h$; so that the number of new infections during the month will be $fsbaim\,h/p$, or nh/p *.

Now each time a healthy person is bitten by an infecting mosquito the number of malarial patients is increased by unity, while the bites inflicted on those who are already suffering from the disease make no difference in the number of cases. This is equivalent to increasing m and decreasing h by h/p for each bite.

[By taking mean values in the above I am assuming that if

$$z = F(x_1, x_2, \ldots x_n),$$

then $\bar{z}$ may be taken $= F(\bar{x}_1, \bar{x}_2, \ldots \bar{x}_n)$, and I have been led to make this assumption in a *first* investigation by finding that the modal values of the distribution approximate very closely to the means in a number of particular cases fully worked out.]

Recovery Rate. The ratio of the number of recoveries to the total number of malaria cases during any period is called the *Recovery Rate* for that period. Professor Ross has estimated that of a given number of infected persons 50 °/。 are ill after three months, that is, the recovery rate is ·5 per three months.

If m_0 be the number of cases at the beginning of the period, m_1, m_2, m_3, at the ends of the first, second and third months respectively, and r the recovery rate per month,

$$\begin{aligned} m_1 &= m_0 - m_0 r \;\; = m_0(1-r), \\ m_2 &= m_1(1-r) = m_0(1-r)^2, \\ m_3 &= m_2(1-r) = m_0(1-r)^3. \end{aligned}$$

---

* Professor Ross assumes that to a first approximation n will be the number of new cases in the month.

But
$$m_3 = m_0(1 - {\cdot}5),$$
$$\therefore \quad (1-r)^3 = {\cdot}5,$$
$$\log(1-r) = \tfrac{1}{3}\log {\cdot}5$$
$$= \bar{1}{\cdot}899657,$$
$$\therefore \quad 1 - r = {\cdot}7937,$$
and
$$\therefore \quad r = {\cdot}2063 \text{ per month.}$$

In the numerical examples it will be necessary to know the recovery rate for the average period between two consecutive bites. Suppose there are n bites in a month and r is the recovery rate for the period between two consecutive bites, then, as before,
$$(1-r)^n = 1 - {\cdot}2063$$
$$= {\cdot}7937,$$
$$\log(1-r) = \frac{1}{n}\log {\cdot}7937$$
$$= \frac{1}{n} \times \bar{1}{\cdot}899657$$
$$= -\frac{{\cdot}100343}{n},$$
whence the value of r is readily obtained.

I will now consider a formula for giving the number of cases at the end of a month in terms of the number at the beginning, the total population, the number of anophelines and the recovery rate.

Let m_0 be the number of cases at the beginning of the month out of a population p; m_t the number after the t^{th} bite and r the recovery rate when there are n infecting bites in the month; then
$$m_t = m_{t-1} + \frac{p - m_{t-1}}{p} - rm_{t-1},$$
or
$$m_t = m_{t-1}\left(1 - \frac{1}{p} - r\right) + 1.$$

Put R for $\left(1 - \frac{1}{p} - r\right)$ and this equation may be written
$$m_t = m_{t-1}R + 1.$$

Now r remains constant for the month under consideration and therefore also R. Hence
$$m_{t-1} = m_{t-2}R + 1,$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$
$$m_2 = m_1 R + 1,$$
$$m_1 = m_0 R + 1.$$

From these equations
$$m_t = m_0 R^t + \frac{1 - R^t}{1 - R},$$

and therefore, for the end of the month

$$m_n = m_0 R^n + \frac{1 - R^n}{1 - R} \quad \text{...............................(A).}$$

Now, for any particular month $n = fsbaim$ where m is the number of cases at the beginning of the month, (or the average number for the first half of the month and the second half of the previous month); and Professor Ross has estimated the approximate values of the constants f, s, b, i to be $\frac{1}{4}, \frac{1}{3}, \frac{1}{4p}$ and $\frac{1}{4}$ respectively; hence

$$n = \frac{1}{4} \cdot \frac{1}{3} \cdot \frac{1}{4p} \cdot \frac{1}{4} \cdot am$$

$$= \frac{am}{192p},$$

also $$r = \cdot 2063 \text{ when } n \not> 1,$$

and $$\log (1 - r) = -\frac{\cdot 100343}{n} \text{ when } n \not< 1.$$

We can thus by successive applications of the formula (A) obtain a series of values of m at the ends of consecutive months when we know the value at the beginning of the period, the total population and the average number of anophelines in the neighbourhood. Although it is impossible to estimate this last quantity with any degree of accuracy, yet by comparing the results of numerical examples, using different values of a, we get an idea of the effect on the malaria rate of reducing the anophelines in any given proportion.

Condition for a Stable Population. Suppose that a certain locality has a population p and m_0 cases of malaria. This latter will remain stationary if the number of new cases is equal to the number of recoveries. With a given value of m_0 and p the number of new infections depends on the number of anophelines in the neighbourhood. Hence with the anophelines present in a certain proportion to the human population, the malaria rate is stationary and if they are present in a greater or less proportion there will be a corresponding increase or decrease in the number of cases.

Let m_1 be the number of cases at the end of the month; then, by Equation (A)

$$m_1 = m_0 R^n + \frac{1 - R^n}{1 - R}.$$

The value of m is stationary if $m_1 = m_0 = m$, i.e.

$$m (1 - R^n) = \frac{1 - R^n}{1 - R},$$

or $$\left(m - \frac{1}{1 - R}\right)(1 - R^n) = 0,$$

$\therefore$ either $$1 - R^n = 0 \quad \text{......................................(i),}$$

or $$m - \frac{1}{1 - R} = 0 \quad \text{.....................................(ii).}$$

If $1 - R^n = 0$, $n = 0$ for $R = 1 - \frac{1}{p} - r$ and is therefore fractional; also if $n = 0$ the case reduces to that in which there is an entire absence of mosquitoes: hence in this enquiry $1 - R^n \neq 0$ and we must proceed with Equation (ii). This may be written

$$r = \frac{1}{m} - \frac{1}{p}.$$

Now when $n > 1$

$$\log(1 - r) = -\frac{\cdot 100343}{n}.$$

Hence by giving a series of values to m we may find the number of anophelines corresponding to any stable population.

Taking $p = 1000$, $m = 5$, we get

$$r = \cdot 2 - \cdot 001 = \cdot 199,$$

$$\therefore \quad \log(1 - r) = - \cdot 096367,$$

$$\therefore \quad \frac{\cdot 100343}{n} = \cdot 096367,$$

and

$$n = 1 \cdot 04125.$$

But

$$n = \frac{am}{192p},$$

$$\therefore \quad a = \frac{192 \times 1000 \times 1 \cdot 04125}{5}$$

$$= 39984.$$

Similarly we get the table of corresponding values of m and a for stable populations given below.

TABLE A.

Table of Corresponding Values of m and a for a Stable Population of a Thousand.

m	*a*	*m*	*a*	*m*	*a*
5	39984	*150*	52032	*600*	110914
8	41886	*175*	53648	*650*	126720
10	42553	*200*	55339	*700*	147892
15	43539	*250*	59062	*750*	177525
20	44148	*300*	63308	*800*	221752
25	44605	*350*	68193	*850*	295897
50	46251	*400*	73883	*900*	444118
75	47662	*450*	80612	*950*	885583
100	49069	*500*	88680		
125	50520	*550*	98534		

Again, suppose $m = 5$ and that a has such a value as will make $n = 1$, i.e.

$$\frac{5a}{192000} = 1 \text{ or } a = 38400.$$

We now have $r = \cdot 2063$ and $rm = 1\cdot 0315$; hence $rm > n$; or the number of recoveries exceeds the number of new cases and the value of m decreases continuously.

From the curve connecting these quantities (Diagrams I and II), we see at a glance the value of m corresponding to any given value of a and we gather that any appreciable change in the number of anophelines will involve a corresponding gradual change in the number of cases; or, in other words, that, given any definite number of anophelines per thousand of the population there is a particular malaria rate which will ultimately be reached by a gradual change in the value of m.

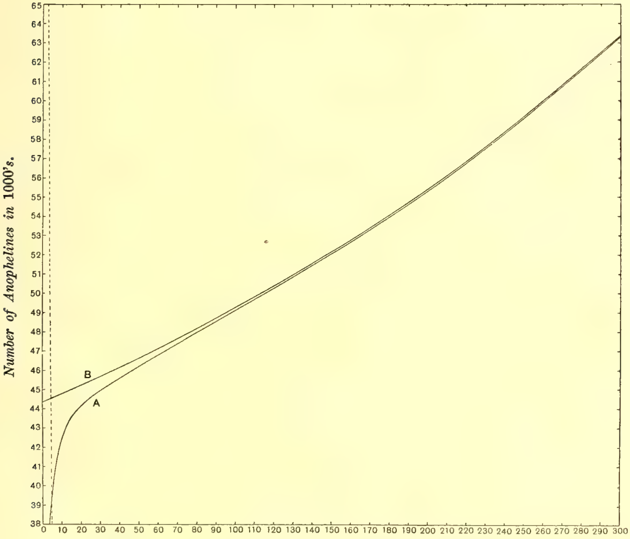

DIAGRAM I.* A. Stable Population Curve, Part 1. B. Curve of Equation (ii) p. 429.

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* See p. 429, for the construction of these curves.

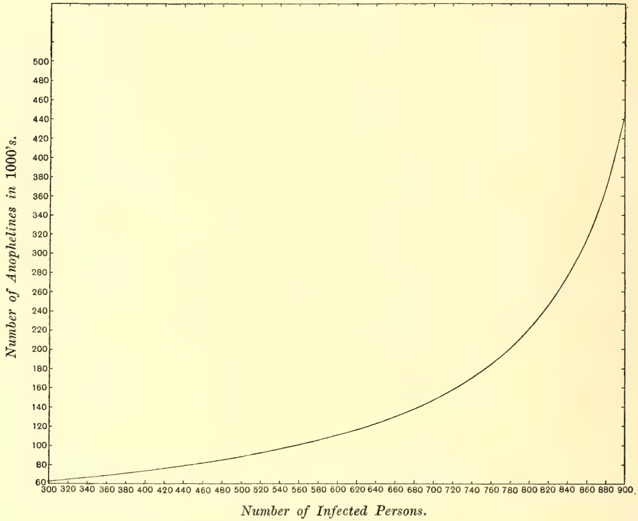


Diagram II.* Stable Population Curve, Part 2.

The direct relation between m and a (these symbols denoting corresponding values for a stable population) may be investigated as follows:

When $n \not< 1$, we have (p. 424)

$$(1-r)^n = \cdot 7937,$$

$$\therefore \quad \log_e(1-r) = \frac{1}{n}\log_e \cdot 7937$$

$$= -\frac{\cdot 23104}{n},$$

or

$$1-r = e^{-\frac{\cdot 23104}{n}}.$$

Now for a stable population

$$r = \frac{1}{m} - \frac{1}{p},$$

also

$$n = \frac{am}{192p},$$

$$\therefore \quad 1 + \frac{1}{p} - \frac{1}{m} = e^{-\frac{\cdot 23104 \times 192p}{am}}$$

$$= e^{-\frac{44 \cdot 36p}{am}}.$$

---

* See p. 429 for the construction of this curve.

Put c for 44·36 and the equation is*

$$1+\frac{1}{p}-\frac{1}{m}=e^{-\frac{pc}{am}} \quad \text{..............................(i).}$$

Expanding the right-hand side we get, to a first approximation,

$$1+\frac{1}{p}-\frac{1}{m}=1-\frac{pc}{am},$$

which reduces to

$$a=\frac{p^2c}{p-m} \quad \text{..................................(ii).}$$

A table of values of a and m found from Equation (ii) is given in Table B below; the graph is the upper curve in Diagram I, the portion of the graph common to this equation and to Table A, is given in Diagram II.

The graph of Equation (ii) fits very closely that of Table A when m exceeds 100, and accurately when m approaches p; for small values of m however the two

---

* The general form of this curve is given below. It has three branches :—(a) in the first quadrant, the line $m=p$ being an asymptote ; this branch ends in a *point d'arrêt* at $\left(\frac{p}{p+1}, 0\right)$ on the m axis ; (b) in the second quadrant with the axes of coordinates as asymptotes ; (c) in the fourth quadrant, the axis of m and the line $m=p$ being asymptotes. That part of the branch (a) which is above the hyperbola $am=192p$, alone has any direct application to the present problem.

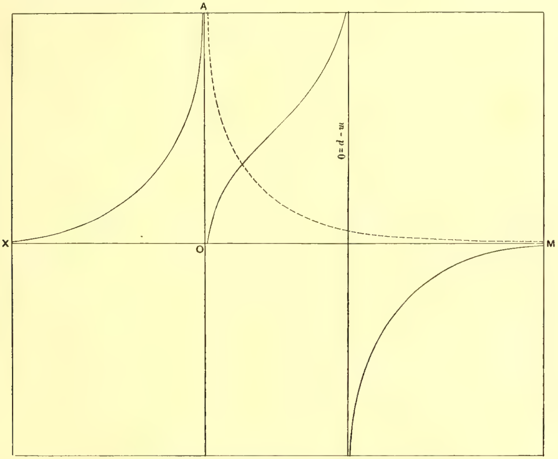


DIAGRAM III. General Form of Curve of Equation (i). The dotted line is the hyperbola which limits the practical solution of the part of the curve in the quadrant AOM.

curves differ considerably and it is necessary to proceed with Equation (i) to a higher degree of approximation.

TABLE B. (*Equation* (ii).)

m	a
0	44360
10	44808
50	46695
100	49288
150	52188
200	55450
250	59146
300	63371

Writing Equation (i) in the form

$$-\frac{pc}{am} = \log_e\left(1 + \frac{1}{p} - \frac{1}{m}\right)$$

and expanding the right-hand side, we have

$$-\frac{pc}{am} = \left(\frac{1}{p} - \frac{1}{m}\right) - \frac{1}{2}\left(\frac{1}{p} - \frac{1}{m}\right)^2 + \frac{1}{3}\left(\frac{1}{p} - \frac{1}{m}\right)^3 - + \dots,$$

or

$$\frac{pc}{am} = \frac{p-m}{pm} + \frac{1}{2}\frac{(p-m)^2}{p^2m^2} + \frac{1}{3}\frac{(p-m)^3}{p^3m^3} + \dots.$$

Putting z for $\frac{p-m}{pm}$, this may be written:

$$\frac{pc}{am} = z + \frac{1}{2}z^2 + \frac{1}{3}z^3 \text{ approx.,}$$

or

$$\frac{am}{pc} = \frac{1}{z}\left(1 + \frac{1}{2}z + \frac{1}{3}z^2\right)^{-1}$$

$$= \frac{1}{z}\left\{1 - \frac{1}{2}z - \frac{1}{3}z^2 + \left(\frac{1}{2}z + \frac{1}{3}z^2\right)^2 - + \dots\right\}$$

$$= \frac{1}{z}\left\{1 - \frac{1}{2}z - \frac{1}{3}z^2 + \frac{1}{4}z^3 + \dots\right\}$$

$$= \frac{1}{z} - \frac{1}{2} - \frac{z}{12} \text{ approx.,}$$

i.e.

$$a = \frac{p^2c}{p-m} - \frac{pc}{2m} - \frac{c(p-m)}{12m^2} \text{ ....(iii).}$$

The above relations cease to hold good when $n < 1$, i.e. when $am < 192p$; hence the limit of applicability of the *Stable Population Curve*, obtained from Equation (iii), is given by its intersection with the hyperbola

$$am = 192p.$$

Part of this hyperbola is shown on the left of Diagram I as a broken line from which it appears that in a population of a thousand, there can be no stable condition with a smaller average number of infected persons than about 4·5.

The numbers in Table A are all found for a population of a thousand, but the general form of Equation (iii) is applicable to a population of any magnitude so long as the number of infecting bites does not fall below one per month. For example, when $p = 20000$, and $m = 200$, Equation (iii) gives $a = 893941$, while the formulae on p. 426 give $a = 893966$, the difference being less than ·003 per cent.

It will be of interest to compare the values of a found from Equation (iii) with those of Table A in a few typical cases:

m	a, by Equation (iii)	a, from Table A
10	42554	42553
50	46249	46251
100	49067	49069
500	88676	88680

In the last three cases the term $\frac{c(p-m)}{12m^2}$ is negligible, while in the last case the term $\frac{pc}{2m}$ is only about ·01 per cent. of the whole.

Numerical Examples. In the numerical illustrations which follow, the increase or decrease in malaria, month by month, has been obtained by successive applications of Equation (A), p. 425, in the form

$$m_s = m_{s-1}R_s^{n_s} + \frac{1 - R_s^{n_s}}{1 - R_s},$$

where

$$n_s = \frac{am_{s-1}}{192p};$$

$$R_s = 1 - \frac{1}{p} - r_s,$$

$$\log(1 - r_s) = -\frac{\cdot 100343}{n_s} \qquad [n > 1],$$

$$r_s = \cdot 2063 \qquad [n \not> 1].$$

Seven-figure tables have been used throughout in the calculations but the results are given to one place of decimals only.

[*Note.* It is assumed, in all the examples, that Professor Ross' recovery rate holds. This would rapidly reduce a large number of cases to the vanishing point in the absence of new infections, whereas it is well known that a few persons suffer from relapses after living many years in countries where there is no possibility of re-infection. (See *Report,* p. 16.)]

Example 1, Case (i). $p = 1000$, $m_0 = 50$, $a = 64000$. (The number of anophelines for a stable population is about 46000.)

TABLE C.

(*Giving the value of m for consecutive months.*)

Months	1st Year	2nd Year	3rd Year	4th Year	5th Year	6th Year
1	53·9	118·0	198·8	258·5	286·5	297·7
2	58·0	124·6	205·0	261·7	287·9	298·2
3	62·4	131·3	211·0	264·9	289·2	298·5
4	67·0	138·0	216·8	267·8	290·3	299·0
5	71·7	144·9	222·4	270·5	291·4	299·4
6	76·7	151·8	227·7	273·1	292·4	299·6
7	82·0	158·6	233·0	275·5	293·3	300·1
8	87·5	165·5	238·0	277·4	294·3	300·4
9	93·2	172·6	242·6	279·5	295·0	300·6
10	99·1	179·5	247·0	281·5	295·7	301·0
11	105·2	186·1	251·2	283·3	296·5	
12	111·6	192·6	254·9	284·9	297·1	

Example 1, Case (ii). $p = 1000$, $m_0 = 50$, $a = 24000$.

TABLE D.

Months	1st Year	2nd Year	3rd Year	4th Year
1	45·1	13·8	5·8	4·98
2	40·7	12·6	5·7	4·96
3	36·8	11·5	5·5	4·94
4	33·2	10·5	5·4	4·92
5	30·1	9·6	5·3	4·91
6	27·2	8·8	5·3	4·90
7	24·7	8·0	5·2	4·89
8	22·4	7·4	5·1	4·88
9	20·3	6·9	5·1	4·87
10	18·3	6·5	5·1	4·87
11	16·6	6·2	5·0	4·86
12	15·2	6·0	5·0	4·86

It will be seen from this example that with a comparatively small number of infected persons and a quantity of anophelines somewhat in excess of the number which would keep the population stable, we get a steady rise in the malaria rate until nearly one-third of the whole population are infected; on the other hand, the destruction of rather more than half the mosquitoes will produce a fall in the rate until the cases are so few that they can be easily and effectively dealt with.

For the graphs see Diagram IV.

Example 2. $p = 1000$, $m_0 = 500$, $a =$ (i) 96000, (ii) 48000, (iii) 24000, (iv) 0. (The number of anophelines corresponding to a stable population would be about 89600.)

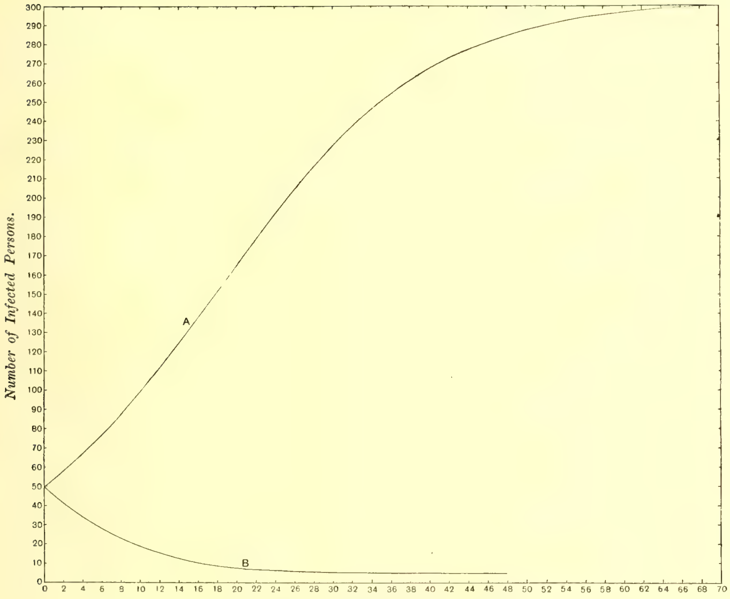


DIAGRAM IV. Curves illustrating Example 1. A. Case (i). B. Case (ii).

In Case (i) we get an increase and in each of the other cases a decrease in the value of m according to the following Tables:

TABLE E.

Case (i), $a=96000$.

TABLE F.

Case (ii), $a=48000$.

Months	1st Year	1st Year	2nd Year	3rd Year	4th Year	5th Year
1	507·9	455·3	235·0	169·7	138·9	121·3
2	513·9	418·7	227·0	166·3	137·0	120·2
3	518·7	388·3	219·6	163·1	135·3	119·1
4	522·7	362·4	212·8	160·2	133·6	118·0
5	526·3	340·2	206·5	157·3	132·0	117·0
6	528·8	321·0	200·7	154·6	130·5	116·0
7	530·7	304·1	195·3	152·0	129·0	115·1
8	532·3	289·3	190·3	149·6	127·6	114·2
9	533·5	276·0	185·6	147·2	126·2	113·3
10	534·4	264·2	181·3	145·0	124·9	112·5
11	535·2	253·5	177·2	142·9	123·6	111·7
12	535·8	243·9	173·3	140·8	122·4	110·9

TABLE G. — *Case* (iii), $a = 24000$.

TABLE H. — *Case* (iv), $a = 0$.

	TABLE G.				TABLE H.		
Months	1st Year	2nd Year	3rd Year	4th Year	1st Year	2nd Year	3rd Year
1	427·0	94·8	27·3	8·8	396·9	24·8	1·6
2	369·8	85·0	24·7	8·1	315·0	19·7	
3	320·8	76·2	22·4	—	250·0	15·6	
4	279·8	68·5	20·3	—	198·4	12·4	
5	245·3	61·6	18·5	—	157·5	9·8	
6	215·8	55·5	16·8	—	125·0	7·8	
7	190·6	50·0	15·3	—	99·2	6·2	
8	168·8	45·2	13·9	—	78·7	4·9	
9	149·9	40·8	12·7	—	62·5	3·9	
10	132·9	36·8	11·6	—	49·6	3·1	
11	118·5	33·3	10·5	—	39·4	2·5	
12	105·9	30·1	9·6	—	31·3	1·9	

This example serves to show the effect (i) of leaving the mosquitoes alone, (ii) of destroying half, (iii) of destroying three-quarters of the mosquitoes, (iv) of exterminating them (but see *Note* on p. 431); the comparison is best seen by means of the curves in Diagram V.

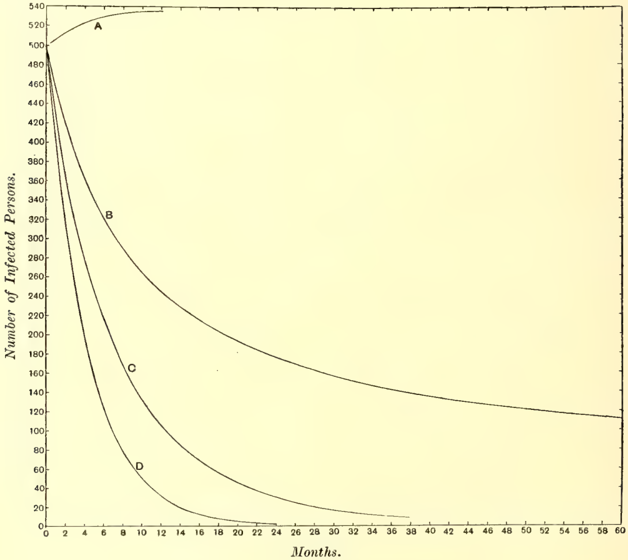


DIAGRAM V. Curves illustrating Example 2.

A. Case (i). B. Case (ii). C. Case (iii). D. Case (iv).

The foregoing results point to a stable population for any given number of mosquitoes, but it is perfectly clear that this number is itself subject to constant change, often of considerable magnitude, and that, in consequence, a stable population is rarely, if ever, attained in practice.

The minor contributory causes mentioned on p. 421, will also affect the malaria rate to an extent which cannot be neglected, while another important item to consider is the distribution both of the human population and of the mosquitoes; also whether the most densely populated districts are at the same time those most infected with mosquitoes. These and many other points will require careful investigation with the aid to be derived from the most recent and complete observations before the subject can be said to be exhausted, and I am quite well aware that the present paper merely traverses the outskirts of an important and extensive field of research.

But still it seems to me to establish two very important points, i.e., (i) Given a number of mosquitoes greater than that corresponding to the "stable population value," the number of malaria cases will tend to increase until a stable population value is reached, (ii) Given a number of mosquitoes less than that corresponding to the stable population value, the number of malaria cases will tend to decrease until that stable population is established.

In both cases the number of malaria cases tends to asymptote to the stable value. The amount of malaria does not increase or decrease indefinitely, but tends to attain a definite prevalence. Where the "stable value" means a large number of malaria cases, the right step seems to be the reduction of the number of mosquitoes; on the other hand, where it means a small number of cases it should be possible to segregate and isolate these cases.

In conclusion I wish to express my grateful thanks to Professor Karl Pearson for much valuable assistance in the preparation of this paper.

[*Postscript.* Since completing this paper, I have, through the kindness of Professor Ross, been able to see proofs of part of his forthcoming treatise on Malaria. From a comparison of the two accounts it will be seen that there is complete agreement between us on the essential and fundamental points, but some difference in numerical details. This difference is due to an attempt on my part to treat the question by fuller and more rigid mathematical methods which would be out of place in a treatise written from the medical standpoint, while the simpler methods employed by Professor Ross give results which are sufficiently accurate for practical purposes.

The principal points of agreement are: (a) for a given number of anophelines per unit of the population the number of malaria cases will gradually rise or fall to a fixed value at which it will remain stationary, and (b) when the anophelines are less than a certain number (about forty per unit of the population) there can be no stable condition and the malaria cases will gradually decrease and finally disappear.

The differences are: (*c*) the number of cases in the "stable population," corresponding to a given number of anophelines, is higher in Professor Ross' treatment than in mine, and (*d*) the changes in the malaria rate are more rapid; e.g., in a particular example, where the population is 1000, the number of anophelines per person 100 and the initial number of cases 500, we have:

Months......	0	1	2	3	4...	finally
Cases (Ross)	500	525	544	560	571	600
,, (Waite)	500	512	521	529	535	556

The divergence seems to be chiefly due to the difference in the time units employed in the two methods of treatment. Professor Ross has used the *month* throughout and has taken the value of m constant during each month, while I have used the average time between two consecutive infecting bites as my unit. The fact that m is increased by unity each time a healthy person is bitten by an infected mosquito and is continually being diminished owing to recoveries, fully justifies, in my opinion, the adoption of this unit.

Part of the difference (*d*) is easily accounted for as follows: Suppose m is increasing; then in finding m_{s+1} from m_s, the number of new cases obtained by Professor Ross is too great; for m_s is the smallest value of m during the month, and therefore the chance of malarial sufferers being bitten again is greater and that of healthy persons being infected less than that found. Again, the number of recoveries found for the month is, for the same reason, too small, and thus both causes combine to make the value of m_{s+1} too great.

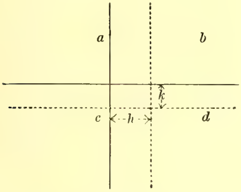
Similar reasoning shows that when the malaria is decreasing the value found for m_{s+1} is too small; i.e. in both cases the changes in the malaria rate are too rapid. It is evident, however, that the difference on this account becomes smaller as the stable value is approached. H. W.]

TABLES OF THE TETRACHORIC FUNCTIONS FOR FOURFOLD CORRELATION TABLES.

By P. F. EVERITT, B.Sc.

Explanation of the Tables.

The method of the fourfold table for determining correlation was described by Pearson in *Phil. Trans.* A. vol. 195, pp. 1—47.



a	b	$a+b$
c	d	$c+d$
$a+c$	$b+d$	N

Following his nomenclature, the normal correlation surface is divided into four parts by two planes at right angles to the axes of x and y at distances h', k' from the origin and these distances h' and k', when measured in terms of the standard deviations of their respective variables, are called h and k. The volumes or frequencies in the four divisions are represented by a, b, c, d in the manner shown in the plan and it will be seen, that $b+d$ and $c+d$, owing to the position given to the point of intersection of the traces of the dividing planes, cannot exceed $\frac{1}{2}N$.

In the paper referred to, the correlation coefficient r is determined from the equation

$$\frac{d}{N} = \frac{b+d}{N} \cdot \frac{c+d}{N} + \overset{\infty}{\underset{1}{S}} \left(\frac{r^n}{n!} HK \, \bar{v}_{n-1} \bar{w}_{n-1} \right),$$

where H, K are the ordinates of the normal curve of area N corresponding to the abscissae h and k and consequently dividing the curve into areas, of which the proportions to the whole are $\frac{b+d}{N}$ and $\frac{c+d}{N}$ respectively, while $\bar{v}$ and $\bar{w}$ are given by

$$\bar{v}_n = h\bar{v}_{n-1} - (n-1)\,\bar{v}_{n-2}, \qquad \bar{w}_n = k\bar{w}_{n-1} - (n-1)\,\bar{w}_{n-2},$$
$$\bar{v}_0 = 1, \quad \bar{v}_1 = h, \qquad \bar{w}_0 = 1, \quad \bar{w}_1 = k.$$

Now let

$$\tau_n = \frac{H\bar{v}_{n-1}}{\sqrt{n\,!}} \text{ and } \tau_n' = \frac{K\bar{w}_{n-1}}{\sqrt{n\,!}},$$

then the equation becomes

$$\frac{d}{N} = \frac{b+d}{N} \cdot \frac{c+d}{N} + \overset{\infty}{\underset{1}{S}} (\tau_n \tau_n' r^n).$$

It is clear from the above, that τ_n' is the same function of $\frac{c+d}{N}$ as τ_n is of $\frac{b+d}{N}$ and that one table of these functions will serve for both, if we enter the table with $\frac{b+d}{N}$ as argument for the latter, and with $\frac{c+d}{N}$ for the former. It should be noted that these quantities $\frac{b+d}{N}$, $\frac{c+d}{N}$ are identical with $\frac{1}{2}(1-\alpha)$, where $\frac{1}{2}(1+\alpha)$ and α are used as arguments by Sheppard* in his published tables and to avoid ambiguity have been tabulated under that heading.

In the present tables the values of the first six τ functions, henceforth to be termed *tetrachoric* functions, have been computed for values of $\frac{1}{2}(1-\alpha)$ from ·001 to ·500 by successive increments of ·001; the last column contains the values of h† (or k) corresponding to the value of $\frac{1}{2}(1-\alpha)$ given in the first column, and required for computing the functions of higher order than the sixth as well as for the probable error of r.

In the auxiliary table are given the values of p_n and q_n required to compute the functions of the seventh to twelfth orders by means of the difference formula

$$\tau_n = hp_n\tau_{n-1} - q_n\tau_{n-2}.$$

Illustrations of the Use of the Tables.

(*a*) With interpolation.

Consider the hypothetical table given below:

1668	131	1799
137	64	201
1805	195	2000

Here $\frac{b+d}{N} = \cdot 09750$, $\frac{c+d}{N} = \cdot 10050$, $\frac{d}{N} = \cdot 032000$.

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* W. F. Sheppard, "New Tables of the Probability Integral," *Biometrika*, Vol. II. p. 174.

† x of Sheppard's Tables.

Now enter the table twice using interpolation, once with $\frac{1}{2}(1-\alpha)$ equal to ·09750 and a second time with it equal to ·10050, and we have

$\frac{1}{2}(1-\alpha)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6
·09750	+ ·17228	+ ·15787	+ ·04779	− ·06018	− ·06693	+ ·00854
·10050	+ ·17614	+ ·15926	+ ·04567	− ·06275	− ·06652	+ ·01111

Multiplying the numbers in each column together, we find the equation

$$\cdot032000 = \cdot009799 + \cdot030345r + \cdot025142r^2 + \cdot002183r^3 + \cdot003776r^4 + \cdot004452r^5 + \cdot000095r^6;$$

whence, solving by Newton's method,

$$r = \cdot501.$$

An illustration of the calculation of the probable error of r when found as above is given in Pearson's paper on p. 36.

(*b*) Without interpolation.

Using the same table as in example (*a*), we have as before

$$\frac{b+d}{N} = \cdot09750, \quad \frac{c+d}{N} = \cdot10050, \quad \frac{d}{N} = \cdot032000.$$

Entering the table with $\frac{1}{2}(1-\alpha)$ equal to ·098 and ·101 we have

$\frac{1}{2}(1-\alpha)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6
·098	+ ·17292	+ ·15811	+ ·04744	− ·06061	− ·06687	+ ·00897
·101	+ ·17678	+ ·15948	+ ·04531	− ·06317	− ·06644	+ ·01153

Multiplying out as before

$$\cdot032000 = \cdot009898 + \cdot030569r + \cdot025215r^2 + \cdot002150r^3 + \cdot003829r^4 + \cdot004443r^5 + \cdot000103r^6;$$

whence, solving by Newton's method,

$$r = \cdot498.$$

This particular illustration, chosen so that the true values of $\frac{1}{2}(1-\alpha)$ fall midway between tabulated values, shows the maximum error caused by working without interpolation for values of $\frac{1}{2}(1-\alpha)$ of ·100, and in practically every case this error will be negligible when compared with the probable error of r.

A study of the differences of the functions shows that for values of $\frac{1}{2}(1-\alpha)$ greater than ·100 the error introduced by working without interpolation will not be appreciably greater than in the example given and may quite easily be less; for values of $\frac{1}{2}(1-\alpha)$ less than ·100 it will be desirable to use interpolation if the greatest accuracy attainable is desired, but even in very unfavourable cases such errors will rarely become as large as the probable error of the result.

The tetrachoric functions of higher order than the sixth (easily found by means of the auxiliary table) will be chiefly required when r is large and the series consequently converges less rapidly. In connection with such cases, it is well to recollect that in the series on the right-hand side of the equation successive terms need not necessarily converge but that *alternate* terms are absolutely convergent*.

Another Use for the Tables.

The tables can also be used with advantage in the work of computing the frequencies in the divisions of any frequency table, when the correlation coefficient r is known and the frequency is normal. Such problems can be classified into three cases according to the fineness of the grouping and a brief outline of the methods to be employed is given.

Case I. Fourfold table. According to the purpose to be served, the dividing planes are fixed either by h and k or $b+d$ and $c+d$ being known; the value of d is then calculated from

$$\frac{d}{N}=\frac{b+d}{N}\cdot\frac{c+d}{N}+\overset{\infty}{\underset{1}{S}}(\tau_n\tau_n' r^n);$$

when d is known, b, c and a are also known from the total column and row.

Case II. Ninefold table. It is required to divide a normal frequency surface into nine parts $A, B, \ldots L$ as shown in the diagram. Then it will always be possible

A	B	C	
D	E	F	
G	I	L	
			N

to consider one or more of A, C, G, L in turn as the d of a standard fourfold table and find its value from the equation given in Case I. The column and row totals are either known or obtained directly from values of h and k. From the known divisions and the column and row totals the values of other divisions and groups of divisions are next obtained; the table is then divided afresh, if necessary, according to the particular circumstances of the case, the values of other groups of divisions obtained, and so ultimately the value of each separate division not already known may be found by differences.

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* Pearson, *loc. cit.* p. 10.

Case III. More than three columns or rows.

Find the value of as many of the corner divisions as may be treated as the *d* of a fourfold table by the method of Case I; then find the column and row totals, if not already known, and from them the values of other divisions and groups of divisions. The table is then divided afresh and the values of other groups of divisions obtained and this process repeated until the values of the remaining separate divisions may be found by differences. This case is really an extension of Case II.

In conclusion I desire to express my indebtedness to Prof. Pearson for suggesting the construction of the tables and for much valuable advice as to methods of calculation and arrangement. I also wish to express my thanks to Mr Sheppard for allowing me to use a hitherto unpublished table of the probability integral, from which the values of h corresponding to given values of $\frac{1}{2}(1-\alpha)$ have been taken, the published tables not providing sufficient decimal places without troublesome interpolation.

Auxiliary Table for calculating Tetrachoric Functions beyond those Tabled.

n	p_n	q_n
7	·37796	·77152
8	·35355	·80178
9	·33333	·82496
10	·31623	·84327
11	·30151	·85812
12	·28868	·87039

$\tau_n = hp_n\tau_{n-1} - q_n\tau_{n-2}$.

TABLE OF THE TETRACHORIC FUNCTIONS.

$\frac{1}{2}(1-a)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	h
·001	·00337	·00736	+ ·01175	+ ·01391	+ ·01134	+ ·00415	3·09023
·002	·00634	·01290	+ ·01885	+ ·01968	+ ·01269	+ ·00053	2·87816
·003	·00915	·01778	+ ·02446	+ ·02335	+ ·01228	− ·00328	2·74778
·004	·01185	·02222	+ ·02918	+ ·02587	+ ·01111	− ·00687	2·65207
·005	·01446	·02634	+ ·03326	+ ·02764	+ ·00952	− ·01017	2·57583
·006	·01700	·03020	+ ·03686	+ ·02887	+ ·00770	− ·01318	2·51214
·007	·01949	·03386	+ ·04008	+ ·02970	+ ·00575	− ·01592	2·45726
·008	·02192	·03734	+ ·04298	+ ·03021	+ ·00371	− ·01841	2·40892
·009	·02431	·04066	+ ·04561	+ ·03047	+ ·00164	− ·02067	2·36562
·010	·02665	·04384	+ ·04800	+ ·03053	− ·00044	− ·02271	2·32635
·011	·02896	·04690	+ ·05020	+ ·03041	− ·00253	− ·02457	2·29037
·012	·03123	·04985	+ ·05221	+ ·03014	− ·00460	− ·02625	2·25713
·013	·03348	·05270	+ ·05406	+ ·02975	− ·00664	− ·02777	2·22621
·014	·03569	·05545	+ ·05577	+ ·02926	− ·00866	− ·02914	2·19729
·015	·03787	·05811	+ ·05735	+ ·02867	− ·01064	− ·03037	2·17009
·016	·04003	·06069	+ ·05880	+ ·02801	− ·01259	− ·03147	2·14441
·017	·04216	·06320	+ ·06015	+ ·02727	− ·01449	− ·03246	2·12007
·018	·04427	·06564	+ ·06139	+ ·02647	− ·01636	− ·03334	2·09693
·019	·04635	·06801	+ ·06254	+ ·02562	− ·01818	− ·03411	2·07485
·020	·04842	·07031	+ ·06361	+ ·02472	− ·01996	− ·03479	2·05375
·021	·05046	·07256	+ ·06459	+ ·02378	− ·02170	− ·03538	2·03352
·022	·05249	·07475	+ ·06549	+ ·02280	− ·02340	− ·03589	2·01409
·023	·05449	·07688	+ ·06633	+ ·02179	− ·02505	− ·03632	1·99539
·024	·05648	·07897	+ ·06709	+ ·02074	− ·02666	− ·03667	1·97737
·025	·05845	·08100	+ ·06780	+ ·01968	− ·02823	− ·03696	1·95996
·026	·06040	·08299	+ ·06844	+ ·01858	− ·02976	− ·03718	1·94313
·027	·06233	·08493	+ ·06903	+ ·01747	− ·03125	− ·03734	1·92684
·028	·06425	·08682	+ ·06956	+ ·01634	− ·03270	− ·03744	1·91104
·029	·06615	·08868	+ ·07005	+ ·01520	− ·03411	− ·03749	1·89570
·030	·06804	·09049	+ ·07048	+ ·01404	− ·03547	− ·03749	1·88079
·031	·06992	·09227	+ ·07087	+ ·01287	− ·03680	− ·03744	1·86630
·032	·07177	·09400	+ ·07122	+ ·01168	− ·03810	− ·03734	1·85218
·033	·07362	·09570	+ ·07153	+ ·01049	− ·03935	− ·03720	1·83842
·034	·07545	·09737	+ ·07179	+ ·00929	− ·04057	− ·03702	1·82501
·035	·07727	·09900	+ ·07202	+ ·00809	− ·04176	− ·03680	1·81191
·036	·07908	·10060	+ ·07221	+ ·00688	− ·04291	− ·03654	1·79912
·037	·08087	·10216	+ ·07237	+ ·00566	− ·04402	− ·03624	1·78661
·038	·08265	·10370	+ ·07249	+ ·00444	− ·04510	− ·03592	1·77438
·039	·08442	·10520	+ ·07258	+ ·00322	− ·04615	− ·03556	1·76241
·040	·08617	·10668	+ ·07264	+ ·00200	− ·04717	− ·03517	1·75069
·041	·08792	·10812	+ ·07268	+ ·00077	− ·04815	− ·03475	1·73920
·042	·08965	·10954	+ ·07268	− ·00045	− ·04910	− ·03431	1·72793
·043	·09137	·11093	+ ·07266	− ·00167	− ·05003	− ·03384	1·71689
·044	·09309	·11229	+ ·07261	− ·00290	− ·05092	− ·03335	1·70604
·045	·09479	·11363	+ ·07253	− ·00412	− ·05178	− ·03283	1·69540
·046	·09648	·11495	+ ·07243	− ·00534	− ·05261	− ·03229	1·68494
·047	·09816	·11623	+ ·07231	− ·00656	− ·05342	− ·03173	1·67466
·048	·09983	·11750	+ ·07217	− ·00778	− ·05420	− ·03115	1·66456
·049	·10149	·11874	+ ·07200	− ·00899	− ·05495	− ·03055	1·65463
·050	·10314	·11996	+ ·07181	− ·01020	− ·05567	− ·02994	1·64485

τ_1, τ_2 and h are essentially positive.

TABLE—(*continued*).

$\frac{1}{2}(1-a)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	h
·051	·10478	·12115	+ ·07160	− ·01140	− ·05637	− ·02931	1·63523
·052	·10641	·12232	+ ·07138	− ·01260	− ·05704	− ·02866	1·62576
·053	·10803	·12347	+ ·07113	− ·01380	− ·05769	− ·02799	1·61644
·054	·10964	·12460	+ ·07087	− ·01499	− ·05831	− ·02732	1·60725
·055	·11124	·12571	+ ·07058	− ·01618	− ·05891	− ·02662	1·59819
·056	·11284	·12680	+ ·07028	− ·01736	− ·05949	− ·02592	1·58927
·057	·11442	·12787	+ ·06997	− ·01854	− ·06004	− ·02520	1·58047
·058	·11600	·12892	+ ·06964	− ·01971	− ·06057	− ·02447	1·57179
·059	·11756	·12995	+ ·06929	− ·02087	− ·06107	− ·02373	1·56322
·060	·11912	·13096	+ ·06893	− ·02203	− ·06155	− ·02298	1·55477
·061	·12067	·13196	+ ·06855	− ·02318	− ·06202	− ·02222	1·54643
·062	·12222	·13293	+ ·06816	− ·02433	− ·06246	− ·02145	1·53820
·063	·12375	·13389	+ ·06775	− ·02547	− ·06288	− ·02068	1·53007
·064	·12528	·13483	+ ·06734	− ·02660	− ·06328	− ·01989	1·52204
·065	·12679	·13575	+ ·06690	− ·02773	− ·06365	− ·01910	1·51410
·066	·12830	·13666	+ ·06646	− ·02884	− ·06401	− ·01830	1·50626
·067	·12981	·13754	+ ·06601	− ·02996	− ·06435	− ·01749	1·49851
·068	·13130	·13842	+ ·06554	− ·03106	− ·06467	− ·01668	1·49085
·069	·13279	·13927	+ ·06506	− ·03216	− ·06498	− ·01586	1·48328
·070	·13427	·14011	+ ·06457	− ·03325	− ·06526	− ·01504	1·47579
·071	·13574	·14094	+ ·06407	− ·03433	− ·06552	− ·01421	1·46838
·072	·13720	·14175	+ ·06356	− ·03541	− ·06577	− ·01337	1·46106
·073	·13866	·14254	+ ·06304	− ·03648	− ·06600	− ·01253	1·45381
·074	·14011	·14332	+ ·06251	− ·03754	− ·06621	− ·01169	1·44663
·075	·14156	·14409	+ ·06197	− ·03859	− ·06641	− ·01085	1·43953
·076	·14299	·14484	+ ·06142	− ·03963	− ·06659	− ·01000	1·43250
·077	·14442	·14558	+ ·06086	− ·04067	− ·06675	− ·00915	1·42554
·078	·14584	·14630	+ ·06029	− ·04170	− ·06690	− ·00829	1·41865
·079	·14726	·14701	+ ·05971	− ·04272	− ·06703	− ·00743	1·41183
·080	·14867	·14771	+ ·05913	− ·04374	− ·06715	− ·00658	1·40507
·081	·15007	·14839	+ ·05854	− ·04474	− ·06725	− ·00572	1·39838
·082	·15146	·14906	+ ·05794	− ·04574	− ·06733	− ·00485	1·39174
·083	·15285	·14971	+ ·05733	− ·04673	− ·06741	− ·00399	1·38517
·084	·15423	·15036	+ ·05671	− ·04771	− ·06746	− ·00312	1·37866
·085	·15561	·15099	+ ·05609	− ·04869	− ·06751	− ·00226	1·37220
·086	·15698	·15160	+ ·05546	− ·04965	− ·06753	− ·00139	1·36581
·087	·15834	·15221	+ ·05483	− ·05061	− ·06755	− ·00053	1·35946
·088	·15970	·15280	+ ·05418	− ·05156	− ·06755	+ ·00034	1·35317
·089	·16105	·15339	+ ·05353	− ·05250	− ·06754	+ ·00120	1·34694
·090	·16239	·15396	+ ·05288	− ·05344	− ·06751	+ ·00207	1·34076
·091	·16373	·15451	+ ·05222	− ·05436	− ·06748	+ ·00294	1·33462
·092	·16506	·15506	+ ·05155	− ·05528	− ·06743	+ ·00380	1·32854
·093	·16639	·15560	+ ·05088	− ·05619	− ·06736	+ ·00467	1·32251
·094	·16770	·15612	+ ·05020	− ·05709	− ·06729	+ ·00553	1·31652
·095	·16902	·15663	+ ·04952	− ·05798	− ·06720	+ ·00639	1·31058
·096	·17033	·15713	+ ·04883	− ·05887	− ·06710	+ ·00725	1·30469
·097	·17163	·15763	+ ·04813	− ·05975	− ·06699	+ ·00811	1·29884
·098	·17292	·15811	+ ·04744	− ·06061	− ·06687	+ ·00897	1·29303
·099	·17421	·15858	+ ·04673	− ·06148	− ·06674	+ ·00982	1·28727
·100	·17550	·15904	+ ·04602	− ·06233	− ·06660	+ ·01068	1·28155

TABLE—(*continued*).

$\frac{1}{2}(1-a)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	h
·101	·17678	·15948	+ ·04531	− ·06317	− ·06644	+ ·01153	1·27587
·102	·17805	·15992	+ ·04459	− ·06401	− ·06628	+ ·01238	1·27024
·103	·17932	·16035	+ ·04387	− ·06484	− ·06610	+ ·01322	1·26464
·104	·18058	·16077	+ ·04315	− ·06566	− ·06592	+ ·01407	1·25908
·105	·18184	·16118	+ ·04242	− ·06647	− ·06572	+ ·01491	1·25357
·106	·18309	·16158	+ ·04169	− ·06727	− ·06551	+ ·01575	1·24808
·107	·18433	·16197	+ ·04095	− ·06807	− ·06530	+ ·01659	1·24264
·108	·18557	·16235	+ ·04021	− ·06886	− ·06507	+ ·01742	1·23723
·109	·18681	·16272	+ ·03947	− ·06964	− ·06484	+ ·01825	1·23186
·110	·18804	·16308	+ ·03872	− ·07041	− ·06459	+ ·01908	1·22653
·111	·18926	·16343	+ ·03797	− ·07117	− ·06434	+ ·01990	1·22123
·112	·19048	·16378	+ ·03721	− ·07193	− ·06408	+ ·02072	1·21596
·113	·19169	·16411	+ ·03646	− ·07268	− ·06381	+ ·02154	1·21073
·114	·19290	·16443	+ ·03570	− ·07342	− ·06353	+ ·02235	1·20553
·115	·19410	·16475	+ ·03493	− ·07415	− ·06324	+ ·02316	1·20036
·116	·19530	·16506	+ ·03417	− ·07488	− ·06294	+ ·02397	1·19522
·117	·19649	·16536	+ ·03340	− ·07559	− ·06264	+ ·02477	1·19012
·118	·19768	·16565	+ ·03263	− ·07630	− ·06233	+ ·02557	1·18504
·119	·19886	·16593	+ ·03186	− ·07700	− ·06201	+ ·02636	1·18000
·120	·20004	·16620	+ ·03108	− ·07770	− ·06168	+ ·02716	1·17499
·121	·20121	·16647	+ ·03030	− ·07838	− ·06134	+ ·02794	1·17000
·122	·20238	·16672	+ ·02952	− ·07906	− ·06100	+ ·02873	1·16505
·123	·20354	·16697	+ ·02874	− ·07973	− ·06065	+ ·02950	1·16012
·124	·20470	·16721	+ ·02796	− ·08039	− ·06029	+ ·03028	1·15522
·125	·20585	·16745	+ ·02717	− ·08105	− ·05992	+ ·03105	1·15035
·126	·20700	·16767	+ ·02638	− ·08169	− ·05955	+ ·03181	1·14551
·127	·20814	·16789	+ ·02559	− ·08233	− ·05917	+ ·03257	1·14069
·128	·20928	·16810	+ ·02480	− ·08297	− ·05878	+ ·03333	1·13590
·129	·21042	·16830	+ ·02401	− ·08359	− ·05839	+ ·03408	1·13113
·130	·21155	·16849	+ ·02321	− ·08421	− ·05799	+ ·03483	1·12639
·131	·21267	·16868	+ ·02241	− ·08482	− ·05758	+ ·03557	1·12168
·132	·21379	·16886	+ ·02162	− ·08542	− ·05717	+ ·03631	1·11699
·133	·21490	·16903	+ ·02082	− ·08601	− ·05675	+ ·03704	1·11232
·134	·21601	·16919	+ ·02001	− ·08660	− ·05632	+ ·03777	1·10768
·135	·21712	·16935	+ ·01921	− ·08718	− ·05589	+ ·03850	1·10306
·136	·21822	·16950	+ ·01841	− ·08775	− ·05546	+ ·03921	1·09847
·137	·21932	·16964	+ ·01760	− ·08831	− ·05501	+ ·03993	1·09390
·138	·22041	·16978	+ ·01680	− ·08887	− ·05456	+ ·04064	1·08935
·139	·22149	·16990	+ ·01599	− ·08942	− ·05411	+ ·04134	1·08482
·140	·22258	·17003	+ ·01518	− ·08996	− ·05365	+ ·04204	1·08032
·141	·22365	·17014	+ ·01437	− ·09050	− ·05318	+ ·04273	1·07584
·142	·22473	·17025	+ ·01356	− ·09103	− ·05271	+ ·04342	1·07138
·143	·22580	·17035	+ ·01275	− ·09155	− ·05224	+ ·04410	1·06694
·144	·22686	·17044	+ ·01194	− ·09206	− ·05176	+ ·04478	1·06252
·145	·22792	·17053	+ ·01113	− ·09257	− ·05127	+ ·04545	1·05812
·146	·22898	·17061	+ ·01032	− ·09307	− ·05078	+ ·04612	1·05374
·147	·23003	·17069	+ ·00950	− ·09356	− ·05028	+ ·04678	1·04939
·148	·23108	·17076	+ ·00869	− ·09405	− ·04978	+ ·04744	1·04505
·149	·23212	·17082	+ ·00788	− ·09452	− ·04928	+ ·04809	1·04073
·150	·23316	·17087	+ ·00706	− ·09499	− ·04877	+ ·04874	1·03643

TABLE—(*continued*).

$\frac{1}{2}(1-a)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	h
·151	·23419	·17092	+ ·00625	− ·09546	− ·04825	+ ·04938	1·03215
·152	·23522	·17097	+ ·00543	− ·09592	− ·04774	+ ·05002	1·02789
·153	·23625	·17100	+ ·00462	− ·09637	− ·04721	+ ·05065	1·02365
·154	·23727	·17103	+ ·00380	− ·09681	− ·04669	+ ·05127	1·01943
·155	·23829	·17106	+ ·00298	− ·09725	− ·04615	+ ·05189	1·01522
·156	·23930	·17108	+ ·00217	− ·09768	− ·04562	+ ·05250	1·01103
·157	·24031	·17109	+ ·00135	− ·09810	− ·04508	+ ·05311	1·00686
·158	·24131	·17110	+ ·00053	− ·09852	− ·04454	+ ·05371	1·00271
·159	·24232	·17110	− ·00028	− ·09892	− ·04399	+ ·05431	·99858
·160	·24331	·17109	− ·00110	− ·09933	− ·04344	+ ·05490	·99446
·161	·24430	·17108	− ·00191	− ·09972	− ·04288	+ ·05549	·99036
·162	·24529	·17107	− ·00273	− ·10011	− ·04232	+ ·05607	·98627
·163	·24628	·17104	− ·00355	− ·10049	− ·04176	+ ·05664	·98220
·164	·24726	·17102	− ·00436	− ·10087	− ·04120	+ ·05721	·97815
·165	·24823	·17098	− ·00518	− ·10124	− ·04063	+ ·05778	·97411
·166	·24921	·17094	− ·00599	− ·10160	− ·04006	+ ·05834	·97009
·167	·25017	·17090	− ·00681	− ·10196	− ·03948	+ ·05889	·96609
·168	·25114	·17085	− ·00762	− ·10231	− ·03890	+ ·05943	·96210
·169	·25210	·17080	− ·00844	− ·10265	− ·03832	+ ·05998	·95812
·170	·25305	·17073	− ·00925	− ·10299	− ·03774	+ ·06051	·95417
·171	·25401	·17067	− ·01007	− ·10332	− ·03715	+ ·06104	·95022
·172	·25495	·17060	− ·01088	− ·10364	− ·03656	+ ·06156	·94629
·173	·25590	·17052	− ·01169	− ·10396	− ·03597	+ ·06208	·94238
·174	·25684	·17044	− ·01251	− ·10427	− ·03537	+ ·06260	·93848
·175	·25778	·17035	− ·01332	− ·10458	− ·03478	+ ·06310	·93459
·176	·25871	·17026	− ·01413	− ·10487	− ·03417	+ ·06360	·93072
·177	·25964	·17016	− ·01494	− ·10517	− ·03357	+ ·06410	·92686
·178	·26056	·17006	− ·01575	− ·10545	− ·03296	+ ·06459	·92301
·179	·26148	·16995	− ·01656	− ·10573	− ·03236	+ ·06507	·91918
·180	·26240	·16984	− ·01737	− ·10601	− ·03175	+ ·06555	·91537
·181	·26331	·16972	− ·01817	− ·10627	− ·03113	+ ·06603	·91156
·182	·26422	·16960	− ·01898	− ·10653	− ·03052	+ ·06649	·90777
·183	·26513	·16948	− ·01979	− ·10679	− ·02990	+ ·06695	·90399
·184	·26603	·16934	− ·02059	− ·10704	− ·02928	+ ·06741	·90023
·185	·26693	·16921	− ·02140	− ·10728	− ·02866	+ ·06786	·89647
·186	·26782	·16907	− ·02220	− ·10752	− ·02803	+ ·06830	·89273
·187	·26871	·16892	− ·02300	− ·10775	− ·02741	+ ·06874	·88901
·188	·26960	·16877	− ·02380	− ·10798	− ·02678	+ ·06917	·88529
·189	·27049	·16861	− ·02460	− ·10819	− ·02615	+ ·06960	·88159
·190	·27137	·16845	− ·02540	− ·10841	− ·02552	+ ·07002	·87790
·191	·27224	·16829	− ·02620	− ·10861	− ·02489	+ ·07044	·87422
·192	·27311	·16812	− ·02700	− ·10882	− ·02425	+ ·07085	·87055
·193	·27398	·16795	− ·02779	− ·10901	− ·02362	+ ·07125	·86689
·194	·27485	·16777	− ·02859	− ·10920	− ·02298	+ ·07165	·86325
·195	·27571	·16759	− ·02938	− ·10939	− ·02234	+ ·07204	·85962
·196	·27657	·16740	− ·03018	− ·10956	− ·02170	+ ·07243	·85600
·197	·27742	·16721	− ·03097	− ·10974	− ·02106	+ ·07281	·85239
·198	·27827	·16701	− ·03176	− ·10990	− ·02041	+ ·07319	·84879
·199	·27912	·16681	− ·03255	− ·11007	− ·01977	+ ·07356	·84520
·200	·27996	·16661	− ·03334	− ·11022	− ·01912	+ ·07392	·84162

TABLE—(*continued*).

$\frac{1}{2}(1-a)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	h
·201	·28080	·16640	−·03412	−·11037	−·01848	+·07428	·83805
·202	·28164	·16619	−·03491	−·11051	−·01783	+·07464	·83450
·203	·28247	·16597	−·03569	−·11065	−·01718	+·07498	·83095
·204	·28330	·16575	−·03648	−·11079	−·01653	+·07532	·82742
·205	·28413	·16553	−·03726	−·11091	−·01587	+·07566	·82389
·206	·28495	·16530	−·03804	−·11104	−·01522	+·07599	·82038
·207	·28577	·16506	−·03882	−·11115	−·01457	+·07632	·81687
·208	·28658	·16483	−·03959	−·11126	−·01391	+·07664	·81338
·209	·28739	·16459	−·04037	−·11137	−·01326	+·07695	·80990
·210	·28820	·16434	−·04114	−·11147	−·01260	+·07726	·80642
·211	·28901	·16409	−·04192	−·11157	−·01194	+·07756	·80296
·212	·28981	·16384	−·04269	−·11166	−·01129	+·07786	·79950
·213	·29060	·16358	−·04346	−·11174	−·01063	+·07815	·79606
·214	·29140	·16332	−·04423	−·11182	−·00997	+·07844	·79262
·215	·29219	·16305	−·04499	−·11189	−·00931	+·07872	·78919
·216	·29298	·16279	−·04576	−·11196	−·00865	+·07899	·78577
·217	·29376	·16251	−·04652	−·11203	−·00799	+·07926	·78237
·218	·29454	·16224	−·04728	−·11208	−·00733	+·07952	·77897
·219	·29532	·16196	−·04804	−·11214	−·00667	+·07978	·77557
·220	·29609	·16167	−·04880	−·11218	−·00600	+·08004	·77219
·221	·29686	·16139	−·04956	−·11223	−·00534	+·08028	·76882
·222	·29763	·16110	−·05031	−·11226	−·00468	+·08052	·76546
·223	·29840	·16080	−·05107	−·11230	−·00402	+·08076	·76210
·224	·29916	·16050	−·05182	−·11233	−·00335	+·08099	·75875
·225	·29991	·16020	−·05257	−·11235	−·00269	+·08122	·75541
·226	·30067	·15990	−·05332	−·11237	−·00203	+·08144	·75208
·227	·30142	·15959	−·05406	−·11238	−·00136	+·08165	·74876
·228	·30216	·15927	−·05481	−·11239	−·00070	+·08186	·74545
·229	·30291	·15896	−·05555	−·11239	−·00004	+·08207	·74214
·230	·30365	·15864	−·05629	−·11239	+·00063	+·08226	·73885
·231	·30439	·15832	−·05703	−·11238	+·00129	+·08246	·73556
·232	·30512	·15799	−·05777	−·11237	+·00195	+·08265	·73228
·233	·30585	·15766	−·05851	−·11235	+·00262	+·08283	·72900
·234	·30658	·15733	−·05924	−·11233	+·00328	+·08301	·72574
·235	·30730	·15699	−·05997	−·11230	+·00394	+·08318	·72248
·236	·30802	·15665	−·06070	−·11227	+·00461	+·08334	·71923
·237	·30874	·15631	−·06143	−·11224	+·00527	+·08351	·71599
·238	·30945	·15596	−·06215	−·11220	+·00593	+·08366	·71275
·239	·31017	·15561	−·06288	−·11215	+·00659	+·08381	·70952
·240	·31087	·15526	−·06360	−·11210	+·00726	+·08396	·70630
·241	·31158	·15490	−·06432	−·11205	+·00792	+·08410	·70309
·242	·31228	·15454	−·06504	−·11199	+·00858	+·08423	·69988
·243	·31298	·15418	−·06576	−·11192	+·00924	+·08436	·69668
·244	·31367	·15382	−·06647	−·11185	+·00990	+·08449	·69349
·245	·31436	·15345	−·06718	−·11178	+·01056	+·08461	·69031
·246	·31505	·15308	−·06789	−·11170	+·01122	+·08472	·68713
·247	·31574	·15270	−·06860	−·11162	+·01188	+·08483	·68396
·248	·31642	·15232	−·06931	−·11154	+·01253	+·08494	·68080
·249	·31710	·15194	−·07001	−·11145	+·01319	+·08504	·67764
·250	·31778	·15156	−·07071	−·11135	+·01385	+·08513	·67449

TABLE—(*continued*).

$\frac{1}{2}(1-a)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	h
·251	·31845	·15117	− ·07141	− ·11125	+ ·01450	+ ·08522	·67135
·252	·31912	·15078	− ·07211	− ·11115	+ ·01516	+ ·08530	·66821
·253	·31979	·15039	− ·07280	− ·11104	+ ·01581	+ ·08538	·66508
·254	·32045	·14999	− ·07350	− ·11093	+ ·01647	+ ·08546	·66196
·255	·32111	·14959	− ·07419	− ·11081	+ ·01712	+ ·08553	·65884
·256	·32177	·14919	− ·07488	− ·11069	+ ·01777	+ ·08559	·65573
·257	·32242	·14879	− ·07557	− ·11056	+ ·01842	+ ·08565	·65262
·258	·32307	·14838	− ·07625	− ·11043	+ ·01907	+ ·08571	·64952
·259	·32372	·14797	− ·07693	− ·11030	+ ·01972	+ ·08575	·64643
·260	·32437	·14756	− ·07761	− ·11016	+ ·02037	+ ·08580	·64335
·261	·32501	·14714	− ·07829	− ·11002	+ ·02102	+ ·08584	·64027
·262	·32565	·14672	− ·07897	− ·10987	+ ·02166	+ ·08587	·63719
·263	·32628	·14630	− ·07964	− ·10972	+ ·02231	+ ·08590	·63412
·264	·32691	·14588	− ·08031	− ·10956	+ ·02295	+ ·08593	·63106
·265	·32754	·14545	− ·08098	− ·10940	+ ·02360	+ ·08595	·62801
·266	·32817	·14502	− ·08165	− ·10924	+ ·02424	+ ·08596	·62496
·267	·32879	·14459	− ·08231	− ·10907	+ ·02488	+ ·08597	·62191
·268	·32941	·14415	− ·08298	− ·10890	+ ·02552	+ ·08598	·61887
·269	·33003	·14372	− ·08364	− ·10873	+ ·02616	+ ·08598	·61584
·270	·33065	·14328	− ·08429	− ·10855	+ ·02680	+ ·08598	·61281
·271	·33126	·14283	− ·08495	− ·10837	+ ·02743	+ ·08597	·60979
·272	·33187	·14239	− ·08560	− ·10818	+ ·02807	+ ·08596	·60678
·273	·33247	·14194	− ·08625	− ·10799	+ ·02870	+ ·08594	·60376
·274	·33307	·14149	− ·08690	− ·10779	+ ·02933	+ ·08591	·60076
·275	·33367	·14104	− ·08755	− ·10759	+ ·02997	+ ·08589	·59776
·276	·33427	·14058	− ·08819	− ·10739	+ ·03060	+ ·08586	·59477
·277	·33486	·14012	− ·08883	− ·10718	+ ·03122	+ ·08582	·59178
·278	·33545	·13966	− ·08947	− ·10697	+ ·03185	+ ·08578	·58879
·279	·33604	·13920	− ·09011	− ·10676	+ ·03248	+ ·08573	·58581
·280	·33662	·13873	− ·09074	− ·10654	+ ·03310	+ ·08568	·58284
·281	·33720	·13826	− ·09137	− ·10632	+ ·03372	+ ·08563	·57987
·282	·33778	·13779	− ·09200	− ·10609	+ ·03434	+ ·08557	·57691
·283	·33836	·13732	− ·09263	− ·10587	+ ·03496	+ ·08551	·57395
·284	·33893	·13685	− ·09325	− ·10563	+ ·03558	+ ·08544	·57100
·285	·33950	·13637	− ·09388	− ·10540	+ ·03620	+ ·08536	·56805
·286	·34007	·13589	− ·09450	− ·10516	+ ·03681	+ ·08529	·56511
·287	·34063	·13541	− ·09511	− ·10491	+ ·03743	+ ·08521	·56217
·288	·34119	·13492	− ·09573	− ·10466	+ ·03804	+ ·08512	·55924
·289	·34175	·13443	− ·09634	− ·10441	+ ·03865	+ ·08503	·55631
·290	·34230	·13394	− ·09695	− ·10416	+ ·03926	+ ·08494	·55338
·291	·34286	·13345	− ·09756	− ·10390	+ ·03987	+ ·08484	·55047
·292	·34341	·13296	− ·09816	− ·10364	+ ·04047	+ ·08473	·54755
·293	·34395	·13246	− ·09876	− ·10337	+ ·04107	+ ·08463	·54464
·294	·34449	·13196	− ·09936	− ·10310	+ ·04168	+ ·08451	·54174
·295	·34503	·13146	− ·09996	− ·10283	+ ·04228	+ ·08440	·53884
·296	·34557	·13096	− ·10056	− ·10256	+ ·04287	+ ·08428	·53594
·297	·34611	·13046	− ·10115	− ·10228	+ ·04347	+ ·08415	·53305
·298	·34664	·12995	− ·10174	− ·10199	+ ·04407	+ ·08402	·53016
·299	·34717	·12944	− ·10233	− ·10171	+ ·04466	+ ·08389	·52728
·300	·34769	·12893	− ·10291	− ·10142	+ ·04525	+ ·08375	·52440

TABLE—(*continued*).

$\frac{1}{2}(1-a)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	h
·301	·34822	·12841	−·10349	−·10113	+·04584	+·08361	·52153
·302	·34874	·12790	−·10407	−·10083	+·04643	+·08347	·51866
·303	·34925	·12738	−·10465	−·10053	+·04701	+·08332	·51579
·304	·34977	·12686	−·10522	−·10023	+·04759	+·08316	·51293
·305	·35028	·12634	−·10580	−·09992	+·04818	+·08301	·51007
·306	·35079	·12581	−·10636	−·09961	+·04876	+·08284	·50722
·307	·35129	·12529	−·10693	−·09930	+·04933	+·08268	·50437
·308	·35180	·12476	−·10750	−·09899	+·04991	+·08251	·50153
·309	·35230	·12423	−·10806	−·09867	+·05048	+·08233	·49869
·310	·35279	·12370	−·10862	−·09834	+·05105	+·08216	·49585
·311	·35329	·12316	−·10917	−·09802	+·05162	+·08197	·49302
·312	·35378	·12263	−·10973	−·09769	+·05219	+·08179	·49019
·313	·35427	·12209	−·11028	−·09736	+·05276	+·08160	·48736
·314	·35475	·12155	−·11082	−·09703	+·05332	+·08140	·48454
·315	·35524	·12101	−·11137	−·09669	+·05388	+·08121	·48173
·316	·35572	·12046	−·11191	−·09635	+·05444	+·08101	·47891
·317	·35620	·11992	−·11245	−·09600	+·05500	+·08080	·47610
·318	·35667	·11937	−·11299	−·09566	+·05555	+·08059	·47330
·319	·35714	·11882	−·11353	−·09531	+·05610	+·08038	·47050
·320	·35761	·11827	−·11406	−·09495	+·05665	+·08016	·46770
·321	·35808	·11771	−·11459	−·09460	+·05720	+·07994	·46490
·322	·35854	·11716	−·11512	−·09424	+·05775	+·07972	·46211
·323	·35900	·11660	−·11564	−·09388	+·05829	+·07949	·45933
·324	·35946	·11604	−·11616	−·09351	+·05883	+·07926	·45654
·325	·35991	·11548	−·11668	−·09315	+·05937	+·07902	·45376
·326	·36037	·11492	−·11720	−·09278	+·05991	+·07878	·45099
·327	·36082	·11436	−·11771	−·09240	+·06044	+·07854	·44821
·328	·36126	·11379	−·11822	−·09203	+·06097	+·07829	·44544
·329	·36171	·11322	−·11873	−·09165	+·06150	+·07804	·44268
·330	·36215	·11265	−·11923	−·09127	+·06203	+·07779	·43991
·331	·36259	·11208	−·11974	−·09088	+·06255	+·07753	·43715
·332	·36302	·11151	−·12024	−·09049	+·06308	+·07727	·43440
·333	·36346	·11093	−·12073	−·09010	+·06360	+·07701	·43164
·334	·36389	·11036	−·12123	−·08971	+·06412	+·07674	·42889
·335	·36431	·10978	−·12172	−·08932	+·06463	+·07647	·42615
·336	·36474	·10920	−·12221	−·08892	+·06514	+·07620	·42340
·337	·36516	·10862	−·12270	−·08852	+·06565	+·07592	·42066
·338	·36558	·10804	−·12318	−·08811	+·06616	+·07564	·41793
·339	·36600	·10745	−·12366	−·08771	+·06667	+·07535	·41519
·340	·36641	·10687	−·12414	−·08730	+·06717	+·07507	·41246
·341	·36682	·10628	−·12461	−·08689	+·06767	+·07477	·40974
·342	·36723	·10569	−·12509	−·08647	+·06817	+·07448	·40701
·343	·36764	·10510	−·12555	−·08606	+·06867	+·07418	·40429
·344	·36804	·10451	−·12602	−·08564	+·06916	+·07388	·40157
·345	·36844	·10391	−·12649	−·08522	+·06965	+·07358	·39886
·346	·36884	·10332	−·12695	−·08479	+·07014	+·07327	·39614
·347	·36923	·10272	−·12741	−·08437	+·07062	+·07296	·39343
·348	·36962	·10212	−·12786	−·08394	+·07110	+·07264	·39073
·349	·37001	·10152	−·12831	−·08351	+·07158	+·07232	·38802
·350	·37040	·10092	−·12876	−·08307	+·07206	+·07200	·38532

TABLE—(*continued*).

$\frac{1}{2}(1-a)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	h
·351	·37078	·10032	− ·12921	− ·08264	+ ·07254	+ ·07168	·38262
·352	·37116	·09971	− ·12966	− ·08220	+ ·07301	+ ·07135	·37993
·353	·37154	·09911	− ·13010	− ·08176	+ ·07348	+ ·07102	·37723
·354	·37192	·09850	− ·13054	− ·08131	+ ·07395	+ ·07069	·37454
·355	·37229	·09789	− ·13097	− ·08087	+ ·07441	+ ·07035	·37186
·356	·37266	·09728	− ·13140	− ·08042	+ ·07487	+ ·07002	·36917
·357	·37303	·09667	− ·13183	− ·07997	+ ·07533	+ ·06967	·36649
·358	·37340	·09606	− ·13226	− ·07952	+ ·07579	+ ·06933	·36381
·359	·37376	·09544	− ·13269	− ·07906	+ ·07624	+ ·06898	·36113
·360	·37412	·09483	− ·13311	− ·07861	+ ·07669	+ ·06863	·35846
·361	·37447	·09421	− ·13353	− ·07815	+ ·07714	+ ·06827	·35579
·362	·37483	·09359	− ·13394	− ·07768	+ ·07758	+ ·06792	·35312
·363	·37518	·09297	− ·13436	− ·07722	+ ·07803	+ ·06756	·35045
·364	·37553	·09235	− ·13477	− ·07675	+ ·07847	+ ·06719	·34779
·365	·37588	·09173	− ·13517	− ·07629	+ ·07890	+ ·06683	·34513
·366	·37622	·09111	− ·13558	− ·07582	+ ·07934	+ ·06646	·34247
·367	·37656	·09048	− ·13598	− ·07534	+ ·07977	+ ·06609	·33981
·368	·37690	·08985	− ·13638	− ·07487	+ ·08020	+ ·06571	·33715
·369	·37724	·08923	− ·13677	− ·07439	+ ·08062	+ ·06534	·33450
·370	·37757	·08860	− ·13717	− ·07391	+ ·08105	+ ·06496	·33185
·371	·37790	·08797	− ·13756	− ·07343	+ ·08147	+ ·06458	·32921
·372	·37823	·08734	− ·13794	− ·07295	+ ·08188	+ ·06419	·32656
·373	·37855	·08671	− ·13833	− ·07246	+ ·08230	+ ·06380	·32392
·374	·37888	·08607	− ·13871	− ·07198	+ ·08271	+ ·06341	·32128
·375	·37920	·08544	− ·13909	− ·07149	+ ·08312	+ ·06302	·31864
·376	·37951	·08480	− ·13946	− ·07100	+ ·08352	+ ·06262	·31600
·377	·37983	·08416	− ·13984	− ·07050	+ ·08392	+ ·06222	·31337
·378	·38014	·08353	− ·14021	− ·07001	+ ·08432	+ ·06182	·31074
·379	·38045	·08289	− ·14057	− ·06951	+ ·08472	+ ·06142	·30811
·380	·38076	·08225	− ·14094	− ·06901	+ ·08512	+ ·06101	·30548
·381	·38106	·08160	− ·14130	− ·06851	+ ·08551	+ ·06061	·30286
·382	·38136	·08096	− ·14166	− ·06801	+ ·08589	+ ·06019	·30023
·383	·38166	·08032	− ·14201	− ·06750	+ ·08628	+ ·05978	·29761
·384	·38196	·07967	− ·14236	− ·06700	+ ·08666	+ ·05936	·29499
·385	·38225	·07903	− ·14271	− ·06649	+ ·08704	+ ·05895	·29237
·386	·38254	·07838	− ·14306	− ·06598	+ ·08742	+ ·05853	·28976
·387	·38283	·07773	− ·14340	− ·06547	+ ·08779	+ ·05810	·28715
·388	·38312	·07708	− ·14374	− ·06495	+ ·08816	+ ·05766	·28454
·389	·38340	·07643	− ·14408	− ·06444	+ ·08853	+ ·05725	·28193
·390	·38368	·07578	− ·14442	− ·06392	+ ·08889	+ ·05682	·27932
·391	·38396	·07513	− ·14475	− ·06340	+ ·08925	+ ·05638	·27671
·392	·38423	·07447	− ·14508	− ·06288	+ ·08961	+ ·05595	·27411
·393	·38451	·07382	− ·14540	− ·06236	+ ·08997	+ ·05551	·27151
·394	·38478	·07316	− ·14573	− ·06183	+ ·09032	+ ·05507	·26891
·395	·38504	·07251	− ·14604	− ·06131	+ ·09067	+ ·05463	·26631
·396	·38531	·07185	− ·14636	− ·06078	+ ·09101	+ ·05419	·26371
·397	·38557	·07119	− ·14668	− ·06025	+ ·09136	+ ·05374	·26112
·398	·38583	·07053	− ·14699	− ·05972	+ ·09170	+ ·05329	·25853
·399	·38609	·06987	− ·14730	− ·05919	+ ·09203	+ ·05284	·25594
·400	·38634	·06921	− ·14760	− ·05866	+ ·09237	+ ·05239	·25335

TABLE—(*continued*).

$\frac{1}{2}(1-a)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	h
·401	·38659	·06855	− ·14790	− ·05812	+ ·09270	+ ·05193	·25076
·402	·38684	·06789	− ·14820	− ·05758	+ ·09303	+ ·05148	·24817
·403	·38709	·06722	− ·14850	− ·05705	+ ·09335	+ ·05102	·24559
·404	·38734	·06656	− ·14879	− ·05651	+ ·09367	+ ·05056	·24301
·405	·38758	·06589	− ·14908	− ·05596	+ ·09399	+ ·05010	·24043
·406	·38782	·06522	− ·14937	− ·05542	+ ·09430	+ ·04963	·23785
·407	·38805	·06456	− ·14965	− ·05488	+ ·09462	+ ·04916	·23527
·408	·38829	·06389	− ·14993	− ·05433	+ ·09493	+ ·04869	·23269
·409	·38852	·06322	− ·15021	− ·05378	+ ·09523	+ ·04822	·23012
·410	·38875	·06255	− ·15049	− ·05323	+ ·09553	+ ·04775	·22754
·411	·38897	·06188	− ·15076	− ·05268	+ ·09583	+ ·04728	·22497
·412	·38920	·06121	− ·15103	− ·05213	+ ·09613	+ ·04680	·22240
·413	·38942	·06053	− ·15130	− ·05158	+ ·09642	+ ·04632	·21983
·414	·38964	·05986	− ·15156	− ·05102	+ ·09671	+ ·04584	·21727
·415	·38985	·05919	− ·15182	− ·05047	+ ·09700	+ ·04536	·21470
·416	·39007	·05851	− ·15208	− ·04991	+ ·09728	+ ·04488	·21214
·417	·39028	·05784	− ·15233	− ·04935	+ ·09756	+ ·04439	·20957
·418	·39049	·05716	− ·15258	− ·04879	+ ·09784	+ ·04390	·20701
·419	·39069	·05648	− ·15283	− ·04823	+ ·09811	+ ·04341	·20445
·420	·39089	·05580	− ·15308	− ·04767	+ ·09838	+ ·04292	·20189
·421	·39109	·05513	− ·15332	− ·04711	+ ·09865	+ ·04243	·19934
·422	·39129	·05445	− ·15356	− ·04654	+ ·09891	+ ·04194	·19678
·423	·39149	·05377	− ·15380	− ·04598	+ ·09918	+ ·04144	·19422
·424	·39168	·05309	− ·15403	− ·04541	+ ·09943	+ ·04094	·19167
·425	·39187	·05240	− ·15426	− ·04484	+ ·09969	+ ·04044	·18912
·426	·39206	·05172	− ·15449	− ·04427	+ ·09994	+ ·03994	·18657
·427	·39224	·05104	− ·15471	− ·04370	+ ·10019	+ ·03944	·18402
·428	·39243	·05036	− ·15493	− ·04313	+ ·10043	+ ·03894	·18147
·429	·39261	·04967	− ·15515	− ·04256	+ ·10067	+ ·03843	·17892
·430	·39279	·04899	− ·15537	− ·04198	+ ·10091	+ ·03793	·17637
·431	·39296	·04830	− ·15558	− ·04141	+ ·10115	+ ·03742	·17383
·432	·39313	·04761	− ·15579	− ·04083	+ ·10138	+ ·03691	·17128
·433	·39330	·04693	− ·15599	− ·04026	+ ·10161	+ ·03640	·16874
·434	·39347	·04624	− ·15620	− ·03968	+ ·10183	+ ·03589	·16620
·435	·39364	·04555	− ·15640	− ·03910	+ ·10205	+ ·03537	·16366
·436	·39380	·04486	− ·15659	− ·03852	+ ·10227	+ ·03486	·16112
·437	·39396	·04418	− ·15679	− ·03794	+ ·10249	+ ·03434	·15858
·438	·39411	·04349	− ·15698	− ·03735	+ ·10270	+ ·03382	·15604
·439	·39427	·04280	− ·15717	− ·03677	+ ·10291	+ ·03330	·15351
·440	·39442	·04211	− ·15735	− ·03619	+ ·10311	+ ·03278	·15097
·441	·39457	·04141	− ·15753	− ·03560	+ ·10331	+ ·03226	·14843
·442	·39472	·04072	− ·15771	− ·03502	+ ·10351	+ ·03174	·14590
·443	·39486	·04003	− ·15789	− ·03443	+ ·10371	+ ·03121	·14337
·444	·39501	·03934	− ·15806	− ·03384	+ ·10390	+ ·03069	·14084
·445	·39514	·03864	− ·15823	− ·03325	+ ·10409	+ ·03016	·13830
·446	·39528	·03795	− ·15840	− ·03266	+ ·10427	+ ·02963	·13577
·447	·39542	·03726	− ·15856	− ·03207	+ ·10446	+ ·02910	·13324
·448	·39555	·03656	− ·15872	− ·03148	+ ·10463	+ ·02858	·13072
·449	·39568	·03587	− ·15888	− ·03089	+ ·10481	+ ·02804	·12819
·450	·39580	·03517	− ·15904	− ·03030	+ ·10498	+ ·02751	·12566

TABLE—(*continued*).

$\frac{1}{2}(1-a)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	h
·451	·39593	·03447	− ·15919	− ·02970	+ ·10515	+ ·02698	·12314
·452	·39605	·03378	− ·15934	− ·02911	+ ·10532	+ ·02644	·12061
·453	·39617	·03308	− ·15948	− ·02851	+ ·10548	+ ·02591	·11809
·454	·39629	·03238	− ·15962	− ·02792	+ ·10564	+ ·02537	·11556
·455	·39640	·03168	− ·15976	− ·02732	+ ·10579	+ ·02484	·11304
·456	·39651	·03099	− ·15990	− ·02673	+ ·10594	+ ·02430	·11052
·457	·39662	·03029	− ·16003	− ·02613	+ ·10609	+ ·02376	·10799
·458	·39673	·02959	− ·16016	− ·02553	+ ·10624	+ ·02322	·10547
·459	·39683	·02889	− ·16029	− ·02493	+ ·10638	+ ·02268	·10295
·460	·39694	·02819	− ·16041	− ·02433	+ ·10652	+ ·02214	·10043
·461	·39703	·02749	− ·16053	− ·02373	+ ·10665	+ ·02159	·09791
·462	·39713	·02679	− ·16065	− ·02313	+ ·10678	+ ·02105	·09540
·463	·39723	·02609	− ·16077	− ·02253	+ ·10691	+ ·02051	·09288
·464	·39732	·02539	− ·16088	− ·02193	+ ·10704	+ ·01996	·09036
·465	·39741	·02469	− ·16099	− ·02132	+ ·10716	+ ·01941	·08784
·466	·39749	·02398	− ·16109	− ·02072	+ ·10727	+ ·01887	·08533
·467	·39758	·02328	− ·16120	− ·02012	+ ·10739	+ ·01832	·08281
·468	·39766	·02258	− ·16130	− ·01951	+ ·10750	+ ·01777	·08030
·469	·39774	·02188	− ·16139	− ·01891	+ ·10761	+ ·01722	·07778
·470	·39781	·02117	− ·16149	− ·01830	+ ·10771	+ ·01668	·07527
·471	·39789	·02047	− ·16158	− ·01770	+ ·10781	+ ·01613	·07276
·472	·39796	·01977	− ·16166	− ·01709	+ ·10791	+ ·01558	·07024
·473	·39803	·01906	− ·16175	− ·01648	+ ·10801	+ ·01502	·06773
·474	·39809	·01836	− ·16183	− ·01588	+ ·10810	+ ·01447	·06522
·475	·39816	·01765	− ·16191	− ·01527	+ ·10818	+ ·01392	·06271
·476	·39822	·01695	− ·16198	− ·01466	+ ·10827	+ ·01337	·06020
·477	·39828	·01625	− ·16206	− ·01405	+ ·10835	+ ·01281	·05768
·478	·39834	·01554	− ·16212	− ·01344	+ ·10842	+ ·01226	·05517
·479	·39839	·01484	− ·16219	− ·01284	+ ·10850	+ ·01171	·05266
·480	·39844	·01413	− ·16225	− ·01223	+ ·10857	+ ·01115	·05015
·481	·39849	·01342	− ·16231	− ·01162	+ ·10864	+ ·01060	·04764
·482	·39854	·01272	− ·16237	− ·01101	+ ·10870	+ ·01004	·04513
·483	·39858	·01201	− ·16242	− ·01040	+ ·10876	+ ·00949	·04263
·484	·39862	·01131	− ·16247	− ·00979	+ ·10882	+ ·00893	·04012
·485	·39866	·01060	− ·16252	− ·00918	+ ·10887	+ ·00837	·03761
·486	·39870	·00990	− ·16257	− ·00857	+ ·10892	+ ·00782	·03510
·487	·39873	·00919	− ·16261	− ·00796	+ ·10896	+ ·00726	·03259
·488	·39876	·00848	− ·16265	− ·00734	+ ·10901	+ ·00670	·03008
·489	·39879	·00778	− ·16268	− ·00673	+ ·10905	+ ·00614	·02758
·490	·39882	·00707	− ·16271	− ·00612	+ ·10908	+ ·00559	·02507
·491	·39884	·00636	− ·16274	− ·00551	+ ·10912	+ ·00503	·02256
·492	·39886	·00566	− ·16277	− ·00490	+ ·10914	+ ·00447	·02005
·493	·39888	·00495	− ·16279	− ·00429	+ ·10917	+ ·00391	·01755
·494	·39890	·00424	− ·16281	− ·00367	+ ·10919	+ ·00335	·01504
·495	·39891	·00354	− ·16283	− ·00306	+ ·10921	+ ·00279	·01253
·496	·39892	·00283	− ·16284	− ·00245	+ ·10923	+ ·00224	·01003
·497	·39893	·00212	− ·16285	− ·00184	+ ·10924	+ ·00168	·00752
·498	·39894	·00141	− ·16286	− ·00122	+ ·10925	+ ·00112	·00501
·499	·39894	·00071	− ·16287	− ·00061	+ ·10925	+ ·00056	·00251
·500	·39894	·00000	− ·16287	·00000	+ ·10925	·00000	·00000

ON THE SELECTIVE ELIMINATION OCCURRING DURING THE DEVELOPMENT OF THE FRUITS OF STAPHYLEA.

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I. INTRODUCTION.

Volumes have been written concerning Natural Selection—its existence or non-existence, its potency or its inefficiency—but much to the discredit of biologists, thousands of pages of this literature may be explored without finding a single oasis of quantitative data to relieve the monotony of the polemic desert.

Fortunately there are a few notable exceptions, among which memoirs by Weldon[*][†], Bumpus[‡], Beeton and Pearson[§], Weldon[||], Di Cesnola[¶], Crampton[**], Di Cesnola[††], Kellogg and Bell[‡‡] and perhaps a few others may be mentioned as contributing to the direct quantitative evidence concerning the potency of natural selection.

These researches, highly valuable in themselves, are only a slight beginning in the scientific investigation of a field of cardinal biological importance. Perhaps in the earlier days of Darwinism it was not possible to do much more than collect qualitative evidence and to reason from analogy; one great value of the researches just cited is that they show that the exceedingly complex problems of natural selection can be made the field of quantitative investigation, and now that this fact has been demonstrated I think that we should stigmatize as merely pseudo-scientific discussions of selection not based on quantitative data.

Some years ago it occurred to me that it might be possible to ascertain some facts concerning selective elimination by comparing the characters of ovaries which develop into mature fruits with those which do not. That this is a distinct problem from that of the selective elimination of individuals I am quite aware. It is a consideration of the fitness of certain (morphological) types of organs produced by individuals rather than of the fitness of the individuals with given peculiarities. All will admit, I believe, that from the morphological and physiological view-point this problem is well worthy of careful study. In comparison with the results which may be obtained in quantitative investigation of selective elimination of individuals, the results should have real interest to students of evolution.

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[*] Weldon, W. F. R.: "An attempt to measure the death-rate due to the selective elimination of *Carcinus moenas* with respect to a particular dimension." *Proc. Roy. Soc. Lond.* Vol. LVII. pp. 360—379, 1895.

[†] Weldon, W. F. R.: Presidential address, Zoological Section. *Trans. Brit. Assoc. Bristol*, 1898, pp. 887—902.

[‡] Bumpus, H. C.: "The elimination of the unfit as illustrated by the introduced sparrow, *Passer domesticus*." *Biol. Lect. Del. Mar. Biol. Lab. Wood's Holl*, 1897—98, pp. 209—226, 1899.

[§] Beeton, Mary, and K. Pearson: "Data for the problem of evolution in man. II. A first study of the inheritance of longevity, and the selective death-rate in man." *Proc. Roy. Soc.* Vol. LXV. pp. 290—305, 1899.

[||] Weldon, W. F. R.: "A first study of natural selection in *Clausilia laminata*." *Biometrika*, Vol. I. pp. 109—124, 1901.

[¶] Di Cesnola, A. P.: "Preliminary note on the protective value of colour in *Mantis religiosa*." *Biometrika*, Vol. III. pp. 58, 59, 1903.

[**] Crampton, H. E.: "Experimental and statistical studies upon Lepidoptera. I. Variation and elimination in *Philosamia cynthia*." *Biometrika*, Vol. III. pp. 113—130, 1904. See also *Journ. Exp. Zool.* Vol. II. pp. 425—430, 1905.

[††] Di Cesnola, A. P.: "A first study of natural selection in *Helix arbustorum*." *Biometrika*, Vol. V. pp. 387—399, 1907.

[‡‡] Kellogg, V. L. and Ruby G. Bell: "Studies of variation in insects." *Proc. Wash. Acad. Sci.* Vol. VI. pp. 324—329, 1905.

A first study was made of the ovaries of the leguminous plant *Cercis**. Here the results were negative, but there were certain limitations to the material which I have never succeeded in removing. The results were published for two reasons. First, I wanted if possible to interest others in a problem which needs the results of several independent researches for its solution. Second, I had the present investigation nearly far enough along to see what the outcome would be, and I wanted to draw the conclusions from my first set of data quite independently and entirely unbiased by the evidence of a second study.

II. MATERIAL.

The fruit of the American Bladder Nut, *Staphylea trifolia*, is familiar to botanists as an inflated, membranaceous capsule of (generally) three locules, the dorsal sutures of which project to form three lobes. The carpels are (generally) united throughout the greater portion of the length of the fruit. From 4 to

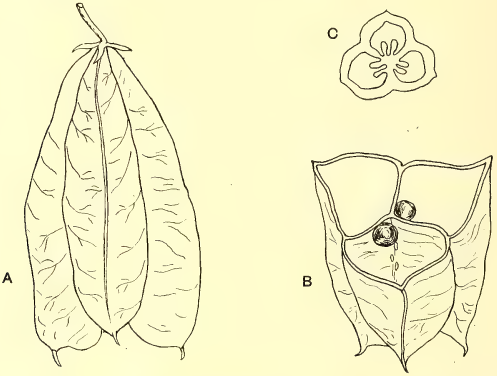

FIG. 1. Diagram showing the external appearance (A) and cross section (B) of fruit of *Staphylea*. Note the one seed and the arrangement of the aborted ovules in the front locule. The cross section of a young ovary (C) shows diagrammatically the disposition of the ovules of each locule in two rows along the inner angle.

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* Harris, J. Arthur: "Is there a Selective Elimination in the Fruiting of the Leguminosae?" *Am. Nat.* Vol. XLIII. pp. 556—559, 1909.

12 ovules are borne in two series along the inner angle of the locule. None, one or a few of the ovules in each cell or carpel develop into smooth, bony seeds.

The flowers are borne in racemes or panicles. Only a small percentage of those formed in an inflorescence mature fruits. I am able to present no reliable statistics on this point, but I would say that in the individuals which I have studied not over 10 per cent. of the flowers of an inflorescence which ripens fruits at all mature their ovaries. What proportion of inflorescences fail entirely to ripen fruits I am quite unable to say.

It has not seemed worth while to secure quantitative data on this point. The exact proportion varies from season to season. It is always so small that a very stringent elimination is obvious.

This memoir is limited to two series of data, (1) that especially collected for the purpose in 1908, and (2) a small lot of ovaries in different stages of development taken for other purposes in the spring of 1906.

1. *The* 1908 *Collections.*

The collection of this material and the counting of the number of ovules per locule was carried out by my two assistants, Miss Eva N. Dixon and Miss Rose M. Pechmann, and my sister, Miss Nellie L. Harris. I regret not having been able to attend to the sampling personally, but the necessity for an investigation of the problem of selective elimination in *Staphylea* became apparent to me after I had left the Missouri Botanical Garden, and the necessity of devoting the spring season to experimental plantings at the Station for Experimental Evolution, precluded my visiting the Garden at the flowering season for *Staphylea* to make the collections myself.

As for the countings, I cannot state too highly my indebtedness to these assistants for the faithful way in which the arduous task was carried out. In all over 21,000 locules had to be opened and the number of ovules counted. For the mature fruits the opening was simple, but the counting of the aborted ovules is not always an easy task. For the immature ovaries the work was much more trying to the patience and eyesight. It was quite out of the question to deal with the fresh material, and the difficulty of dissection and counting was increased by preservation in alcohol. First the ovaries had to be dissected out of the flowers and then each of the locules opened with a needle and the number of ovules counted. Possibly some of my fellow botanists would have had the deftness of fingers and the patience to carry through the countings, but I would have found the task impossible, and even Miss Pechmann and Miss Dixon occasionally nearly lost heart during the several months the work was under way.

The collection of material to determine whether or no there is a selective elimination is a difficult problem in a form like *Staphylea*. The number of flowers produced is very large, and it is quite impossible to take account of even a small proportion of them. The inflorescence is long enough to allow the lower ovaries to be developing into young fruits while the upper ones are still in the unopened buds. A collection of the flowers before any elimination has taken place cannot be made for comparison with the ripened fruits, for before the more distal flowers are mature enough for the ovules to be counted with safety, the more proximal ones may be falling from the plant or developing into young fruits.

It is desirable that the sample represent not only all parts of the plant, but also *all regions of the inflorescence*, since, should the ovaries be correlated with their position on the axis, the constants from material carelessly collected might not at all represent the population from which it is drawn.

The labour of gathering ovaries with a record of their position on the inflorescence seemed to be prohibitive. After careful consideration it became apparent that the collection of the spring material must be carried out in a rather rough and ready manner. The following was adopted.

When a shrub was well in flower it was shaken gently to dislodge the flowers which had ceased to develop and were ready to fall. Sometimes flowers were on the ground which had been dislodged by wind and rain, but these were never taken for fear that they had been removed by too great violence. After the tree had been shaken and the fallen sample collected, a gathering as nearly random as possible of the opened flowers which remained was taken.

In a few cases only a single collection of fallen and apparently developing flowers was made from an individual, but usually the shrub was visited again after a few days and a second gathering was made in the same manner as the first.

Finally a series of matured fruits from these individuals was taken in the autumn.

In the two spring series only ovaries dissected from well-opened flowers were examined. All buds were discarded for fear that the ovules might not yet all be laid down. By discarding any young fruits which fell we avoided one danger of overweighting the proximal region of the inflorescence.

From the foregoing remarks it appears that the 1908 ovaries fall into three classes:

A. Ovaries from opened flowers which were eliminated from the shrub.

B. Ovaries from opened flowers which were apparently continuing their development at the time of sampling.

C. Ovaries which had completed their development to mature fruits.

Series B should represent the original population of ovaries more nearly than A, since it contains many which would be eliminated later. Because of the methods necessarily employed in collecting the samples, I do not look upon the comparison of Series A and Series B as very critical. Only a relatively small proportion of the ovaries develop to maturity; Series A doubtless contains many which are non-selectively eliminated, while B includes many which a little later would have fallen from the tree and been classified as A.

It seems worth while, however, to carry out the comparison between Series A and B. The critical tests are those between B and C and A and C, the most significant being the latter.

In *Staphylea* the normal type of ovary is 3-merous. Occasionally individuals are found which produce a large proportion of 4-merous fruits. Such a one is shrub 36, which was omitted in all these comparisons because I wanted to deal with as nearly a "normal" population of 3-merous ovaries as possible. I feared that the 3-merous ovaries produced on a shrub which bore a very high percentage of 4-merous fruits might be in some characteristics—morphological or physiological—different from those formed by the growing points of an individual with almost exclusively trimerous tendencies.

Shrub 28 bore no flower in 1908.

In dealing with the raw data it has seemed desirable to treat the material for each individual separately, and for the following reason. Rather extensive experience in dealing with biometric constants calculated from the fruits of individuals has shown that there may be very real differences between those of different plants. If each individual did not contribute the same quantity of material to the three collections studied, it is quite possible that in dealing with lumped samples differences in constants might arise quite independently of any influence of selective elimination. I think that with 28 individuals the danger of error from this source is not great, but conclusions based on the results from 28 individual small samples will be stronger than those resting upon merely one large composite series.

2. *The* 1906 *Collections.*

As indicated above these collections were not made primarily for a study of selective elimination at all, but for a study of some questions of fertility and fecundity. Constants calculated from them indicated the necessity for the detailed study attempted in 1908. The results are treated here as supplementary to those from the more detailed investigations made later.

The collection comprises 270 inflorescences* bearing partly developed fruits, none of which had reached a length greater than 20 mm., and was made primarily to determine the change in the correlation between the number of ovules formed and the number developing into seeds at different stages in the development of the fruit. The fruit increases in size very rapidly during its early stages of development, and only a rough measure of its size was considered worth while. Three classes, 6—10 mm., 11—15 mm. and 16—20 mm. in length, were recognized†. The shape of the fruit varies considerably, and the length affords only a rough measure of the size, but it is ample for present purposes.

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* The correlation between the number of flowers formed and the number of fruits still continuing to develop at the time the inflorescences were examined is discussed in *Biometrika*, Vol. VI. pp. 440, 441, 1909.

† These are exclusive of the short styles.

Fruits which have reached a length of 11—15 mm. may be considered to be a selected class as compared with those 6—10 mm. in length—if selection occurs at all—and those of 16—20 mm. in length may be regarded as a selected class as compared with both of the lower classes.

III. STATEMENT OF PROBLEMS AND METHODS OF INVESTIGATION.

In this memoir five characters of the ovary are considered. They are:

(1) Number of ovules per locule.

(2) Total number of ovules per ovary.

(3) Radial asymmetry of the ovary with respect to the number of ovules per locule.

(4) Number of locules per ovary with an "odd" number of ovules.

(5) Number of locules per ovary.

To demonstrate that there is a selective elimination for any or all of these characters, it is necessary to show that there are differences between the constants for the ovaries which develop into fruits and those which do not, of such an order of magnitude that they cannot reasonably be referred to the probable errors of random sampling. In these pages I follow the rather common example of statisticians in regarding differences of at least 2·5 times their probable errors as significant. In addition I have demanded reasonably constant results from the series of individual plants.

The working hypothesis of the existence of a selective elimination for any character whatever may be tested by determining the difference between the constants for:

(*a*) A sample from an original population before elimination and a sample of the eliminated individuals.

(*b*) A sample from an original population before elimination and a sample of the individuals remaining after elimination.

(*c*) A sample of eliminated individuals and a sample of individuals unaffected by the elimination.

Here "individual," "sample" and "population" are used in their commonly accepted and convenient statistical sense. By individual we understand a particular ovary, by sample the collection of ovaries with which the limitations of time permitted us to deal, and by population the total ovaries produced by a given shrub, or the total produced by all the shrubs investigated, as the case may be.

A constant calculated from a sample does not necessarily describe precisely the population from which it was drawn, for it may be too large or too small by an amount depending upon the chance errors of random sampling. As biologists we are interested in our samples only in so far as they indicate for us the characteristics of the population. The degree of significance to be attached to the constant calculated from a sample as a description of a population as a whole is determined by the probable error of the constant. The probable error of random sampling has nothing whatever to do with the care of the naturalist in doing his work, as some biologists seem to believe. Be the observer never so keen and conscientious, the significance to be attached to his conclusions is always to some extent dependent upon the number of observations before him on his work-table. This is merely a conclusion drawn by common sense from common observation. Fortunately in many fields of work the mathematician can give us formulae for determining the relative weight to be attached to constants calculated from different masses of data. These words of explanation are inserted for the benefit of fellow-biologists who may still be unacquainted with the true nature of the probable error of random sampling so much used by biometricians, for throughout these pages I find it necessary to return continually to this fundamental safeguard in the interpretation of the significance, as descriptions of populations, of constants drawn from samples of the population.

In the investigation of the problem of selective elimination on the *Cercis* material, I was necessarily limited to the comparison of the constants for the original population with those for the eliminated ovaries, since a severe frost prevented any of the ovaries maturing. Here we are able to compare eliminated ovaries, the original population after partial elimination, and the matured fruits.

Further details concerning problems and methods will be taken up under the individual problems, to which we now pass.

IV. DISCUSSION OF PROBLEMS ON BASIS OF ACTUAL DATA FOR 1908.

Problem 1. Is there a difference in the mean number of ovules per locule, or per fruit, in the ovaries which are eliminated and those which develop to maturity?

Seriations of the number of ovules per locule are given for these collections of material in Appendix, Tables XXVIII—XXX. The data for the total ovules per fruit are given for the three collections from the 28 individuals in Tables XXXI—XXXIII. The means, standard deviations and coefficients of variation for the number of ovules per locule, and the standard deviations and coefficients of variation of the total ovules per ovary are given for the 28 individuals of the three collections and their totals in Tables I—III.

In comparing the means for the three series it is unnecessary to treat both ovules per locule and total ovules per fruit, for the latter is necessarily three times

TABLE I.

Physical Constants for Series A.

Number of Shrub	Ovules per Locule			Ovules per Ovary	
	Average and Probable Error	Standard Deviation and Probable Error	Coefficient of Variation	Standard Deviation and Probable Error	Coefficient of Variation
11	8·117 ± ·022	·574 ± ·016	7·07 ± ·20	1·143 ± ·054	4·70 ± ·22
12	7·237 ± ·036	·920 ± ·025	12·72 ± ·36	2·060 ± ·098	9·49 ± ·46
13	7·693 ± ·047	·856 ± ·033	11·13 ± ·46	1·842 ± ·124	7·98 ± ·54
14	6·440 ± ·041	·752 ± ·029	11·69 ± ·46	1·816 ± ·122	9·40 ± ·64
15	7·893 ± ·033	·842 ± ·023	10·67 ± ·30	1·737 ± ·083	7·34 ± ·35
16	7·226 ± ·040	·965 ± ·028	13·35 ± ·39	2·327 ± ·117	10·74 ± ·55
17	7·873 ± ·035	·908 ± ·025	11·53 ± ·32	2·144 ± ·102	9·08 ± ·44
18	7·467 ± ·039	·708 + ·028	9·49 ± ·37	1·414 ± ·095	6·31 ± ·43
19	7·181 ± ·038	·936 ± ·027	13·03 ± ·38	2·007 ± ·101	9·31 ± ·47
20	6·658 ± ·045	1·025 ± ·032	15·39 ± ·48	2·091 ± ·111	10·47 ± ·56
21	7·453 ± ·043	·788 ± ·031	10·58 ± ·42	1·775 ± ·120	7·94 ± ·54
22	7·570 ± ·038	·732 ± ·027	9·67 ± ·36	1·557 ± ·100	6·86 ± ·44
23	6·060 ± ·045	·818 ± ·032	13·50 ± ·53	1·894 ± ·128	10·42 ± ·71
24	7·128 ± ·045	·888 ± ·032	12·46 ± ·45	1·907 ± ·117	8·92 ± ·55
25	8·154 ± ·061	1·256 ± ·043	15·40 ± ·54	3·329 ± ·198	13·61 ± ·82
26	6·953 ± ·046	·843 ± ·033	12·13 ± ·18	1·844 ± ·124	8·84 ± ·60
27	7·067 ± ·043	·894 ± ·030	12·66 ± ·44	2·128 ± ·126	10·04 ± ·60
29	6·663 ± ·030	·781 ± ·021	11·72 ± ·33	1·763 ± ·084	8·82 ± ·42
30	5·917 ± ·029	·659 ± ·020	11·14 ± ·35	1·220 ± ·065	6·87 ± ·37
31	7·237 ± ·030	·783 ± ·022	10·82 ± ·30	1·687 ± ·080	7·77 ± ·37
32	7·713 ± ·035	·723 ± ·025	9·38 ± ·32	1·557 ± ·092	6·73 ± ·40
33	6·648 ± ·035	·856 ± ·025	12·87 ± ·38	1·946 ± ·098	9·76 ± ·49
34	7·053 ± ·046	·838 ± ·033	11·88 ± ·47	2·082 ± ·140	9·84 ± ·67
35	7·490 ± ·029	·741 ± ·020	9·90 ± ·27	1·513 ± ·072	6·73 ± ·32
37	8·337 ± ·050	1·292 ± ·036	15·50 ± ·44	3·254 ± ·155	13·01 ± ·63
38	6·020 ± ·025	·648 ± ·018	10·76 ± ·30	1·489 ± ·071	8·24 ± ·40
39	7·933 ± ·041	·668 ± ·029	8·42 ± ·37	1·470 ± ·111	6·17 ± ·47
40	7·118 ± ·043	·884 ± ·030	12·42 ± ·43	1·973 ± ·117	9·24 ± ·55

the former. The frequencies and constants for total ovules per ovary are conveniently tabled here for a later problem.

For purposes of comparison we may now take the differences between the three series. We use:

B—A, or the constant for the developing ovules minus the constant for the eliminated ovaries.

C—B, or the constant for the matured fruits minus the constant for those in early stages of development.

C—A, or the constant for the matured fruits minus the constant for the eliminated ovaries.

The three comparisons are made in Table IV.

TABLE II.

Physical Constants for Series B.

Number of Shrub	OVULES PER LOCULE			OVULES PER OVARY	
	Average and Probable Error	Standard Deviation and Probable Error	Coefficient of Variation	Standard Deviation and Probable Error	Coefficient of Variation
11	8·157 ± ·026	·672 ± ·018	8·24 ± ·23	1·300 ± ·062	5·31 ± ·25
12	6·913 ± ·038	·990 ± ·027	14·32 ± ·40	2·427 ± ·011	11·70 ± ·57
13	7·827 ± ·024	·624 ± ·017	7·97 ± ·22	1·109 ± ·053	4·72 ± ·23
14	6·573 ± ·050	·912 ± ·035	13·87 ± ·55	2·289 ± ·154	11·61 ± ·79
15	8·087 ± ·031	·786 ± ·022	9·73 ± ·27	1·487 ± ·071	6·13 ± ·29
16	7·141 ± ·043	1·038 ± ·030	14·53 ± ·43	2·608 ± ·131	12·17 ± ·62
17	8·137 ± ·033	·847 ± ·023	10·41 ± ·29	1·795 ± ·086	7·35 ± ·35
18	7·080 ± ·035	·906 ± ·025	12·79 ± ·36	2·168 ± ·103	10·21 ± ·49
19	7·246 ± ·043	·993 ± ·031	13·70 ± ·43	2·257 ± ·120	10·38 ± ·56
20	7·107 ± ·037	·915 ± ·027	12·87 ± ·38	2·049 ± ·103	9·61 ± ·49
21	7·772 ± ·025	·604 ± ·018	7·77 ± ·23	1·366 ± ·071	5·86 ± ·30
22	7·620 ± ·031	·640 ± ·022	8·40 ± ·29	1·252 ± ·074	5·48 ± ·32
23	6·140 ± ·040	·721 ± ·028	11·75 ± ·46	3·544 ± ·239	19·24 ± ·34
24	7·026 ± ·046	·947 ± ·032	13·48 ± ·47	2·269 ± ·134	10·77 ± ·64
25	8·883 ± ·046	1·190 ± ·033	13·40 ± ·38	2·517 ± ·120	9·45 ± ·45
26	7·093 ± ·049	·882 ± ·034	12·44 ± ·49	1·929 ± ·130	9·07 ± ·62
27	7·433 ± ·042	·960 ± ·030	12·91 ± ·40	2·135 ± ·114	9·58 ± ·52
29	6·383 ± ·034	·874 ± ·024	13·69 ± ·38	2·007 ± ·096	10·48 ± ·50
30	6·047 ± ·032	·760 ± ·023	12·57 ± ·38	1·674 ± ·087	9·23 ± ·48
31	6·860 ± ·032	·833 ± ·023	12·14 ± ·34	1·960 ± ·093	9·53 ± ·46
32	7·677 ± ·029	·738 ± ·020	9·62 ± ·27	1·729 ± ·082	7·51 ± ·36
33	6·357 ± ·036	·864 ± ·026	13·59 ± ·41	1·896 ± ·098	9·94 ± ·52
34	7·089 ± ·042	1·036 ± ·030	14·61 ± ·43	2·322 ± ·117	10·92 ± ·55
35	7·730 ± ·022	·557 ± ·015	7·21 ± ·20	1·046 ± ·050	4·51 ± ·22
37	8·763 ± ·041	1·046 ± ·029	11·94 ± ·33	2·442 ± ·116	9·29 ± ·45
38	5·833 ± ·026	·668 ± ·018	11·45 ± ·32	1·411 ± ·067	8·06 ± ·39
39	8·240 ± ·027	·699 ± ·019	8·49 ± ·24	1·429 ± ·068	5·78 ± ·28
40	7·780 ± ·034	·874 ± ·024	11·24 ± ·31	2·006 ± ·096	8·59 ± ·41

The means of the eliminated ovaries will be sensibly identical with those of the developing series if there is no selective elimination. If there is a tendency to the elimination of those with high numbers of ovules, all three of these differences will have the negative sign, while, if it is the ovaries with the lower number of ovules which are the more likely to be eliminated, the difference will carry the positive sign. These statements apply only to such differences as are greater than those liable to arise through the errors of random sampling. When differences are slight, the errors due to this source may be considerable enough to change the sign of a constant from positive to negative. The probable errors of the differences are also given, and to facilitate comparison the ratios of the differences to their probable error, are also tabulated.

Before considering the ratios of the individual differences to their probable errors we may examine the table as a whole, basing our conclusions concerning the presence or the absence of a selective elimination merely upon the signs of the differences in the mean number of ovules per locule.

If there be no selective elimination whatever, the means calculated from the three collections should be sensibly identical—that is to say, identical except for the differences due to the errors of random sampling. If our series of shrubs were very large we should expect half of the differences between the values of any two constants to be negative and half to be positive, if these differences were due merely to chance differences in the gathering of the fruits composing the samples, just as we would expect 50 per cent. heads and 50 per cent. tails in tossing a coin a few hundred times. We have only 28 throws of our coin, to use our figure, and so we cannot expect to get exactly 14 of each sign, but we ought not to get very wide divergences from these numbers unless there is some biological factor at work to modify the proportions.

That there must be some such factor is quite evident from a casual examination of the tables. Taking first the differences between the eliminated ovaries and those which are continuing their development, we find that the mean number of

TABLE III.

Physical Constants for Series C.

Number of Shrub	Ovules per Locule			Ovules per Ovary	
	Average and Probable Error	Standard Deviation and Probable Error	Coefficient of Variation	Standard Deviation and Probable Error	Coefficient of Variation
11	8·360 ± ·030	·760 ± ·021	9·09 ± ·25	1·804 ± ·086	7·19 ± ·34
12	7·480 ± ·031	·793 ± ·022	10·61 ± ·28	1·657 ± ·079	7·38 ± ·35
13	6·757 ± ·036	·915 ± ·025	13·54 ± ·38	1·891 ± ·090	9·33 ± ·45
14	6·588 ± ·035	·746 ± ·025	11·32 ± ·38	1·664 ± ·096	8·42 ± ·49
15	8·713 ± ·032	·836 ± ·023	9·59 ± ·27	1·980 ± ·094	7·57 ± ·36
16	8·190 ± ·033	·860 ± ·024	10·51 ± ·29	2·127 ± ·101	8·66 ± ·42
17	8·730 ± ·041	1·051 ± ·029	12·03 ± ·34	2·671 ± ·127	10·20 ± ·49
18	7·587 ± ·029	·754 ± ·021	9·94 ± ·28	1·861 ± ·089	8·18 ± ·39
19	7·850 ± ·027	·689 ± ·019	8·77 ± ·24	1·639 ± ·078	6·96 ± ·33
20	7·643 ± ·028	·709 ± ·019	9·28 ± ·26	1·538 ± ·073	6·71 ± ·32
21	8·067 ± ·025	·654 ± ·018	8·11 ± ·22	1·544 ± ·074	6·38 ± ·31
22	7·903 ± ·020	·511 ± ·014	6·47 ± ·18	1·151 ± ·055	4·86 ± ·23
23	6·261 ± ·039	·777 ± ·028	12·41 ± ·45	1·644 ± ·101	8·75 ± ·54
24	7·537 ± ·028	·713 ± ·020	9·46 ± ·26	1·378 ± ·066	6·09 ± ·29
25	9·597 ± ·077	1·990 ± ·055	20·74 ± ·59	1·930 ± ·092	6·70 ± ·32
26	7·618 ± ·030	·668 ± ·021	8·77 ± ·28	1·429 ± ·078	6·25 ± ·34
27	8·103 ± ·027	·688 ± ·019	8·49 ± ·24	1·347 ± ·064	5·54 ± ·26
29	6·917 ± ·031	·802 ± ·022	11·59 ± ·32	1·763 ± ·084	8·49 ± ·41
30	6·387 ± ·026	·666 ± ·018	10·43 ± ·29	1·294 ± ·062	6·75 ± ·32
31	7·463 ± ·029	·750 ± ·021	10·05 ± ·28	1·679 ± ·080	7·50 ± ·36
32	7·810 ± ·026	·664 ± ·018	8·50 ± ·24	1·471 ± ·070	6·28 ± ·30
33	6·833 ± ·030	·783 ± ·022	11·45 ± ·32	1·775 ± ·085	8·66 ± ·42
34	7·647 ± ·029	·740 ± ·020	9·68 ± ·27	1·541 ± ·073	6·72 ± ·32
35	7·830 ± ·022	·578 ± ·016	7·38 ± ·20	1·127 ± ·054	4·80 ± ·23
37	8·533 ± ·040	1·021 ± ·028	11·97 ± ·33	2·112 ± ·101	8·25 ± ·40
38	7·003 ± ·035	·908 ± ·025	12·96 ± ·36	2·343 ± ·112	11·15 ± ·53
39	8·290 ± ·029	·743 ± ·020	8·97 ± ·25	1·540 ± ·073	6·19 ± ·30
40	8·233 ± ·028	·721 ± ·020	8·75 ± ·24	1·546 ± ·074	6·26 ± ·30

TABLE IV.

Difference between Mean Number of Ovules per Locule for three Series of Ovaries.

Number of Shrub	B—A		C—B		C—A	
	Difference and Probable Error	Diff. / *E* diff.	Difference and Probable Error	Diff. / *E* diff.	Difference and Probable Error	Diff. / *E* diff.
11	·040 ± ·034	1·16	·203 ± ·039	5·14	·243 ± ·037	6·55
12	– ·323 ± ·053	6·15	·567 ± ·049	11·47	·243 ± ·047	5·14
13	·133 ± ·053	2·51	– 1·070 ± ·043	24·82	– ·937 ± ·059	15·85
14	·133 ± ·065	2·05	·015 ± ·061	·24	·148 ± ·054	2·73
15	·193 ± ·045	4·31	·627 ± ·046	13·71	·820 ± ·046	17·75
16	– ·085 ± ·058	1·46	1·049 ± ·045	23·21	·964 ± ·052	18·58
17	·263 ± ·048	5·44	·593 ± ·052	11·30	·857 ± ·054	15·83
18	·333 ± ·053	6·34	– ·213 ± ·046	4·65	·120 ± ·049	2·46
19	·064 ± ·058	1·11	·604 ± ·051	11·89	·669 ± ·047	14·29
20	·449 ± ·058	7·70	·536 ± ·047	11·50	·985 ± ·052	18·83
21	·319 ± ·050	6·35	·294 ± ·035	8·50	·613 ± ·050	12·19
22	·051 ± ·049	1·03	·283 ± ·037	7·68	·334 ± ·043	7·70
23	·080 ± ·060	1·33	·121 ± ·056	2·17	·201 ± ·060	3·37
24	– ·102 ± ·064	1·60	·511 ± ·054	9·53	·409 ± ·053	7·77
25	·729 ± ·076	9·55	·713 ± ·090	7·90	1·443 ± ·098	14·66
26	·140 ± ·074	1·88	·525 ± ·057	9·21	·665 ± ·055	12·03
27	·367 ± ·060	6·10	·670 ± ·050	13·51	1·037 ± ·051	20·40
29	– ·280 ± ·033	8·51	·533 ± ·046	11·57	·253 ± ·044	5·81
30	·130 ± ·043	3·03	·340 ± ·041	8·24	·470 ± ·039	12·18
31	– ·377 ± ·044	8·46	·603 ± ·044	13·84	·227 ± ·042	5·37
32	– ·036 ± ·045	·80	·133 ± ·039	3·45	·097 ± ·043	2·24
33	– ·291 ± ·039	7·37	·476 ± ·048	10·01	·185 ± ·046	3·98
34	·036 ± ·063	·57	·558 ± ·051	10·87	·593 ± ·054	10·93
35	·240 ± ·036	6·65	·100 ± ·031	3·19	·340 ± ·037	9·29
37	·427 ± ·065	6·59	– ·230 ± ·057	4·04	·197 ± ·065	3·03
38	– ·187 ± ·036	5·16	1·170 ± ·044	26·65	·983 ± ·043	22·66
39	·307 ± ·049	6·22	·050 ± ·040	1·26	·357 ± ·050	7·10
40	·662 ± ·055	12·13	·453 ± ·044	10·26	1·115 ± ·051	21·83

ovules is higher in the developing series in 20 cases and higher in the eliminated series in 8 cases. This is a deviation of only 6 shrubs from the theoretical 14 of each class and hence cannot be regarded as very strongly significant. So far as the evidence goes, it indicates that ovaries bearing a smaller number of ovules are less likely to continue their development towards maturity than those having a larger number.

Even if there were a strong selective elimination of ovaries with a smaller number of ovules per locule, we would not expect to detect the full intensity of such a selective elimination in a comparison of the two series A and B, for the simple reason that we are comparing an eliminated series of ovaries with a series in which further elimination is to be expected.

If it is true that the elimination is not yet complete in the B series, and that the failure of ovaries to develop is not random, but selective, we would expect to find differences between the B series and the matured fruits—series C—of the same prevailing sign as between the series A and B.

The results obtained by taking the difference C—B bear out in a most striking manner the considerations drawn from the differences B—A, for only 3 out of the 28 differences bear the negative sign!

Finally the most critical comparison is that between the sample of eliminated ovaries and the fruits which reach maturity, i.e. C—A. The results bear out fully the conclusions drawn from the two foregoing comparisons, for only *one* of the 28 differences has the negative sign.

If any biologist wishes to test the number of cases in which twenty-seven positive and one negative sign would be expected according to purely chance causes, he might take 28 coins and toss them, noting the number of heads and tails he gets in each toss in order to determine the number of times that he will find 27 heads and 1 tail in a few hundred tosses. Mathematical theory says it will be about 46 times in 100,000,000.

The reader may take these chances for what he considers them worth. Personally I feel that the evidence is rather strongly in favour of the conclusion that there is a selective elimination of ovaries and that on the average it is those with the larger number of ovules which reach maturity.

In Diagram 1 I have tried to make patent to the eye the difference between the eliminated and the matured ovaries for each of the 28 individuals and for all of the material examined. Here the circles show the mean of the eliminated ovaries and the solid dots the mean of those which reach maturity. The length of

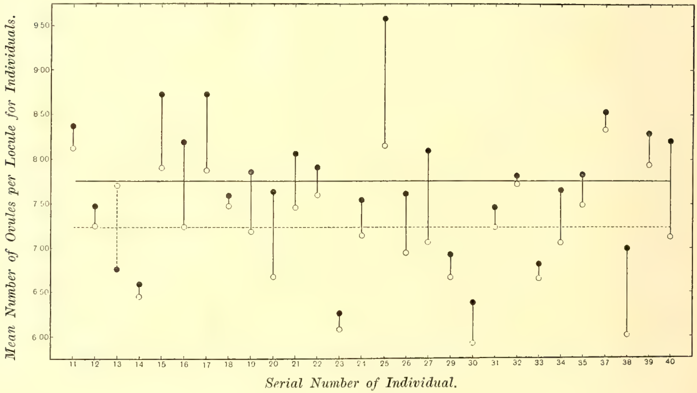

DIAGRAM 1. Showing difference in mean of eliminated and matured ovaries for the 28 individuals and for the population. Circles = mean of eliminated locules for individuals; solid dots = mean of matured locules for individuals. Broken transverse line = mean of all eliminated locules; solid transverse line = mean of all matured locules.

the line connecting these shows the difference between the two samples for each individual. The transverse lines show the position of the means for the whole population, the broken one being for the eliminated and the continuous one for the matured ovaries.

The diagram shows one further point—the great difference in the means of the individuals. The two transverse lines may be regarded as in a way smoothing these irregularities. The solid line is really the mean position of the solid dots on the scale while the broken line shows the mean position of the circles, each individual being weighted with the number of ovaries which it furnished. I think the differences shown on the diagram are conspicuous enough to carry considerable weight.

Let us now consider the significance of the constants from the individual trees. We shall not expect this to be very high as judged by a comparison of the probable errors, because of the smallness of the samples of material taken. Unfortunately the excessive labour of the dissection and the counting of the ovaries, even in the mature state, precluded the examination of more extensive collections.

The trustworthiness of the difference between any two constants is indicated by its probable error. The probable error of the difference between any two uncorrelated* constants, say x and y, may be calculated by the formula

$$E(x-y)=\sqrt{Ex^2+Ey^2}.$$

Conventionally, a constant is not considered significant unless it is 2·5 or more times its probable error. This is the degree of divergence from 0 demanded in the following discussion. For convenience of comparison the ratio

$$\frac{\text{Difference}}{\text{Probable Error of Difference}}$$

has been tabled for the three comparisons for each tree (see the third column, Table IV.).

For the comparison B—A it appears that there are 13 significantly positive and 5 significantly negative differences. For the comparison C—B there are 22 significantly positive and 3 significantly negative differences.

Finally for the most critical test of selective elimination, C—A, there are 25 significantly positive and 1 significantly negative differences.

The thing which strikes one particularly about these conclusions is the high number of cases in which the differences between the constants are significant with

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[*] I am afraid that here our constants are not perfectly independent. As Professor Pearson suggested to me, this is certainly not true where the size of samples is at all large as compared with the total number of ovaries produced, for a random excess in a first sample would be associated with a random deficit in the second.

I do not believe that all of my samples combined equal one-tenth of the total ovaries produced, and I suspect that the influence of the correlation of the samples would not be large, but I have no statistical proof of this. Indeed in a form like *Staphylea* it would be difficult to get suitable data for the correction of the probable error of the differences in two constants, even if the formulae were available. I think these constants affect in no way the validity of the conclusions drawn in this paper, but they may render some of the probable errors somewhat questionable.

respect to their probable error. In 18, 25 and 26 cases out of the 28 the differences exceed 2·5 times their probable errors, and often reach many times their probable error. I think relationships of significantly positive to significantly negative differences, such as 13 : 5, 22 : 3, 25 : 1 can leave no doubt in the mind of the most sceptical that there is a very potent selective elimination, by which the mean number of ovules per ovary is very materially increased.

The combination of the material from all the individuals gives the results for the differences in the mean number of ovules per locule presented in Table V.

TABLE V.

Comparison of Means for total Material.

Comparison	Difference and Probable Error	Ratio of Difference to its Probable Error	Percentage Increase in Mean
B—A	·1426 ± ·0129	11·054	1·971
C—B	·3693 ± ·0120	30·775	5·005
C—A	·5119 ± ·0121	42·306	7·075

Here the absolute differences are given in the first column, and the percentage increase of the higher over the lower mean in the last column. The results emphasize the conclusions drawn from the constants for the individual trees.

The difference between the eliminated sample and the sample of ovaries remaining on the tree when they were taken is only about 2 per cent. of the former, but the difference is about 11 times its probable error and thus certainly significant.

Doubtless the very slight divergence between the series A and B is due to the presence of a large number of ovaries in B that would a few days later fall from the tree and be classed with the A's, for we note a much greater increase from B to C than from A to B, amounting to no less than 5 per cent.

As is to be expected the difference between the eliminated and the matured fruits is greatest of all, the latter having about 7 per cent. more ovules per locule than the former.

The reader will note that in the last two comparisons the differences are 31 and 42 times their probable errors.

As a further demonstration of the differences in number of ovules per locule or per fruit in the three series I give (Diagram 2), the data in the form of an integral polygon in which the frequency on each grade is the sum of its own frequency and that of all lower grades. This form is one of the most convenient for purposes of comparison.

It is unnecessary to table the actual frequencies for they can easily be computed by anyone who requires them from the totals of our tables. The percentage frequencies for number of ovules per locule are given in Table VI, and for total number of ovules per fruit in Table VII. This method of presenting the data shows very clearly the selective elimination. For instance, in the comparison of

TABLE VI.

Percentile Frequencies of Ovules per Locule in three Series.

Number of Ovules	A	B	C
2	—	·01	—
3	·03	·03	—
4	·22	·35	·04
5	4·25	4·27	·74
6	27·86	25·19	15·78
7	54·64	48·30	31·39
8	91·50	88·00	59·79
9	98·01	96·59	93·11
10	99·94	99·50	99·78
11	100·00	99·93	99·99
12	—	100·00	100·00

TABLE VII.

Percentile Frequencies of total Ovules per Ovary in three Series.

Number of Ovules	A	B	C
11	—	·04	—
12	·05	·04	·04
13	·05	·12	·04
14	·19	·24	·04
15	·76	·77	·04
16	2·10	2·60	·18
17	6·01	6·33	·96
18	16·13	15·58	6·92
19	23·72	22·60	11·69
20	33·17	30·30	18·19
21	43·44	38·54	24·15
22	57·42	47·99	33·28
23	71·88	62·27	45·12
24	88·50	83·04	76·18
25	93·75	89·86	84·95
26	96·18	93·87	89·79
27	97·57	95·94	92·71
28	98·90	97·73	94·75
29	99·67	99·03	97·15
30	99·90	99·80	99·63
31	100·00	99·96	99·96
32	—	99·96	100·00
33	—	100·00	—

number of ovules per locule we note that locules with 7 ovules or less form 54·6 per cent. of the collection, while in the collection of matured fruits they form only 31·4 per cent. In all three series we are dealing with over 2000 fruits, or over 6000 individual locules, so the results can be looked upon with considerable confidence.

Diagrams 2 and 3 show these results graphically.

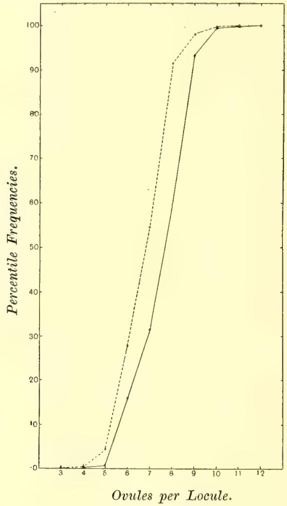

Diagram 2. Percentile curve showing frequency of different numbers of ovules per locule in the eliminated series and the series developing to maturity. Dots and broken lines = Eliminated series; dots and firm lines = Matured series.

Problem 2. Is there a difference in the variability in number of ovules per locule and per fruit in the eliminated ovaries and those which develop to maturity?

The results of the preceding section indicate clearly that the chances of an ovary developing to maturity are not independent of its number of ovules but that those with the smaller numbers are apt to be eliminated.

It is quite thinkable that a strong selective elimination might exist without bringing about any sensible difference in the mean values of a character in the population before and after elimination. If the elimination were not of the larger or the smaller fruits as measured by the number of ovules per locule or per fruit, but of *both* largest and smallest fruits, i.e. of the *extreme* variates of both kinds, the mean values of the number of ovules in the matured fruits might not differ sensibly from that for the original ovaries, but the variability in number would be reduced.

On the other hand it is not impossible that a selective elimination might effect chiefly the modal region of the curve of distribution, materially flattening it off and so increasing the variability as measured by the standard deviation. To the biologist this seems theoretically less probable than the elimination of either the largest or the smallest ovaries, or the extreme variates of both types, but of course all possibilities must be tested out on the basis of actual data.

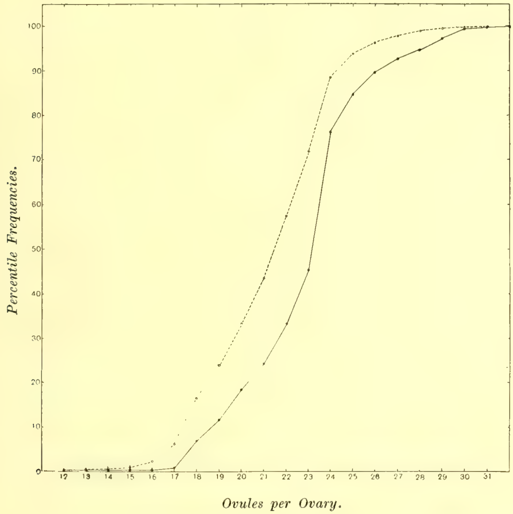

DIAGRAM 3. Percentile curve showing frequency of different numbers of ovules per ovary in the eliminated series and the series developing to maturity. Dots and broken line = Eliminated series; dots and firm line = Matured series.

In comparing the means of the eliminated and uneliminated series, it was only necessary to determine the values for either ovules per locule or ovules per fruit, since one is necessarily three times the other. For the variabilities, however, it seems best to examine the standard deviation for both ovules per locule and total ovules per ovary, since one cannot readily be got from the other.

We are dealing with integral variates, and hence Sheppard's modification for the second moment was not applied.

Standard Deviations.

First, let us deal with absolute variabilities. The standard deviations for the number of ovules per locule in the three collections are given in Tables I—III, and are compared in Table VIII.

TABLE VIII.

Comparison of Standard Deviations of Number of Ovules per Locule in the three Series.

Number of Shrub	B—A		C—B		C—A	
	Difference and Probable Error	$\frac{\text{Diff.}}{E\ \text{diff.}}$	Difference and Probable Error	$\frac{\text{Diff.}}{E\ \text{diff.}}$	Difference and Probable Error	$\frac{\text{Diff.}}{E\ \text{diff.}}$
11	·098 ± ·024	4·04	·088 ± ·028	3·14	·186 ± ·026	7·09
12	·069 ± ·037	1·87	− ·196 ± ·035	5·61	− ·127 ± ·033	3·79
13	− ·232 ± ·037	6·19	·291 ± ·030	9·54	·059 ± ·042	1·40
14	·159 ± ·046	3·46	− ·166 ± ·043	3·83	− ·007 ± ·038	·17
15	− ·156 ± ·032	1·75	·049 ± ·032	1·56	− ·006 ± ·033	·20
16	·073 ± ·041	1·78	− ·177 ± ·038	4·63	− ·104 ± ·037	2·84
17	− ·061 ± ·034	1·78	·204 ± ·037	5·50	·143 ± ·038	3·73
18	·197 ± ·037	5·31	− ·152 ± ·032	4·68	·046 ± ·035	1·32
19	·057 ± ·041	1·50	− ·304 ± ·036	8·45	− ·247 ± ·033	7·44
20	− ·110 ± ·041	2·67	− ·205 ± ·033	6·22	− ·315 ± ·037	8·50
21	− ·185 ± ·036	5·18	·051 ± ·025	2·02	− ·134 ± ·036	3·76
22	− ·092 ± ·035	2·62	− ·129 ± ·026	4·97	− ·221 ± ·031	7·22
23	− ·097 ± ·042	2·28	·056 ± ·039	1·42	− ·041 ± ·042	·97
24	·059 ± ·045	1·31	− ·234 ± ·038	6·20	− ·175 ± ·037	4·71
25	− ·065 ± ·054	1·21	·800 ± ·064	12·51	·734 ± ·070	10·55
26	·039 ± ·047	·81	− ·214 ± ·040	5·31	− ·175 ± ·039	4·49
27	·065 ± ·042	1·53	− ·272 ± ·035	7·74	− ·207 ± ·036	5·75
29	·092 ± ·032	2·86	− ·072 ± ·033	2·20	·020 ± ·031	·66
30	·101 ± ·030	3·32	− ·094 ± ·029	3·24	·007 ± ·028	·25
31	·050 ± ·031	1·57	− ·083 ± ·031	2·69	− ·033 ± ·030	1·11
32	·015 ± ·032	·47	− ·074 ± ·027	2·72	− ·059 ± ·031	1·93
33	·008 ± ·036	·23	− ·081 ± ·034	2·42	− ·073 ± ·033	2·23
34	·198 ± ·044	4·67	− ·296 ± ·036	8·13	− ·097 ± ·038	2·54
35	− ·184 ± ·025	7·23	·021 ± ·022	·96	− ·163 ± ·026	6·33
37	− ·246 ± ·046	5·37	− ·025 ± ·040	·62	− ·271 ± ·045	5·98
38	·020 ± ·026	·78	·240 ± ·031	7·74	·260 ± ·031	8·47
39	·031 ± ·035	·90	·044 ± ·028	1·56	·075 ± ·036	2·12
40	− ·010 ± ·039	·25	− ·154 ± ·031	4·95	− ·163 ± ·036	4·53

The differences are taken in the same way as for the average number of ovules per locule, i.e. B—A, C—B, and C—A. If selection tends to reduce variability the negative sign should predominate in the differences; if selection tends to increase variability the differences with the positive sign should be the more frequent.

To render the comparisons more critical, the probable errors of the standard deviations and the probable errors of their differences have been tabled. The ratio of the difference to its probable error is also given in the same manner as for the means.

Studying the tables first for the prevalence of positive and negative signs we find the results:

B—A, 11 negative and 17 positive differences,
C—B, 18 negative and 10 positive differences,
C—A, 19 negative and 9 positive differences.

Taking up next the significance of the differences for the constants for individuals as measured by their ratio to their probable error, we find results as follows.

For the comparison B—A only 12 of the 28 differences are significant as compared with 18 out of 28 in the case of the means. Of these 6 are positive and 6 negative in sign. The difference C—B shows 20 significant constants, of which 5 are positive and 15 negative. C—A shows 17 significant constants, of which 4 are positive and 13 negative. The first comparison indicates that selective elimination has no influence upon variability. The second and third both suggest that the variability is sensibly less after elimination has taken place.

The differences in the variabilities for the lumped collections may now be examined in Table IX.

TABLE IX.

Comparison	Difference and Probable Error	$\frac{\text{Diff.}}{E \text{ diff.}}$
B—A	$+ \cdot 0756 \pm \cdot 0091$	8·31
C—B	$- \cdot 0938 \pm \cdot 0086$	10·91
C—A	$- \cdot 0182 \pm \cdot 0081$	2·25

At first these results puzzled me very greatly; and the following interpretation is offered only tentatively and as a working hypothesis for further investigation.

The collection B probably comes the nearest to representing the original population of ovaries of any that we have, since it still contains a high percentage of ovaries which will be eliminated later*.

Now it is quite clear that if the ovaries with the *lower* numbers of ovules per locule are eliminated, their variability will be expected to be less than that of the original population. Likewise if the ovaries with the *higher* numbers of ovaries develop to maturity it is clear that *their* variability also should be less than that of the population as a whole. In short, both the samples of eliminated ovaries and the samples of matured fruits represent the more extreme portions of the curve of variation. If this be true, we should expect to find the variability of B greater than that of A or C, while an inevitable consequence of this condition of affairs would be that the difference in the variability of C and A would be less than that of either of the other two differences.

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* I am sorry not to be able to give definite quantitative data bearing upon this point, but this is quite out of the question in a ligneous plant like *Staphylea*, where the number of flowers produced is immensely large and where it is impossible to determine with certainty from the mature inflorescence the number of flowers originally produced.

This is precisely the condition which we find; for the total material the differences B—A and C—B are in opposite senses and clearly significant with regard to the probable errors of their differences, while the difference between C and A cannot safely be regarded as significant.

Returning now to the results for the individual shrubs, we note that so far as the signs give evidence it is that A and C are less variable than B, just as our hypothesis demands. For the differences for individuals which are significant with regard to their probable errors, there are 6 (A < B) and 6 (A > B). For C—B there are 5 (C > B) and 15 (C < B). Finally C—A gives 4 (C > A) and 13 (C < A). These results contain only a partial substantiation for our hypothesis, but no evidence directly against it.

Only further research will show whether these conditions are generally found.

Take next the results for total ovules per ovary. The comparison is made in Table X.

TABLE X. *Comparison of Standard Deviations of total Ovules per Ovary in the three Series.*

Number of Shrub	B—A		C—B		C—A	
	Difference and Probable Error	$\frac{\text{Diff.}}{E \text{ diff.}}$	Difference and Probable Error	$\frac{\text{Diff.}}{E \text{ diff.}}$	Difference and Probable Error	$\frac{\text{Diff.}}{E \text{ diff.}}$
11	·156 ± ·082	1·89	·504 ± ·106	4·76	·660 ± ·101	6·51
12	·367 ± ·099	3·71	− ·770 ± ·080	9·65	− ·040 ± ·126	·32
13	− ·733 ± ·135	5·43	·782 ± ·104	7·49	·005 ± ·153	·03
14	·473 ± ·197	2·40	− ·626 ± ·182	3·43	− ·015 ± ·156	·10
15	− ·250 ± ·109	2·30	·493 ± ·118	4·18	·243 ± ·125	1·94
16	·280 ± ·175	1·60	− ·480 ± ·165	2·90	− ·020 ± ·155	·13
17	− ·035 ± ·133	·26	·876 ± ·153	5·72	·053 ± ·163	·32
18	·754 ± ·141	5·36	− ·308 ± ·136	2·26	·045 ± ·130	·34
19	·250 ± ·157	1·59	− ·617 ± ·143	4·31	− ·037 ± ·127	·29
20	− ·043 ± ·152	·28	− ·511 ± ·126	4·05	− ·055 ± ·133	·41
21	− ·409 ± ·139	2·94	·178 ± ·102	1·74	− ·231 ± ·140	1·65
22	− ·305 ± ·124	2·45	− ·100 ± ·192	1·09	− ·041 ± ·114	·35
23	1·650 ± ·271	6·09	− 1·900 ± ·260	7·32	− ·250 ± ·163	1·53
24	·362 ± ·178	2·03	− ·891 ± ·149	5·97	− ·530 ± ·134	3·94
25	− ·811 ± ·231	3·52	− ·587 ± ·151	3·88	− 1·399 ± ·217	6·44
27	·085 ± ·180	·47	− ·500 ± ·152	3·30	− ·041 ± ·147	·28
28	·007 ± ·170	·04	− ·789 ± ·130	6·05	− ·078 ± ·141	·55
29	·243 ± ·127	1·91	− ·244 ± ·127	1·92	− ·000 ± ·119	·00
30	·454 ± ·108	4·20	− ·380 ± ·106	3·58	·074 ± ·090	·83
31	·273 ± ·123	2·22	− ·282 ± ·123	2·29	·008 ± ·114	·07
32	·172 ± ·124	1·39	− ·257 ± ·108	2·38	− ·086 ± ·116	·74
33	− ·050 ± ·138	·36	− ·121 ± ·129	·94	− ·171 ± ·129	1·33
34	·241 ± ·183	1·32	− ·781 ± ·138	5·67	− ·540 ± ·158	3·41
35	− ·467 ± ·188	5·32	·081 ± ·173	1·10	− ·386 ± ·090	4·29
37	− ·812 ± ·194	4·19	− ·331 ± ·154	2·15	− 1·142 ± ·185	6·18
38	− ·078 ± ·098	·80	·932 ± ·130	7·15	·854 ± ·132	6·46
39	− ·041 ± ·130	·31	·112 ± ·100	1·12	·071 ± ·133	·53
40	·033 ± ·151	·22	− ·460 ± ·121	3·81	− ·427 ± ·138	3·82

The frequencies of positive and negative differences, both for all differences and significant differences, for the individuals are shown in Table XI.

TABLE XI.

Comparison	ALL DIFFERENCES		SIGNIFICANT DIFFERENCES	
	Positive	Negative	Positive	Negative
B—A	16	12	5	5
C—B	8	20	4	13
C—A	9	19	2	6

The comparison of the variabilities calculated from the ovaries for all 28 trees is made in Table XII.

TABLE XII.

Comparison	Difference and Probable Error	$\frac{\text{Diff.}}{E\text{ diff.}}$	Percentage Increase or Decrease*
B—A	$+\cdot 2630 \pm \cdot 0410$	6·41	+9·49
C—B	$-\cdot 2235 \pm \cdot 0389$	5·75	−7·37
C—A	$+\cdot 0395 \pm \cdot 0387$	1·02	+1·43

From these tables the same conclusion is to be drawn as from that for the comparison for the variability of the number of ovules per locule, namely that both the eliminated ovaries, A, and the matured fruits, C, show a less variability than the general population from which a portion of the ovaries have been eliminated. These results are very clear in the comparison for the totals of the three series; they are not so evident on the small individual samples with their higher probable errors.

Coefficients of Variation.

In the preceding section it was shown that a selective elimination brings about a different mean number of ovules per locule or per fruit in the eliminated and the matured ovaries. The ovaries which develop to maturity have a higher average number of ovules than those which are eliminated. Possibly, then, our conclusions concerning variability may be open to question because of the fact that we are comparing the variabilities of samples with generally higher means with those with generally lower means. The absolute variability is generally roughly proportional to the mean value of a character.

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* Increase or decrease from the eliminated to the uneliminated or from the less mature to the more mature.

To make our comparisons more comprehensive and to free them in so far as possible from the influence of the actual magnitudes, we may now make use of the relative variabilities as expressed by the coefficients of variation.

The differences between the three series for the coefficients of variation for both ovules per locule and ovules per fruit appear in Tables XIII and XIV*.

TABLE XIII.

Comparison of Coefficients of Variation of Number of Ovules per Locule in the three Series.

Number of Shrub	B—A		C—B		C—A	
	Coefficient of Variation	$\frac{\text{Diff.}}{E\text{ diff.}}$	Coefficient of Variation	$\frac{\text{Diff.}}{E\text{ diff.}}$	Coefficient of Variation	$\frac{\text{Diff.}}{E\text{ diff.}}$
11	1·17 ± ·30	3·90	·85 ± ·34	2·50	2·02 ± ·32	6·31
12	1·60 ± ·54	2·96	−3·71 ± ·49	7·57	−2·11 ± ·46	4·58
13	−3·16 ± ·51	6·20	5·57 ± ·44	12·66	2·41 ± ·50	4·82
14	2·19 ± ·72	3·04	−2·55 ± ·67	3·81	− ·36 ± ·60	·72
15	− ·94 ± ·40	2·35	− ·13 ± ·38	·34	−1·08 ± ·40	2·70
16	1·82 ± ·58	3·14	−4·03 ± ·52	7·75	−2·84 ± ·49	5·80
17	−1·12 ± ·43	2·60	1·62 ± ·45	3·60	·50 ± ·47	1·06
18	3·31 ± ·52	6·36	−2·85 ± ·46	6·19	·45 ± ·46	·98
19	·67 ± ·57	1·17	−4·93 ± ·49	10·06	−4·26 ± ·45	9·47
20	2·52 ± ·61	4·13	−3·59 ± ·46	7·80	−6·11 ± ·55	11·11
21	−2·81 ± ·47	5·98	·34 ± ·32	1·06	−2·46 ± ·46	5·35
22	−1·27 ± ·46	2·76	−1·93 ± ·34	5·68	−3·20 ± ·40	8·00
23	−1·76 ± ·70	2·51	·67 ± ·64	1·05	−1·09 ± ·69	1·58
24	1·02 ± ·65	1·57	−4·02 ± ·54	7·44	−3·00 ± ·52	5·78
25	−2·00 ± ·66	3·03	7·34 ± ·70	10·48	5·33 ± ·80	6·66
26	·31 ± ·69	·45	3·66 ± ·56	6·53	−3·36 ± ·56	6·00
27	·25 ± ·59	·42	−4·42 ± ·47	9·40	−4·17 ± ·50	8·32
29	1·96 ± ·50	3·92	−2·10 ± ·50	4·20	− ·13 ± ·46	·28
30	1·43 ± ·52	2·67	−2·14 ± ·48	4·50	− ·71 ± ·45	1·58
31	1·32 ± ·45	2·93	−2·09 ± ·44	4·52	− ·77 ± ·41	1·88
32	·24 ± ·42	·57	−1·12 ± ·36	3·11	− ·88 ± ·40	2·20
33	·72 ± ·56	1·28	−2·14 ± ·52	4·11	−1·42 ± ·50	2·84
34	2·74 ± ·64	4·22	−4·93 ± ·51	9·67	−2·20 ± ·54	4·07
35	−2·69 ± ·32	8·41	·18 ± ·28	·64	−2·52 ± ·32	7·87
37	−3·56 ± ·55	6·47	·03 ± ·47	·06	−3·53 ± ·55	6·42
38	·69 ± ·44	1·57	1·51 ± ·48	3·14	2·20 ± ·47	4·68
39	·07 ± ·44	·16	·48 ± ·35	1·37	·55 ± ·45	1·22
40	−1·18 ± ·53	2·23	−2·49 ± ·39	6·38	−3·67 ± ·49	7·49

The results may be tersely summarized for the individuals in Table XV. Here the frequencies for positive and negative differences are shown, both for all the differences and for probably significant differences, i.e. for those which are 2·5 or more times their probable error.

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* Naturally the significance of the sign as indicating the tendency of selection to increase or decrease variability is the same for V as for σ.

For the total material we have, for locules,

$$\begin{aligned} B—A &= +\cdot735 \pm \cdot128, \\ C—B &= -\cdot961 \pm \cdot116, \\ C—A &= -\cdot225 \pm \cdot112, \end{aligned}$$

and for total ovules per ovary

$$\begin{aligned} B—A &= +\ \cdot941 \pm \cdot190, \\ C—B &= -1\cdot615 \pm \cdot175, \\ C—A &= -\ \cdot674 \pm \cdot176. \end{aligned}$$

These results are consistent *throughout*, for both ovules per locule and total ovules per fruit. They show that, both for the series of individuals and the three grand totals, the relative as well as the absolute variability is higher in Series B than in Series A or C. The variability of the eliminated series, A, seems to be

TABLE XIV.

Comparison of Coefficients of Variation of Number of Ovules per Ovary in the three Series.

Number of Shrub	B—A		C—B		C—A	
	Coefficient of Variation	$\frac{\text{Diff.}}{E\text{ diff.}}$	Coefficient of Variation	$\frac{\text{Diff.}}{E\text{ diff.}}$	Coefficient of Variation	$\frac{\text{Diff.}}{E\text{ diff.}}$
11	·61 ± ·33	1·85	1·88 ± ·42	4·47	2·50 ± ·40	6·25
12	2·21 ± ·73	3·03	– 4·32 ± ·67	6·44	– 2·11 ± ·58	3·63
13	– 3·26 ± ·59	5·52	4·61 ± ·50	9·22	1·35 ± ·70	1·93
14	2·21 ± ·02	2·17	– 3·19 ± ·93	3·43	– ·98 ± ·81	1·21
15	– 1·20 ± ·45	2·67	1·44 ± ·46	3·13	·24 ± ·50	·48
16	1·44 ± ·83	1·73	– 3·51 ± ·75	4·68	– 2·08 ± ·69	3·02
17	– 1·72 ± ·56	3·07	2·84 ± ·60	4·73	1·12 ± ·66	1·70
18	3·90 ± ·65	6·00	– 2·03 ± ·63	3·22	1·86 ± ·58	3·21
19	1·07 ± ·73	1·46	– 3·42 ± ·65	5·26	– 2·35 ± ·58	4·05
20	– ·86 ± ·74	1·62	– 2·90 ± ·58	5·00	– 3·76 ± ·64	5·87
21	– 2·08 ± ·62	3·35	·52 ± ·43	1·21	– 1·56 ± ·62	2·52
22	– 1·38 ± ·54	2·55	– ·62 ± ·39	1·59	– 2·00 ± ·50	3·33
23	8·82 ± 1·52	16·96	– 10·49 ± ·44	7·28	– 1·66 ± ·89	1·86
24	1·85 ± ·84	2·21	– 4·67 ± ·70	6·67	– 2·83 ± ·62	4·56
25	4·16 ± ·94	4·42	– 2·74 ± ·55	4·98	– 6·90 ± ·88	7·84
26	·23 ± ·86	·27	– 2·81 ± ·71	3·96	– 2·59 ± ·69	3·80
27	– ·46 ± ·79	·58	– 4·04 ± ·58	6·96	– 4·50 ± ·65	6·92
29	1·66 ± ·65	2·55	– 1·98 ± ·65	3·04	– ·33 ± ·58	·57
30	2·36 ± ·61	3·87	– 2·47 ± ·58	4·26	– ·12 ± ·49	·24
31	1·75 ± ·59	2·96	– 2·03 ± ·58	3·48	– ·27 ± ·52	·52
32	·78 ± ·54	1·44	– 1·23 ± ·47	2·62	– ·45 ± ·50	·90
33	·18 ± ·71	2·53	– 1·28 ± ·67	1·91	– 1·10 ± ·64	1·72
34	1·08 ± ·87	1·25	– 4·20 ± ·64	6·56	– 3·12 ± ·74	4·22
35	– 2·22 ± ·39	5·69	·29 ± ·32	·91	– 1·94 ± ·39	4·97
37	– 3·72 ± ·77	4·83	– 1·04 ± ·60	1·73	– 4·76 ± ·75	6·35
38	– ·18 ± ·56	·32	3·09 ± ·66	4·53	2·91 ± ·66	4·41
39	– ·39 ± ·55	·71	·41 ± ·41	1·00	·02 ± ·56	·04
40	– ·64 ± ·69	·93	– 2·34 ± ·51	4·39	– 2·98 ± ·63	4·73

somewhat higher than that of the matured series, C, but the difference in the relative variability may be in part due to the fact that the mean of A is low as compared to that of C.

TABLE XV.

Series Compared	Differences for Ovules per Locule		Differences for Total Ovules per Ovary	
	Positive Differences	Negative Differences	Positive Differences	Negative Differences
All Differences				
B—A	18	10	16	12
C—B	11	17	8	20
C—A	7	21	7	21
Significant Differences				
B—A	10	8	8	7
C—B	6	16	5	17
C—A	5	14	2	15

The results for variability are not so conclusive as those for type, but I think we may safely conclude:

(*a*) That both the eliminated ovaries and the matured fruits have a lower absolute and relative variability than the original population of ovaries.

(*b*) This is due to the ovaries with the lower number of ovules failing to develop, and to those with the higher numbers having the better opportunity to reach maturity.

Problem 3. Is there a difference in the radial asymmetry of the eliminated ovaries and those which develop to maturity?

From the nature of the thin-walled, inflated "bladder" fruit we could not expect it to be other than somewhat irregular in form. The externally visible irregularities in the evaginations of the three locules are of such a nature that no biologist would describe the fruit as asymmetrical. Externally there is no conveniently measurable character from which the amount of irregularity might be measured. The only character which can be determined with certainty is the number of seeds developing or the number of ovules formed per locule.

Opening the three locules of a fruit quite at random—there being no external character to indicate any differentiation whatever between them—we may find such numbers of ovules per locule as:

11—11—11,
10—11—10,
8— 9—11,
9— 7—11,
11— 7—6.

The first of these is clearly enough a radially symmetrical fruit with respect to number of ovules per locule. Any fruit in which the number of ovules is the same in each locule, as 6—6—6, or 7—7—7, is radially symmetrical with respect to this character. Those in which the number of ovules differs from locule to locule are quite as obviously irregular or radially asymmetrical with respect to number of ovules.

Admitting that fruits in which all the locules produce the same number of ovules are radially symmetrical, while those with at least one of the locules differing from the others in its number of ovules are radially asymmetrical, we may be either (*a*) content to divide our ovaries into two classes, radially symmetrical and radially asymmetrical, or (*b*) get some measure of the amount of asymmetry in individual fruits so that the asymmetrical fruits may be subdivided for further analysis.

The measure of asymmetry must be one for the individual fruit, not for a population. The measure must also be independent of the order in which the three locules of the fruit are taken, for there is no constant differentiation between them and they may be opened in any order. Two measures have occurred to me.

First, the sum of the positive difference between the number of ovules in the locules of an ovary might be used. In a fruit of the formula

$$\begin{matrix} (a) & (b) & (c) \\ 7— & 8— & 6, \end{matrix}$$

we have, taking all possible differences,

$$\begin{aligned} a-b&=-1, & b-c&=+2,\\ a-c&=+1, & c-a&=-1,\\ b-a&=+1, & c-b&=-2, \end{aligned}$$

Sum of positive differences $= 4$.

Comparing a fruit of the formula 7—8—7 we find the sum of the positive differences = 2. The first is more asymmetrical than the second. This is obvious in the present case from mere inspection.

The second measure is the square root of the mean square deviation of the number of ovules per locule from the mean number in the whole fruit. For the first illustration the mean number per locule is 7 and the deviations are

$$\begin{aligned} a&=0, & a^2&=0, \qquad \text{Coefficient of asymmetry } = \sqrt{\tfrac{2}{3}} = \cdot 8165.\\ b&=+1, & b^2&=1,\\ c&=-1, & c^2&=1. \end{aligned}$$

For the second illustration: $A = 7\cdot 3333$, the deviations are

$$a = -\cdot 3333, \quad b = +\cdot 6666, \quad c = -\cdot 3333,$$

and the coefficient of asymmetry is

$$\sqrt{\frac{\cdot 3333^2 + \cdot 6666^2 + \cdot 3333^2}{3}} = \cdot 4714.$$

This measure is, in short, the standard deviation of the locules of the fruit around their own mean. According to this measure the asymmetry of a fruit with the formula 7—8—7 bears the ratio to one of the formula 7—8—6 of ·471 : ·817 instead of 2 : 4 as indicated by the interlocular difference method.

Since the locules of a fruit are undifferentiated it does not matter how we arrange them for convenience of treatment. I have always copied the results of original countings off in a descending series. The two constants for a few illustrative ovaries are:

Formula		Interlocular Difference		Coefficient of Asymmetry
11—11—11	=	0	=	·0000
11—10—10	=	2	=	·4714
11—10—9	=	4	=	·8165
11—9—9	=	4	=	·9428
11—9—8	=	6	=	1·2472
11—8—8	=	6	=	1·4142
11—9—7	=	8	=	1·6330
11—8—7	=	8	=	1·6997
10—6—6	=	10	=	1·8856
10—7—5	=	10	=	2·0548
11—7—6	=	10	=	2·1602

These illustrations show that while the interlocular difference does not distinguish between the amount of irregularity of certain fruits, the coefficient of asymmetry, as I have called the standard deviation of the locules of a fruit around their own mean, does. By mere inspection I am quite unable to decide whether a fruit of the formula 10—6—6 is more irregular, or radially asymmetrical, than one of the formula 10—7—5. Both have the same mean number of ovules per locule and the interlocular difference is the same for both, but the coefficient of asymmetry is slightly higher for the formula 10—7—5.

I have adopted the coefficient of asymmetry in this paper for the following reasons.

(*a*) It is merely the standard deviation—so universally employed in modern statistical work—of the number of ovules per locule in an individual fruit.

(*b*) It differentiates—whether with quantitative accuracy or not—between degrees of asymmetry not distinguished by the interlocular difference method.

(*c*) From data tabled in coefficient of asymmetry classes the interlocular differences may be obtained by the use of a table. The converse is not true.

There is one patent objection to the use of the coefficient of asymmetry as defined here: it is not independent of number of seeds per locule. There are reasons for regarding fruits of the formula 10—6—8 and 5—3—4 as equally asymmetrical, but according to our coefficient their asymmetries bear to each other the ratio of the square roots of 8/3 and 2/3.

The simplest method of freeing our constant from the influence of the absolute number of ovules would seem to be to take the ratio of the coefficient of asymmetry to the mean number of ovules per locule. The reason that I have not done this is that I have used the coefficient of asymmetry in a large series of investigations on the relationship between asymmetry and fecundity in this and other species*. In them I could not well use the relative coefficient because of the danger of introducing spurious correlation in determining the interdependence of asymmetry and fecundity. It seems desirable, for reasons which will be apparent when my data for fecundity are ready to publish, to use the same method of describing the irregularity of the ovaries in both studies.

The chief objection to the use of the coefficient of asymmetry will be removed if we show that the magnitude is not greatly dependent upon the number of ovules per locule. This must be done for every species investigated. For the present series I have discussed this point in Section VI (see page 493) where it has been shown that the correlation between the coefficient of asymmetry and the number of ovules per locule is very slight.

Table XVI gives the interlocular difference for all the coefficients of asymmetry found in the present study. Anyone who desires may calculate all the constants dealt with in terms of interlocular differences by the use of this table and the tables of data.

TABLE XVI.

Interlocular Differences for Coefficients of Asymmetry.

Coefficient of Asymmetry	Interlocular Difference
·0000	0
·4714	2
·8165	4
·9428	4
1·2472	6
1·4142	6
1·6330	8
1·6997	8
1·8856	10
2·0548	10
2·1602	10

To determine whether there is a selective elimination depending upon the asymmetry of the fruit, the asymmetry of each individual fruit is determined, and the distribution of the asymmetries of the eliminated and uneliminated ovaries compared. The determination of the asymmetries of individual fruits, a rather laborious process at best, is facilitated by tabling the coefficients for all the different ovule formulae at the outset.

A standard deviation calculated on only three "observations"—the three locules of an ovary—is of little value as a means of predicting the standard deviation of another fruit of the same species because of the large probable error, but as a description of the irregularity of an individual fruit it is perfectly trustworthy to as many places as we care to carry our arithmetic. I believe the four places to which I have tabled are arithmetically correct. For an individual fruit or very small sample this refinement is quite without significance, but where several hundreds of fruits fall in the same class, as is the case in this work, and especially in large series of data on other problems concerning asymmetry to be published later, the final decimal places may be of significance.

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* I hope that some portions of these investigations on fecundity will be ready for publication in a few months.

TABLE XVII.

Mean Asymmetry for Individuals.

Shrub	A	B	C
11	·305	·373	·325
12	·488	·456	·429
13	·479	·346	·536
14	·325	·400	·389
15	·478	·470	·424
16	·457	·470	·328
17	·462	·492	·441
18	·397	·435	·270
19	·539	·538	·296
20	·644	·494	·374
21	·401	·267	·339
22	·419	·387	·185
23	·426	·359	·380
24	·491	·454	·407
25	·487	·676	·340
26	·500	·470	·359
27	·459	·514	·332
29	·422	·430	·466
30	·397	·366	·402
31	·475	·436	·400
32	·372	·360	·287
33	·435	·491	·417
34	·599	·549	·362
35	·421	·326	·275
37	·589	·534	·626
38	·285	·361	·339
39	·361	·403	·425
40	·496	·463	·380

TABLE XVIII.

Comparison of Asymmetry.

Shrub	B—A	C—B	C—A
11	·068	− ·048	·020
12	− ·032	− ·027	− ·059
13	− ·133	·190	·057
14	·075	− ·011	·064
15	− ·008	− ·045	− ·053
16	·012	− ·142	− ·129
17	·030	− ·051	− ·021
18	·038	− ·165	− ·127
19	− ·001	− ·242	− ·243
20	− ·150	− ·120	− ·270
21	− ·134	− ·028	− ·162
22	− ·032	− ·202	− ·235
23	− ·067	·021	− ·046
24	− ·036	− ·047	− ·083
25	·188	− ·335	− ·147
26	− ·030	− ·111	− ·141
27	·056	− ·182	− ·126
29	·008	·036	·043
30	− ·031	·037	·005
31	− ·039	− ·036	− ·075
32	− ·012	− ·073	− ·085
33	·056	− ·074	− ·018
34	− ·051	− ·187	− ·238
35	− ·095	− ·051	− ·146
37	·054	·091	− ·037
38	·077	− ·023	·054
39	·042	·021	·063
40	− ·033	− ·083	− ·116

The frequencies of the different coefficients of radial asymmetry for each of the individual trees are given for the three collections in Tables XXXIV—XXXVI.

The comparison may first be made on the basis of the mean asymmetry of the eliminated and the developing ovaries.

Table XVII gives the mean asymmetry for the three collections from the 28 trees. The excessive labour involved in calculating the probable errors for these 84 means, where the standard deviations must be calculated for classes so irregular as our asymmetry grades, has led me to omit the probable errors for the individuals. The results for Series A and C are shown graphically in Diagram 4.

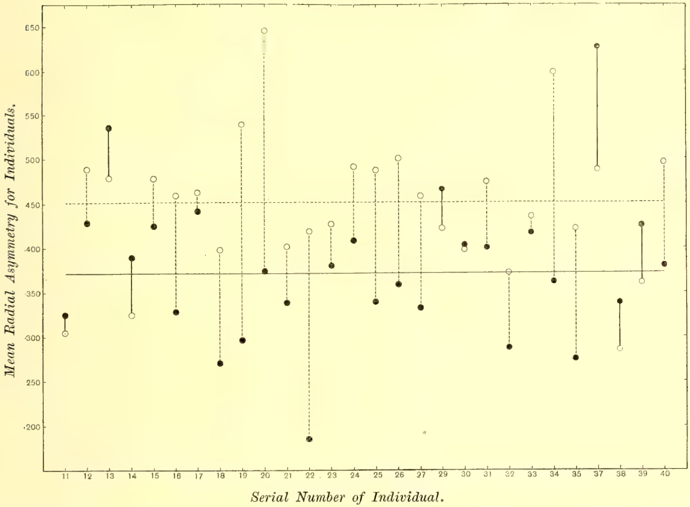

DIAGRAM 4. Showing mean difference in asymmetry of eliminated and matured ovaries for the 28 individuals and for the population. Circles = mean asymmetry of eliminated ovaries for individuals; solid dots = mean asymmetry for matured ovaries for individuals. Broken transverse line = mean asymmetry of all eliminated ovaries; solid transverse line = mean asymmetry of all matured ovaries.

The usual comparisons (B—A, C—B, C—A) are given in Table XVIII. The differences are taken in such a way that if selection tends to eliminate the less symmetrical pods the differences will have the negative sign. For the differences we find:

B—A, 12 positive and 16 negative,
C—B, 6 positive and 22 negative,
C—A, 7 positive and 21 negative.

The mean difference per individual is:

$$B—A = -\cdot 0064,$$
$$C—B = -\cdot 0675,$$
$$C—A = -\cdot 0804.$$

For the grand totals the results are:

Mean asymmetry, Lot A = ·4515 ± ·0051,
Mean asymmetry, Lot B = ·4415 ± ·0048,
Mean asymmetry, Lot C = ·3724 ± ·0045,
B—A = – ·0100 ± ·0070,
C—B = – ·0691 ± ·0066,
C—A = – ·0791 ± ·0068.

The first difference is probably not significant; the second and third are over ten times their probable errors.

The difference in the distributions of asymmetry for the eliminated and the matured series is rather strikingly shown when the frequencies for the two series are reduced to percentages and plotted in a polygon. In Diagram 5 the relatively few asymmetries of 1·2472 and over have been graphed together.

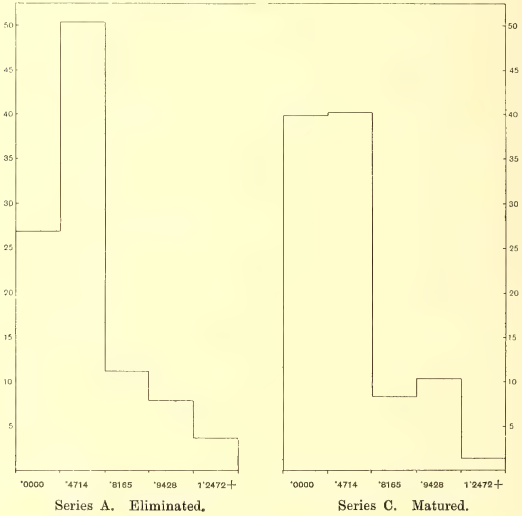

DIAGRAM 5. Percentage frequencies of chief asymmetry classes in the eliminated and matured series.

Whatever way we arrange our data the conclusions flowing from them are the same—that the radial asymmetry of the ovaries which develop to maturity is less than that of those which are eliminated.

Problem 4. Is there a difference in the proportional number of locules with "odd" numbers of ovules in the eliminated ovaries and those which develop to maturity?

Locules with 4, 6, 8, 10 or 12 ovules may be conveniently designated as "even" while those with 5, 7, 9 or 11 ovules may be described as "odd."

With respect to this character the fruits fall into four classes:

3 "even,"
2 "even" + 1 "odd,"
1 "even" + 2 "odd,"
3 "odd."

To the reader it may at first seem to be mere juggling with numbers to seek to determine whether there is any selective elimination of fruits depending upon the number of "odd" or "even" locules per fruit. Nevertheless purely embryological considerations were the incentive to the work.

I have not been able to find any account of the development of the ovary of *Staphylea*, but from the appearance of the matured organ and from our knowledge of the embryology of other forms it would seem quite likely that the ovules of a locule belong to the two margins of the same carpel. If this is the case, locules with "odd" numbers of ovules must have a different number produced on the two margins, while those with an even number almost as surely have the same number on the two carpellary margins. If this be the case no one will deny that structurally "even" locules are more perfectly formed than the "odd," being bilaterally symmetrical with respect to the number of ovules found on the two margins.

The morphology of the fruit can be thoroughly understood only after an embryological investigation, but in the meantime the sensible thing to do seems to be to investigate the question of selective elimination with respect to this point on the basis of the material in hand.

A reason for investigating this point lying within our material itself is the different frequency of fruits with "even" and "odd" locules. For the totals of the three collections the number of fruits belonging to each class is seen in our Table XIX. The predominance of locules with an even number of ovules suggests that this type may represent a point of morphological stability*. If such be the case the determination of whether there is a tendency towards the elimination of those with the larger number of "odd" locules is obviously important.

The test may be made in two ways: (*a*) by the comparison of the percentage of "odd" locules in the three series, and (*b*) by calculating the percentage of each of the four different ovule formulae in the three collections.

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* Compare Sir Francis Galton's suggestion concerning points of organic stability in natural inheritance.

The percentage of total locules with "odd" numbers of ovules in the three series is:

Percentage "odd" in Lot A = 37·407 ± ·412,
Percentage "odd" in Lot B = 36·065 ± ·377,
Percentage "odd" in Lot C = 25·185 ± ·325.

The probable errors for these frequencies were calculated from the formula

$$Ey_s = \cdot 67449 \sqrt{y_s \times \left(1 - \frac{y_s}{m}\right)},$$

where y_s is the frequency in any one of the four classes, and m is the total population. From the probable error for the absolute frequencies so obtained the probable errors for the percentage frequencies were got by taking the ratio Ey_s/m.

Comparing the series among themselves as usual we get:

B—A = − 1·342 ± ·558 °/₀,
C—B = − 10·880 ± ·498 °/₀,
C—A = − 12·222 ± ·524 °/₀.

In all these cases there is an actual decrease in the relative number of "odd" locules as elimination progresses. The decrease of 1·3 per cent. noted from A to B is perhaps not significant, being only about twice its probable error, but the fall of 11 per cent. from B to C is about 22 times its probable error and the 12 per cent. decrease from A to C is over 23 times its probable error.

There can be no reasonable doubt, therefore, that "odd" are more likely to be eliminated than "even" locules. Naturally one locule cannot be eliminated alone, but it has seemed interesting to work through the results for individual locules. We turn now to the more difficult, but more significant, comparison of individual formulae.

The frequencies, absolute and relative, appear in Table XIX. Diagram 6 shows

TABLE XIX. *Frequency of Ovaries of Different Locular Composition in the three Series.*

Formula	A		B		C	
	f	°/₀	*f*	°/₀	*f*	°/₀
3 "even" ...	595	28·401	741	30·061	1265	46·783
2 "even," 1 "odd"	788	37·613	936	37·970	928	34·320
1 "even," 2 "odd"	573	27·351	633	25·680	418	15·458
3 "odd"	139	6·635	155	6·288	93	3·439
Totals	2095	100·000	2465	99·999	2704	100·000
Even Locules ...	3934	62·593	4728	63·935	6069	74·815
Odd Locules ...	2351	37·407	2667	36·065	2043	25·185

the results graphically. The three comparisons are made on the basis of the percentage frequencies in Table XX.

TABLE XX.

Formula	B—A	C—B	C—A
3 "even" ...	+1·660	+16·722	+18·382
2 "even," 1 "odd"	+ ·357	− 3·650	− 3·293
1 "even," 2 "odd"	−1·671	−10·222	−11·893
3 "odd"	− ·347	− 2·849	− 3·196

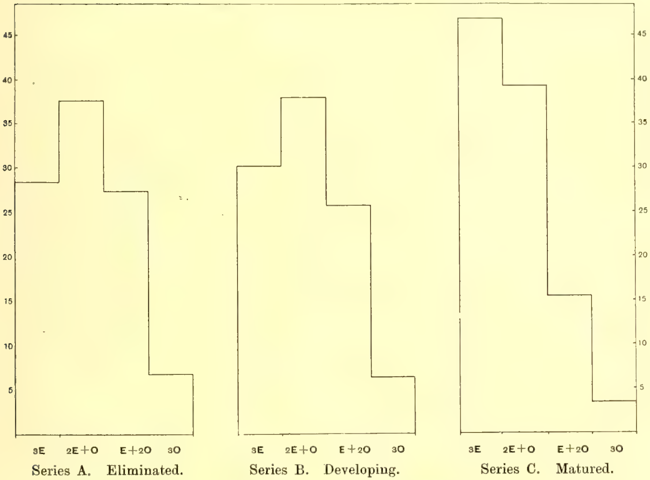

Diagram 6. Percentage frequencies of four types of locular composition of ovaries in the three series.

In both of the critical comparisons it appears that elimination brings about an increase in the per cent. of ovaries with all three locules "even" and a decrease in the per cent. of those which contain one or more locules with an odd number of ovules. The differences are also conspicuously large, amounting to 18·4 per cent. in the case of the "all even" class.

In view of the results secured for the individual locules, it has not seemed necessary to calculate the probable errors of their differences. A better test would seem to be that for the probability against a given system of deviations from theory, in the case of a correlated system of variables having arisen through the errors of random sampling[*]. In making this test we calculate the frequencies to be expected in the classes of a series n if they occurred in the same proportionate frequency as in the series m, and determine the chances against this being due merely to the errors of random sampling from Elderton's Tables[†].

Only the critical comparison C—A need be tested. Table XXI gives the data. χ^2 equals 510·8, a value over seven times as large as Elderton has thought it worth while to table. The chances are thus many millions to one against such a divergence as this occurring by the accidents of random sampling.

TABLE XXI.

Formula	Observed Frequency	Calculated Frequency	$m_r - m_r'$	$\frac{(m_r - m_r')^2}{m_r}$
3 "even" ...	1265	768	-497	321·6
2 "even," 1 "odd"	928	1017	$+89$	7·8
1 "even," 2 "odd"	418	740	$+322$	140·1
3 "odd"	93	179	$+86$	41·3
Totals ...	2704	2704	0	510·8

Why locules with an odd number of ovules should be inferior to those with an even number, or fruits composed in part or altogether of "odd" locules should be inferior to those composed chiefly or altogether of "even" locules, our data do not permit us to determine. There can be no reasonable doubt of the fact of the relationship for our material.

Problem 5. Is there a difference in the percentage of 2- and 4-merous ovaries among the eliminated ovaries and those which develop to maturity?

The answer to this question cannot be expected to be very conclusive because of the rarity of the anomalies in the series under consideration. The available data are given in Table XXXVII. The reason for the frequencies for 3-merous fruits being higher in Series C in this Table than in those given before is that, to determine more accurately the percentage of anomalous fruits, larger samples were examined for this point without counting the numbers of ovules.

For present purposes using only the grand totals, we find the frequencies for 2-loculed fruits:

Series A = 2·477 per cent.
Series B = 2·321 per cent.
Series C = 1·488 per cent.
B—A = − ·156 per cent.
C—B = − ·833 per cent.
C—A = − ·989 per cent.

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[*] Pearson, K.: *Phil. Mag.* Vol. L. pp. 157—175, 1900.

[†] Elderton, W. P.: *Biometrika*, Vol. I. pp. 155—163, 1901.

For 4-merous fruits:

Series A = 1·376 per cent.
Series B = 2·243 per cent.
Series C = 2·461 per cent.
B—A = + ·867 per cent.
C—B = + ·218 per cent.
C—A = + 1·085 per cent.

No probable errors have been calculated for the reason that our percentages are so low that the familiar theory of probable errors cannot be applied.

It is quite impossible to say, therefore, whether or no these differences are really significant, but so far as our data go they indicate that the 2-merous ovaries are more likely to be eliminated than the 3-merous, while the 4-merous ones are less likely to be eliminated.

V. Discussion of Problems on the Basis of Actual Data for 1906.

Problem 1. The seriations of the number of ovules per locule are found in Table XXXVIII; those for total ovules per fruit in Table XXXIX.

For constants we get:

Series A (6—10 mm.), Mean = 7·232 ± ·029.
Series B (11—15 mm.), Mean = 7·660 ± ·014.
Series C (16—20 mm.), Mean = 7·821 ± ·016.
B—A = + ·428 ± ·032.
C—B = + ·161 ± ·021.
C—A = + ·589 ± ·033.

The increase from A to B is 13·4 times its probable error, that from B to C is 8·6 times its probable error and the difference between the mean of the smallest and the largest fruits is 17·8 times its probable error. The increase from the smallest to the largest fruits is 8·14 per cent. of the value for the former.

These results certainly substantiate in the most unequivocal way those secured for means from the 1908 series, and are the more interesting from the fact that the material was collected not merely for a different purpose and with no idea of testing selective elimination, but by a quite different method.

Problem 2. The absolute and relative variabilities for both ovules per locule and total ovules per ovary may again be considered. The complete answer to this problem is given in Table XXII.

As the fruits become larger the variability becomes less. For the absolute variabilities the probable errors show that there is a very high probability that every difference is significant. Probable errors have not been calculated for the coefficients of variation, but the differences are all relatively large and consistent.

TABLE XXII.

Comparison of Absolute and Relative Variabilities for 1906.

Material	Standard Deviation and Probable Error	Coefficient of Variation
Ovules per Locule		
Series A ...	·968 ± ·020	13·382
Series B ...	·774 ± ·010	10·104
Series C ...	·685 ± ·011	8·762
B—A ...	− ·194 ± ·022	− 3·278
C—B ...	− ·089 ± ·015	− 1·342
C—A ...	− ·283 ± ·023	− 4·620
Ovules per Ovary		
Series A ...	2·497 ± ·090	11·528
Series B ...	1·895 ± ·042	8·247
Series C ...	1·594 ± ·046	6·795
B—A ...	− ·602 ± ·099	− 3·281
C—B ...	− ·301 ± ·062	− 1·272
C—A ...	− ·903 ± ·101	− 4·553

Problem 3. The frequency of the different asymmetry classes in the three series is given in Table XL. The three differences of the relative frequencies are given in Table XXIII.

TABLE XXIII.

	B—A	C—B	C—A
·0000	+3·751	+8·495	12·246
·4714	+3·064	−9·790	− 6·726
·8165	−4·877	+2·067	− 2·810
·9428	−1·790	−1·077	− 2·867
1·2472	—	—	—

The results show that as the size of the fruit increases the symmetrical fruits form a higher percentage of the entire population, while the asymmetrical fruits of the various grades become relatively fewer. Our numbers are small and some irregularity is to be expected.

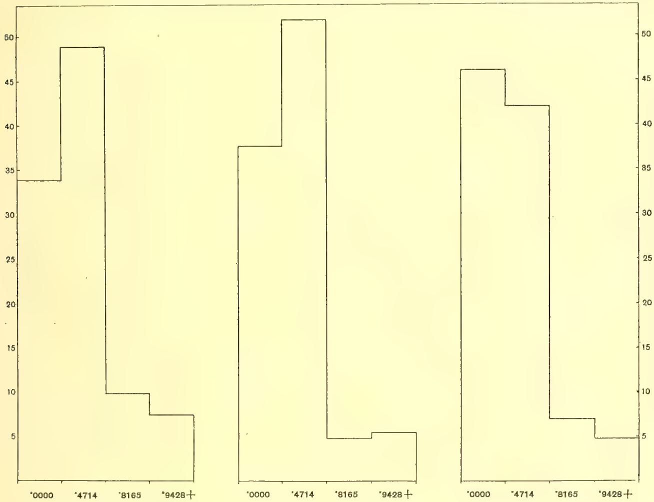
The average asymmetries are:

A, 6—10 mm. long, Mean = ·3822 ± ·0160.
B, 11—15 mm. long, Mean = ·3381 ± ·0091.
C, 16—20 mm. long, Mean = ·3025 ± ·0126.

B—A = − ·044 ± ·018.
C—B = − ·036 ± ·016.
C—A = − ·080 ± ·020.

There is clearly a decrease in asymmetry from A to B and again from B to C. Owing to the smallness of our series the probable errors are high, but all the differences are over twice their probable errors, and the critical difference C—A is four times its probable error.

Diagram 7 shows graphically the frequency of the four chief asymmetry classes.



Series A. Youngest. Series B. Intermediate. Series C. Oldest.

Diagram 7. Percentage frequencies of chief asymmetry classes in the three series of developing fruits for 1906.

Problem 4. The frequency of the four types of ovaries with respect to the number of "odd" locules produced is shown for the three lengths of fruit in Table XXIV. and graphically (with percentage frequencies) in Diagram 8.

TABLE XXIV. *Frequency of Ovaries of Different Locular Composition in the three Series.*

Formula	6—10 mm. = A		11—15 mm. = B		16—20 mm. = C	
	f	°/o	*f*	°/o	*f*	°/o
3 "even" ...	58	33·333	169	35·957	125	45·787
2 "even," 1 "odd"	68	39·080	162	34·468	85	31·136
1 "even," 2 "odd"	35	20·115	107	22·766	51	18·681
3 "odd"	13	7·471	32	6·809	12	3·396

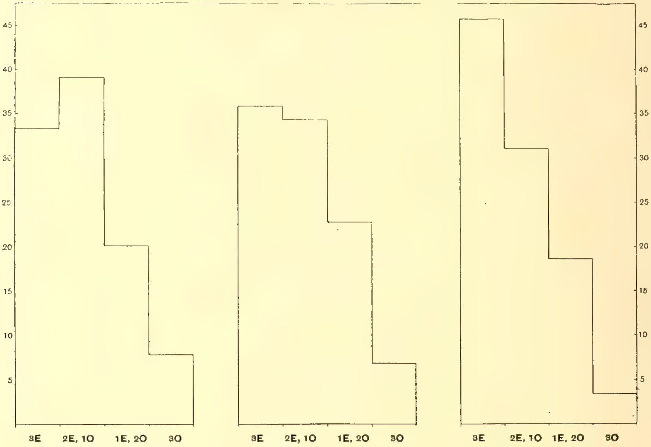
The reader will note, by comparing the percentage frequencies for the smallest and the largest fruits, that the 3 "even" class is much more abundantly represented in the largest than in the smallest fruits. Those with one or more "odd" locules are necessarily less abundant.

The percentage frequency of "odd" locules in the three series is:

Lot A, 6—10 mm. $= 33{\cdot}91 \pm 1{\cdot}40\ {}^\circ/_\circ$.
Lot B, 11—15 mm. $= 33{\cdot}48 \pm {\cdot}85\ {}^\circ/_\circ$.
Lot C, 16—20 mm. $= 27{\cdot}23 \pm 1{\cdot}05\ {}^\circ/_\circ$.
The difference, C—A $= -6{\cdot}68 \pm 1{\cdot}75\ {}^\circ/_\circ$.

In predicting the frequencies of the four classes in Series C from the distribution found in Series A, to ascertain whether the former could possibly be regarded as arising from the latter through the errors of random sampling, we find

$$\chi^2 = 20{\cdot}7,\quad P = {\cdot}00012.$$



Series A. Youngest. Series B. Intermediate. Series C. Oldest.

Diagram 8. Percentage frequencies of four types of locular composition of ovaries in the three series of developing fruits for 1906.

Thus deviations of observation from theory so great as those found here would be expected to arise through the errors of random sampling only about 12 times out of 100,000.

Allowing full weight to the fact that the frequencies in series A have their probable errors, I think we can maintain that series C and series A are differentiated from each other with respect to the number of "odd" and "even" locules in their ovaries.

Problem 5. This problem cannot be discussed, since no data were collected for other than 3-merous fruits.

VI. INTERRELATIONSHIP OF CHARACTERS CONSIDERED.

For convenience and clearness our characters have been treated in the foregoing discussion of problems as though they were quite independent.

Analysis can be carried somewhat further. If a character is quite uncorrelated with others, we shall be confident in regarding any selective elimination which we find associated with this character as arising through some unfitness for continued development associated with it. If, on the contrary, it appears that a character is correlated with some other, we cannot know without further evidence which of a pair is potent in producing a selective elimination.

These points will best be made clear by an examination of the three relationships which seem worth consideration for our material.

1. *The Correlation between the Coefficient of Asymmetry and the Number of "odd" Locules per Ovary.*

The fact that both radial asymmetry as measured by the deviation of the number of ovules per locule from their mean and the composition of the ovary with regard to the "even" and "odd" nature of the individual locules seem to be of significance in determining whether it shall develop into a mature fruit, does not necessarily prove that with *both* of these characteristics there is associated some functional unfitness for development.

There is necessarily *some* correlation between the two characteristics in any fruit. If the three locules have all the same number of ovules and they be even, say 8—8—8, the asymmetry will be ·0000. If it be "2 even, 1 odd," say 8—8—7 or 9—8—8, the asymmetry will be ·4714. Thus a fruit composed of "odd" and "even" must necessarily be somewhat asymmetrical while one of "all even" or "all odd" may be perfectly asymmetrical. If an "all even" or "all odd" fruit is radially asymmetrical with respect to number of ovules per locule, it may be more asymmetrical than a fruit of locules of both types, since there must be a difference of two ovules between two different "even" locules, while an "even" and an "odd" may differ by only a single ovule. For instance, 8—8—6 is more asymmetrical than 8—8—7.

As the reader will see, the problems are very complex. Even the relatively large series of data available for this research are, I fear, insufficient for a detailed study of the questions of the interdependence of these two characteristics of the fruit and their fitness. Later I hope to publish extensive data already on hand that may throw some light on these questions. For the present I shall only show the correlation between the coefficient of asymmetry and the number of "odd" locules per fruit. The data for the three series appear in Tables XLI—XLIII, and the constants by the product moment method* are given in Table XXV.

TABLE XXV.

	Correlation and Probable Error
Series A ...	·2247 ± ·0140
Series B ...	·3296 ± ·0121
Series C ...	·3282 ± ·0116

Clearly there is a definitely significant, though not very large, interdependence between the two characteristics.

It may be possible, therefore, that one of these characteristics has comparatively little independent significance in determining the fitness of an ovary for continuing its development. If only one of the two characteristics is thus potent while the influence of the second is only apparent because of its dependence upon the first, we cannot determine at present which is independent and which is dependent.

Personally, I believe that the coefficients of correlation between them are too low to account entirely for the results for elimination that have been secured, and that therefore they are both concerned in determining whether or no any ovary shall develop. My reasons are as follows:

If the elimination of the ovaries of one characteristic, a, of the two here considered be not primarily due to their possessing their first characteristic but merely apparently connected with it because it is itself correlated with the second characteristic, b†, we should expect no selective elimination with respect to the dependent character within the subgroups of the independent character. Concretely, if the elimination of ovaries with a larger number of "odd" locules is due solely to the fact that these ovaries are also more radially asymmetrical, we would expect to find no elimination with regard to number of "odd" locules when we work within the same asymmetry class. Similarly for elimination with respect to radial asymmetry. Tersely: for constant a is there an elimination depending on b; for constant b is there an elimination depending on a?

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* Calculated without any combination or modification of classes for the coefficients of asymmetry.

† Pearson, K.: *Phil. Trans.* A., Vol. cc. pp. 18, 19, 1902, has termed this indirect selection.

Data are available in Tables XLI—XLIII. Only the critical comparison C—A need be made. Table XXVI. gives the difference for the mean number of "odd" locules for the first five radial asymmetry classes. Beyond this the frequencies are too few to be trustworthy. Table XXVII. gives the difference between the mean radial asymmetry of C and A for the four classes of ovaries with respect to number of "odd" locules per ovary.

TABLE XXVI.

Radial Asymmetry	Mean "Odd" Locules, C—A
·0000	– ·335
·4714	– ·106
·8165	– ·115
·9428	– ·454
1·2472	– ·157

TABLE XXVII.

Character of Ovary	Mean Radial Asymmetry, C—A
3 "even" ...	– ·017
2 "even," 1 "odd"	– ·019
1 "even," 2 "odd"	– ·034
3 "odd"	– ·077

I attach no importance to the numerical value of these differences, for I have not calculated their probable errors, but the negative sign throughout seems to me fairly satisfactory evidence that both of these characters are to some extent of independent significance in determining whether an ovary shall develop to maturity.

2. *The Correlation between the Coefficient of Asymmetry and the Number of Ovules per Locule.*

Both the actual number of ovules per locule—or per ovary—and the radial asymmetry of the fruit with respect to the number of ovules per locule seem to have significance in determining the fate of an ovary. Are they really both significant, or is one dependent upon the other?

This is a fundamental physiological question. I believe a trustworthy answer is given by the coefficient of correlation between the asymmetry of the fruit and the number of ovules per locule. Series B is the most like the original population of ovaries and serves for the determination. The coefficient calculated from the surface given as Table XLIV is

$$r = \cdot 0221 \pm \cdot 0078.$$

Possibly the coefficient is statistically significant, being about thrice its probable error, but it is clearly of so low an order that it can have no appreciable biological significance in the problem now in hand.

Apparently, therefore, both number of ovules and radial asymmetry in the distribution of the ovules in the three locules are independent and physiologically significant factors in determining whether an ovary shall or shall not reach maturity.

3. *The Correlation between the Number of "odd" Locules per Ovary and the Number of Ovules per Locule.*

Again, series B may serve as material. The correlation surface is shown as Table XLV. The constant is

$$r = -\cdot 0368 \pm \cdot 0078.$$

With a correlation so low as this it seems reasonable to think that, in their relation to selective elimination, these two characters are practically independent.

VII. RECAPITULATION.

1. The foregoing pages contain an account of a quantitative study of the selective elimination occurring in the ovaries of *Staphylea trifolia* between the time of flowering and the maturing of the fruit. The scope of the paper is limited strictly to the presentation of observed data and their description by means of biometric constants. No biological theories to explain why one ovary should be less fit than another are suggested, and no hypotheses concerning the influence of the selective elimination demonstrated upon evolutionary progress are brought forward; there will be ample time for this after the much more extensive data already collected are published. A comparison of these results for the selective elimination of *organs* with those secured by others for a selective elimination of *individuals* may also be profitably postponed.

2. The ovaries with relatively low numbers of ovules are more extensively eliminated than those with high numbers. The mean of the population remaining after selection is about 7 or 8 per cent. higher than that of the eliminated individuals.

3. The variability of both the eliminated ovaries and the series remaining after elimination is less than that of the original population. This condition

would be expected as the result of the elimination of the ovaries with the generally lower number of ovules, and the continued development of those with the generally higher numbers, as described immediately above.

4. The ovaries which remain after elimination are more radially symmetrical (as measured by the coefficient of asymmetry, or the standard deviation of the number of ovules per locule around their own mean) than those which are eliminated.

5. Ovaries with one or more locules with an "odd" number of ovules are more likely to be eliminated than those with all the locules bearing an "even" number.

6. Dimerous ovaries seem less likely, and tetramerous ovaries more likely to develop to maturity than the normal trimerous ones. Further data bearing on this point are needed.

7. Radial asymmetry and the composition of the fruit with respect to number of "odd" and "even" locules are not independent, but correlated characters. Evidence is adduced to show that both are to some extent of independent significance in determining whether or no an ovary shall continue its development.

8. Neither radial asymmetry nor the composition of the ovary with respect to number of "odd" and "even" numbers of ovules per locule is closely enough correlated with number of ovules per locule to modify in any way our conclusions concerning them as independent characteristics in their relation to selective elimination.

BIOMETRIC LABORATORY,
UNIVERSITY COLLEGE, LONDON.
January 31, 1910.

VIII. APPENDIX OF TABLES OF DATA.

In publishing these data and the constants calculated from them I have retained for the individuals the original numbers of my notes, 11—40, since I shall later treat other features of the same individuals and it may be a convenience to the reader who cares to make comparisons, as well as to myself.

In titling the Tables the following symbols were used:

L = Number of ovules per locule. O = Total number of ovules per ovary. N = Number of locules per ovary. LC = Locular composition, i.e., the composition of the fruit with respect to number of locules with "odd" and "even" numbers of ovules. A = Asymmetry, as measured by the coefficient of asymmetry.

A, B and C, following the year of collection, refer to the categories explained on p. 456.

TABLE XXVIII—L 1908, A.

Serial Number of Shrub.

Ovules per Locule.	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	29	30	31	32	33	34	35	37	38	39	40	Totals
3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	2
4	—	—	—	1	—	—	—	—	—	1	—	—	2	—	—	—	—	—	4	—	—	—	—	—	—	4	—	—	12
5	—	6	1	3	2	10	—	2	14	30	3	1	32	8	2	6	10	4	46	3	—	18	7	4	5	31	—	5	253
6	1	68	14	90	12	59	20	11	44	80	16	13	79	34	23	39	39	144	161	51	15	109	40	28	26	217	4	47	1484
7	24	89	35	44	65	74	70	54	103	73	44	50	29	68	28	61	77	104	24	123	42	93	49	90	43	41	19	67	1683
8	221	123	82	9	169	114	152	81	97	51	84	93	8	67	64	44	66	45	5	118	122	50	46	173	78	5	78	72	2317
9	47	14	16	3	41	13	45	2	12	5	3	8	—	3	46	—	3	3	—	5	16	—	8	5	88	—	19	4	409
10	7	—	2	—	11	—	12	—	—	—	—	—	—	—	32	—	—	—	—	—	—	—	—	—	57	—	—	—	121
11	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3	—	—	—	4
Totals	300	300	150	150	300	270	300	150	270	240	150	165	150	180	195	150	195	300	240	300	195	270	150	300	300	300	120	195	6285

TABLE XXIX—L 1908, B.

Serial Number of Shrub.

Ovules per Locule.	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	29	30	31	32	33	34	35	37	38	39	40	Totals
2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
4	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6	1	—	3	4	—	—	9	—	—	24
5	—	15	—	7	—	13	—	10	10	6	—	—	23	13	2	6	6	32	30	10	1	27	13	1	1	63	—	1	290
6	2	104	7	75	5	67	5	78	44	69	12	8	90	43	8	33	38	160	160	86	25	130	60	6	4	203	4	21	1547
7	29	75	64	50	47	78	50	94	82	98	46	67	30	68	24	53	65	76	53	142	65	67	81	74	16	19	21	75	1709
8	202	99	207	15	180	96	165	114	88	84	185	111	7	68	73	57	110	26	4	55	188	27	103	211	109	6	186	160	2936
9	54	6	18	—	57	14	63	4	13	13	12	9	—	3	92	1	19	5	—	6	21	1	8	8	98	—	78	32	635
10	13	—	4	2	7	1	13	—	3	—	—	—	—	—	86	—	2	1	—	—	—	—	1	—	61	—	10	11	215
11	—	—	—	1	4	1	4	—	—	—	—	—	—	—	13	—	—	—	—	—	—	—	—	—	8	—	1	—	32
12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	3	—	—	—	5
Totals	300	300	300	150	300	270	300	300	240	270	255	195	150	195	300	150	240	300	255	300	300	255	270	300	300	300	300	300	7395

TABLE XXX—L 1908, C.

Serial Number of Shrub.

Ovules per Locule	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	29	30	31	32	33	34	35	37	38	39	40	Totals
4	—	—	—	—	—	—	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
5	—	—	2	5	—	1	—	1	1	—	—	—	16	—	—	—	—	4	10	3	2	2	2	1	—	7	—	—	57
6	2	52	165	101	—	12	2	38	17	24	9	9	121	39	3	17	8	98	185	27	16	115	35	12	6	100	5	2	1220
7	14	57	37	71	6	16	12	52	40	75	15	26	24	61	1	59	15	117	84	108	38	114	37	36	31	80	14	26	1266
8	190	186	96	27	139	201	121	203	210	186	236	252	18	200	28	147	233	81	21	153	227	69	219	241	125	111	193	191	4304
9	62	5	—	—	93	40	80	5	32	14	27	11	1	—	59	4	26	—	—	8	15	—	7	8	77	2	65	62	703
10	32	—		—	59	30	79	1	—	1	13	2	—	—	200	1	18	—	—	1	2	—	—	2	58	—	23	19	541
11	—	—	—	—	3	—	3	—	—	—	—	—	—	—	9	—	—	—	—	—	—	—	—	—	2	—	—	—	17
12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
Totals	300	300	300	204	300	300	300	300	300	300	300	300	180	300	300	228	300	300	300	300	300	300	300	300	300	300	300	300	8112

TABLE XXXI—O 1908, A.

Serial Number of Shrub.

Total Ovules per Ovary.	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	29	30	31	32	33	34	35	37	38	39	40	Totals
12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
13	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
14	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	3
15	—	—	—	1	—	—	—	—	—	—	—	—	5	—	—	—	1	—	3	—	—	1	—	—	—	1	—	—	12
16	—	—	—	1	—	2	—	—	1	3	—	—	2	—	—	1	1	1	8	—	—	1	—	—	1	6	—	—	28
17	—	1	—	—	—	3	—	—	3	6	1	1	11	2	2	—	2	1	21	1	—	7	—	—	1	17	—	2	82
18	—	9	—	18	1	8	2	1	3	11	2	—	8	3	1	6	3	21	30	2	2	17	8	1	2	49	—	4	212
19	—	4	2	13	—	4	1	1	6	10	1	1	10	2	4	5	6	24	11	12	—	10	5	5	3	11	—	8	159
20	—	18	2	6	—	7	4	3	13	15	4	2	8	15	4	7	8	18	6	8	3	16	6	6	5	8	—	6	198
21	1	8	5	3	12	14	6	5	12	13	4	5	3	9	1	11	12	12	1	14	2	18	8	11	4	4	4	13	215
22	—	23	9	4	9	11	14	16	25	13	7	14	2	10	4	10	14	11	—	28	8	12	8	20	6	1	4	10	293
23	18	13	9	3	20	18	21	10	11	5	17	13	—	10	6	7	9	10	—	21	19	5	6	26	6	1	6	12	302
24	46	19	15	1	32	19	20	14	13	3	13	14	—	7	11	3	7	2	—	13	25	3	8	30	9	—	13	9	349
25	21	3	6	—	12	2	18	—	1	—	1	5	—	2	6	—	2	—	—	1	4	—	1	1	14	—	9	1	110
26	9	2	—	—	8	2	6	—	2	—	—	—	—	—	6	—	—	—	—	—	1	—	—	—	12	—	3	—	51
27	3	—	1	—	4	—	3	—	—	—	—	—	—	—	4	—	—	—	—	—	1	—	—	—	12	—	1	—	29
28	2	—	—	—	2	—	4	—	—	—	—	—	—	—	8	—	—	—	—	—	—	—	—	—	12	—	—	—	28
29	—	—	1	—	—	—	—	—	—	—	—	—	—	—	7	—	—	—	—	—	—	—	—	—	8	—	—	—	16
30	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	4	—	—	—	5
31	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	2
Totals	100	100	50	50	100	90	100	50	90	80	50	55	50	60	65	50	65	100	80	100	65	90	50	100	100	100	40	65	2095

TABLE XXXII—O 1908, B.

Serial Number of Shrub.

Total Ovules per Ovary.	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*29*	*30*	*31*	*32*	*33*	*34*	*35*	*37*	*38*	*39*	*40*	Totals
11	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
13	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1	—	—	2
14	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	1	—	—	3
15	—	1	—	—	—	—	—	—	1	—	—	—	2	—	—	—	—	1	2	—	—	—	2	—	—	4	—	—	13
16	—	3	—	1	—	1	—	2	1	—	—	—	3	2	—	—	1	7	3	2	—	5	1	—	—	13	—	—	45
17	—	3	—	3	—	7	—	2	1	1	—	—	5	4	—	1	—	8	10	6	—	8	4	—	—	29	—	—	92
18	—	13	—	11	—	6	—	10	3	8	—	—	21	5	1	3	6	32	39	4	3	21	5	—	—	35	1	1	228
19	—	15	—	12	—	8	1	9	6	7	3	1	9	4	1	7	2	16	13	12	3	20	8	1	1	13	—	1	173
20	—	12	1	8	—	14	1	16	12	18	1	2	4	8	—	6	4	9	9	28	5	13	9	1	—	2	—	7	190
21	—	15	5	9	3	6	1	10	11	17	5	6	4	10	2	10	14	10	5	21	2	9	16	3	1	1	—	7	203
22	1	12	11	3	6	14	6	14	14	11	5	12	1	15	2	7	10	11	1	8	17	3	12	16	3	—	2	16	233
23	14	7	23	1	20	12	20	19	9	12	24	22	1	6	6	9	15	4	—	13	21	2	17	36	6	—	10	23	352
24	52	15	52	1	32	14	30	18	13	13	40	20	—	9	10	6	21	2	—	3	35	3	13	39	18	1	34	18	512
25	18	3	5	—	21	4	20	—	9	1	5	1	—	2	8	1	4	—	—	2	11	—	3	3	8	—	26	13	168
26	8	1	2	—	13	3	8	—	—	1	1	1	—	—	12	—	2	—	—	1	3	—	—	1	14	—	18	10	99
27	1	—	1	—	2	—	6	—	—	1	1	—	—	—	14	—	1	—	—	—	—	—	—	—	15	—	7	2	51
28	4	—	—	—	1	—	4	—	—	—	—	—	—	—	19	—	—	—	—	—	—	—	—	—	14	—	1	1	44
29	2	—	—	—	2	1	3	—	—	—	—	—	—	—	11	—	—	—	—	—	—	—	—	—	12	—	1	—	32
30	—	—	—	—	—	—	—	—	—	—	—	—	—	—	12	—	—	—	—	—	—	—	—	—	6	—	—	1	19
31	—	—	—	1	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	1	—	—	—	4
32	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
33	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
Totals	100	100	100	50	100	90	100	100	80	90	85	65	50	65	100	50	80	100	85	100	100	85	90	100	100	100	100	100	2465

TABLE XXXIII—O 1908, C.

Serial Number of Shrub.

Total Ovules per Ovary.	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	29	30	31	32	33	34	35	37	38	39	40	Totals
12	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
13	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
14	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
15	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
16	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	1	—	—	—	—	—	—	—	1	—	—	4
17	—	—	1	2	—	—	—	—	—	—	—	—	6	—	—	—	—	1	6	—	1	—	—	—	—	4	—	—	21
18	—	2	23	16	—	1	—	6	1	—	—	1	30	2	1	—	—	10	31	4	1	15	2	—	—	15	—	—	161
19	—	5	13	17	—	1	—	4	5	5	—	1	5	1	—	1	—	11	26	2	—	19	—	—	—	12	1	—	129
20	—	8	27	10	—	1	—	4	1	5	2	2	7	5	—	6	1	24	21	7	3	19	8	2	1	11	—	1	176
21	—	10	4	15	1	1	2	5	2	7	1	1	4	10	—	6	1	16	10	15	4	19	8	3	1	12	2	1	161
22	4	21	18	3	—	7	—	13	9	12	6	2	5	22	—	11	5	26	6	16	10	9	14	14	3	12	2	3	247
23	5	17	7	2	3	9	5	16	15	25	8	15	—	28	—	24	7	11	—	28	16	14	16	17	7	10	5	10	320
24	41	35	7	3	16	42	19	48	44	39	61	68	1	32	1	24	56	6	—	23	53	5	46	57	23	22	31	37	840
25	21	2	—	—	25	14	17	3	18	6	10	8	—	—	6	3	15	—	—	4	8	—	6	4	14	1	30	22	237
26	9	—	—	—	21	9	9	1	4	1	4	2	—	—	4	—	9	—	—	1	4	—	—	2	21	—	16	14	131
27	9	—	—	—	8	5	12	—	1	—	3	—	—	—	5	1	4	—	—	—	—	—	—	1	14	—	9	7	79
28	4	—	—	—	8	3	16	—	—	—	2	—	—	—	10	—	1	—	—	—	—	—	—	—	5	—	2	4	55
29	3	—	—	—	12	3	8	—	—	—	2	—	—	—	30	—	—	—	—	—	—	—	—	—	5	—	2	—	65
30	4	—	—	—	5	4	10	—	—	—	1	—	—	—	36	—	1	—	—	—	—	—	—	—	5	—	—	1	67
31	—	—	—	—	1	—	1	—	—	—	—	—	—	—	6	—	—	—	—	—	—	—	—	—	1	—	—	—	9
32	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	100	100	100	68	100	100	100	100	100	100	100	100	60	100	100	76	100	100	100	100	100	100	100	100	100	100	100	100	2704

TABLE XXXIV—A 1908, A.

Serial Number of Shrub.

Coefficient of Asymmetry.	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*29*	*30*	*31*	*32*	*33*	*34*	*35*	*37*	*38*	*39*	*40*	Totals
·0000	44	27	13	21	29	25	21	17	17	10	15	14	12	15	13	9	14	26	27	17	25	28	8	31	13	48	13	13	565
·4714	47	44	23	23	41	41	61	25	46	36	28	33	31	30	38	26	37	56	38	62	28	40	19	49	56	43	22	34	1057
·8165	6	8	8	2	17	16	9	2	9	13	4	4	3	4	4	11	9	9	6	12	5	10	15	10	14	6	5	12	233
·9428	1	16	3	4	7	4	5	4	13	11	2	3	3	7	7	4	4	9	8	9	6	10	6	6	7	2	—	4	165
1·2472	2	4	3	—	3	4	3	1	2	5	—	—	—	3	2	—	1	—	—	—	1	1	—	3	4	—	—	1	43
1·4142	—	1	—	—	3	—	—	1	1	4	—	1	1	—	1	—	—	—	1	—	—	1	1	1	3	1	—	1	22
1·6330	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	3
1·6997	—	—	—	—	—	—	—	—	1	1	1	—	—	1	—	—	—	—	—	—	—	—	1	—	1	—	—	—	6
1·8856	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
Totals	100	100	50	50	100	90	100	50	90	80	50	55	50	60	65	50	65	100	80	100	65	90	50	100	100	100	40	65	2095

TABLE XXXV—A 1908, B.

Serial Number of Shrub.

Coefficient of Asymmetry.	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*29*	*30*	*31*	*32*	*33*	*34*	*35*	*37*	*38*	*39*	*40*	Totals
·0000	40	28	45	15	26	20	21	28	14	23	43	20	20	18	15	14	21	31	35	21	34	19	21	38	22	35	30	23	720
·4714	40	45	36	26	53	50	55	51	43	38	35	35	20	31	38	24	30	47	34	65	54	41	38	55	44	53	53	56	1190
·8165	12	16	13	6	10	6	13	10	13	16	3	6	7	5	21	4	18	14	10	7	6	14	13	3	19	6	11	6	288
·9428	5	8	3	3	5	12	5	8	3	9	4	4	3	10	10	4	7	4	4	5	6	9	9	3	8	4	5	12	172
1·2472	2	3	2	—	3	2	3	3	5	4	—	—	—	—	9	3	1	3	—	2	—	2	6	—	5	2	—	3	63
1·4142	1	—	—	—	—	—	3	—	1	—	—	—	—	1	3	1	2	—	1	—	—	—	2	1	—	—	—	—	16
1·6330	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1	—	3
1·6997	—	—	1	—	1	—	—	—	1	—	—	—	—	—	2	—	—	—	1	—	—	—	—	—	2	—	—	—	8
1·8856	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
2·0548	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1	—	—	—	—	—	3
2·1602	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	100	100	100	50	100	90	100	100	80	90	85	65	50	65	100	50	80	100	85	100	100	85	90	100	100	100	100	100	2465

TABLE XXXVI—A 1908, C.

Serial Number of Shrub.

Coefficient of Asymmetry.	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	29	30	31	32	33	34	35	37	38	39	40	Totals
·0000	46	37	29	23	24	48	30	56	45	35	61	67	28	38	39	27	54	20	31	29	54	27	48	56	18	41	30	36	1077
·4714	37	33	27	31	61	35	44	30	47	49	26	26	15	36	50	39	22	58	50	55	30	55	27	29	34	44	48	46	1084
·8165	8	10	5	11	6	4	14	3	3	11	5	3	7	6	4	4	8	12	10	12	4	12	7	5	22	8	12	10	226
·9428	9	19	39	3	8	11	10	11	4	3	8	4	8	20	5	6	12	10	9	3	12	5	16	9	15	7	8	6	280
1·2472	—	1	—	—	1	1	2	—	1	2	—	—	2	—	2	—	4	—	—	—	—	1	1	1	8	—	2	2	31
1·4142	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	2	—	—	—	5
1·6330	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
Totals	100	100	100	68	100	100	100	100	100	100	100	100	60	100	100	76	100	100	100	100	100	100	100	100	100	100	100	100	2704

TABLE XXXVII—N 1908, A—C.

Locules per Ovary in three Series.

Tree	A				B				C			
	2	*3*	*4*	*5*	*2*	*3*	*4*	*5*	*2*	*3*	*4*	*5*
11	32	100	1	—	14	100	—	—	5	104	—	—
12	—	100	—	—	—	100	—	—	—	109	1	—
13	—	50	1	—	—	100	3	—	2	118	—	—
14	9	50	—	—	10	50	—	—	14	75	—	—
15	—	100	—	—	1	100	—	—	—	108	2	—
16	—	90	—	—	5	90	—	—	—	121	1	—
17	—	100	—	—	—	100	—	—	—	109	1	—
18	—	50	—	—	2	100	6	—	1	111	3	—
19	5	90	—	—	6	80	—	—	11	153	—	—
20	—	80	—	—	—	90	—	—	—	111	—	—
21	1	50	—	—	—	85	1	—	—	103	2	—
22	—	55	3	—	—	65	1	—	—	102	1	—
23	—	50	2	—	—	50	1	—	—	61	1	—
24	—	60	5	—	2	65	—	—	—	110	—	—
25	—	65	—	—	—	100	1	—	—	163	4	—
26	—	50	—	—	—	50	—	—	—	76	—	—
27	—	65	—	—	—	80	—	—	—	122	—	—
29	—	100	—	—	—	100	4	—	—	123	2	—
*30**	—	80	—	—	—	85	1	—	—	139	2	—
31	—	100	2	—	—	100	—	—	1	105	3	—
32	—	65	3	1	—	100	9	—	—	184	26	—
33	5	90	—	—	2	85	—	—	13	134	—	—
34	—	50	3	—	—	90	15	2	—	118	16	—
35	—	100	10	—	—	100	13	—	—	126	18	—
37	—	100	—	—	—	100	—	—	—	124	—	—
38	2	100	—	—	18	100	—	—	3	153	—	—
39	—	40	—	—	—	100	3	—	1	135	3	—
40	—	65	—	—	—	100	—	—	1	158	—	—
	54	2095	30	1	60	2465	58	2	52	3355	86	—

* Shrub 30, Series C has one fruit intermediate between 3- and 4-loculed condition.

TABLE XXXVIII—L 1906.

Ovules per Locule.

	5	*6*	*7*	*8*	*9*	*10*	*11*	Totals
6—10	12	127	142	214	23	4	—	522
11—15	2	104	393	810	76	24	1	1410
16—20	2	28	168	553	53	15	—	819

TABLE XXXIX—O 1906.

Total Ovules per Ovary.

	17	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	Totals
A ...	7	15	19	18	21	21	21	35	8	4	5	—	—	—	174
B ...	—	6	11	32	40	77	107	146	22	10	7	4	7	1	470
C ...	—	1	3	3	18	41	54	119	15	6	6	5	1	1	273

TABLE XL—A 1906.

	A		B		C	
	f	°/₀	*f*	°/₀	*f*	°/₀
·0000	59	33·908	177	37·659	126	46·154
·4714	85	48·850	244	51·914	115	42·124
·8165	17	9·770	23	4·893	19	6·960
·9428	12	6·896	24	5·106	11	4·029
1·2472	1	·574	2	·425	2	·733
Totals	174	—	470	—	273	—

TABLE XLI—ALC 1908, A.

Coefficient of Asymmetry.

		·0000	*·4714*	*·8165*	*·9428*	*1·2472*	*1·4142*	*1·6330*	*1·6997*	*1·8856*	Totals
Locular Composition.	3 even ...	462	—	—	130	—	—	2	—	1	595
	2 even, 1 odd	—	614	138	—	21	14	—	1	—	788
	1 even, 2 odd	—	443	95	—	22	8	—	5	—	573
	3 odd ...	103	—	—	35	—	—	1	—	—	139
	Totals ...	565	1057	233	165	43	22	3	6	1	2095

TABLE XLII—ALC 1908, B.

Coefficient of Asymmetry.

		·0000	*·4714*	*·8165*	*·9428*	*1·2472*	*1·4142*	*1·6330*	*1·6997*	*1·8856*	*2·0548*	*2·1602*	Totals
Locular Composition.	3 even ...	610	—	—	130	—	—	—	—	1	—	—	741
	2 even, 1 odd	—	739	146	—	35	9	—	6	—	1	—	936
	1 even, 2 odd	—	451	142	—	28	7	—	2	—	2	1	633
	3 odd ...	110	—	—	42	—	—	3	—	—	—	—	155
	Totals ...	720	1190	288	172	63	16	3	8	1	3	1	2465

TABLE XLIII—ALC 1908, C.

Coefficient of Asymmetry.

Locular Composition.	*·0000*	*·4714*	*·8165*	*·9428*	*1·2472*	*1·4142*	*1·6330*	Totals
3 even ...	1001	—	—	263	—	—	1	1265
2 even, 1 odd	—	744	160	—	20	4	—	928
1 even, 2 odd	—	340	66	—	11	1	—	418
3 odd ...	76	—	—	17	—	—	—	93
Totals ...	1077	1084	226	280	31	5	1	2704

TABLE XLIV—AL 1908, B.

Ovules per Locule.

Coefficient of Asymmetry.	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	Totals
·0000	—	—	—	18	555	255	1242	57	33	—	—	2160
·4714	—	—	5	169	652	1089	1191	381	81	2	—	3570
·8165	—	—	7	48	161	248	232	126	33	8	1	864
·9428	—	—	7	26	137	66	206	30	40	4	—	516
1·2472	—	—	4	19	31	36	44	28	18	8	1	189
1·4142	—	—	1	6	3	6	16	6	5	5	—	48
1·6330	—	—	—	1	—	3	—	3	—	2	—	9
1·6997	1	1	—	1	4	3	4	4	2	1	3	24
1·8856	—	—	—	—	2	—	—	—	1	—	—	3
2·0548	—	—	—	2	1	2	1	—	2	1	—	9
2·1602	—	—	—	—	1	1	—	—	—	1	—	3
Totals	1	1	24	290	1547	1709	2936	635	215	32	5	7395

TABLE XLV—LCL 1908, B.

Ovules per Locule.

Locular Composition.	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	Totals
3 "even" ...	—	—	7	—	694	—	1448	—	74	—	—	2223
2 "even," 1 "odd"	1	1	15	104	610	607	1144	214	97	10	5	2808
1 "even," 2 "odd"	—	—	2	141	243	778	344	331	44	16	—	1899
3 "odd" ...	—	—	—	45	—	324	—	90	—	6	—	465
Totals	1	1	24	290	1547	1709	2936	635	215	32	5	7395

A BIOMETRIC STUDY OF PHAGOCYTOSIS WITH SPECIAL REFERENCE TO THE "OPSONIC INDEX."

SECOND MEMOIR. ON THE DISTRIBUTION OF THE MEANS OF SAMPLES.

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(From the Statistical Laboratories of the Lister Institute of Preventive Medicine and the London Hospital.)

IN a recent memoir* we communicated the results of an analysis undertaken with the object of throwing light on difficulties associated with the ordinary method of estimating opsonic indices. The main results of that investigation can be summarised in the following way:

(1) Phagocytic counts have pronouncedly skew frequency distributions and are good examples of Pearson's skew curves.

(2) The means of small samples have likewise a skew distribution.

(3) The use of a thicker bacillary emulsion while diminishing the skewness, does not, under possible experimental conditions, do so sufficiently to allow of testing on the basis of a Gaussian curve.

With respect to (2), we pointed out that results based, as were ours, upon samples extending in no case to more than 2000 cells, were provisional and that we intended to re-investigate this part of the subject with the help of more adequate material.

The present memoir comprises a study of the distribution of a large sample of phagocytic cells, the distributions of the means of sub-samples and some preliminary contributions to the problem of mean distributions in general.

The basis of this work is a count of 20,000 cells made by one of us (J. D. C. W.) who had had 18 months' previous experience in the Inoculation Department of the London Hospital.

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* *Biometrika*, Vol. VI. p. 377.

In undertaking this large count, those methods were adopted which are usually advised in the preparation of films. The mixture, to be subsequently placed on the slides, was made up of three equal parts, (1) serum, (2) corpuscles, (3) bacterial emulsion. The first two were derived from the operator and, in the case of the corpuscles, care was taken that they should be equally distributed, the upper portion of the centrifugalised blood being taken off and thoroughly mixed in a small tube. The bacterial emulsion was prepared as usual and carefully mixed so as to secure the greatest possible homogeneity. Since we desired a large count of a homogeneous character, not a comparison of different mixtures, it was decided to use one pipette only, but of larger calibre than usual, in order to secure the greater amount of mixture requisite for the preparation of a large number of slides. This pipette was throttled for ease of manipulation. The proportions of the three materials were carefully mixed and the pipette placed in an incubator for fifteen minutes, the emulsion having been designed to give an average of three to four bacilli per cell. The pipette was slightly rotated from time to time in order to keep the corpuscles from settling. Sufficient slides being in readiness, the pipette was withdrawn from the incubator, the contents again thoroughly mixed, and films were prepared as quickly as possible. These were then stained with Aniline Fuchsin and Methylene Blue; thereafter cover-slips were applied with Canada Balsam to preserve the slides during the process of counting. The general quality both of films and staining was exceedingly good, very few slides being of an inferior character.

The counting of so large a number of cells as 20,000 was, needless to say, sufficiently laborious, especially in regard to the leucocytes containing more than 10 bacilli apiece. Reasonable accuracy was found possible up to 15 and the few cells containing more than this were marked 16. A mechanical stage was used—the only method, in the operator's opinion, which can secure that the same cell shall not be counted twice over. The only cells omitted were those the outline of which was indistinct, or where for any reason, e.g. the clumping of cells or bacilli, it could not be determined how many bacilli lay in an individual cell; the possible error introduced by this latter criterion will be discussed below.

Definite fragments of bacteria were counted as bacteria; so also were bacilli in definite contact with the periphery of cells. In order to avoid mental prejudice, addition of the rows of figures was postponed generally for days and always at least until the end of the day's work. In this way 20,000 cells were counted in the course of about a month, with occasional intervals. The sheets of figures were then taken and the totals of successive twenty-fives, fifties and hundreds were determined; then the frequency of each number per cell on each sheet of 500 cells was tabulated, this latter process incidentally eliminating any arithmetical mistakes. The frequencies on each of the forty sheets were then added together and the total frequency obtained.

The actual distribution of the twenty thousand cells is set forth in Table I. Proceeding to fit a curve, without using Sheppard's corrections, the constants of

Table II were obtained and Graph 1 shows the theoretical curve and observations. It is evident that, although the curve represents the general trend of the observations with fair accuracy, the goodness of fit as evaluated in the ordinary way is poor. This matter appears to require comment and explanation.

TABLE I.

Actual Frequencies of 20,000 *Cells, together with Corrected Values determined on the Basis of a Random Count of* 2,000 *Cells from the same Material.* (See p. 509 *infra.*)

Number of Bacilli per Cell	Original Value	Corrected Value*
0	1428	1402·75
1	2633	2636·57
2	3351	3355·55
3	3556	3560·83
4	2807	2810·81
5	2038	2040·77
6	1430	1431·94
7	1031	1032·40
8	658	658·89
9	407	407·55
10	271	271·37
11	163	163·22
12	103	103·14
13	61	61·08
14	37	37·05
15	17	17·02
16	9	9·01
Totals	20000	19999·95

It will have been noticed that for the purposes of this count we did not enumerate cells which contained clumps of bacteria. Our reason for this omission was that, to the best of our knowledge, workers ordinarily disregard cells the contents of which cannot be resolved into definite bacilli or fragments of bacilli, and we were most anxious to use data so far as possible identical with those employed in actual practice. It was, however, subsequently pointed out to us, that this process was open to serious objection for the following reasons. In the first place, some workers of great reputation never employ for diagnostic purposes mixtures containing an appreciable number of clumps, say more than two or three per cent. In the second place, the exclusion of clumps would tend arbitrarily to heighten the proportion of cells containing 0 bacilli, since cells containing clumps must be regarded as possessing a phagocytic power at least equal to those which contain one bacillus apiece; indeed, Harvey and McKendrick have adduced evidence[†] which supports a belief that it is rather the number of bacteria than the number of acts of ingestion which should be taken as a measure of phagocytic power.

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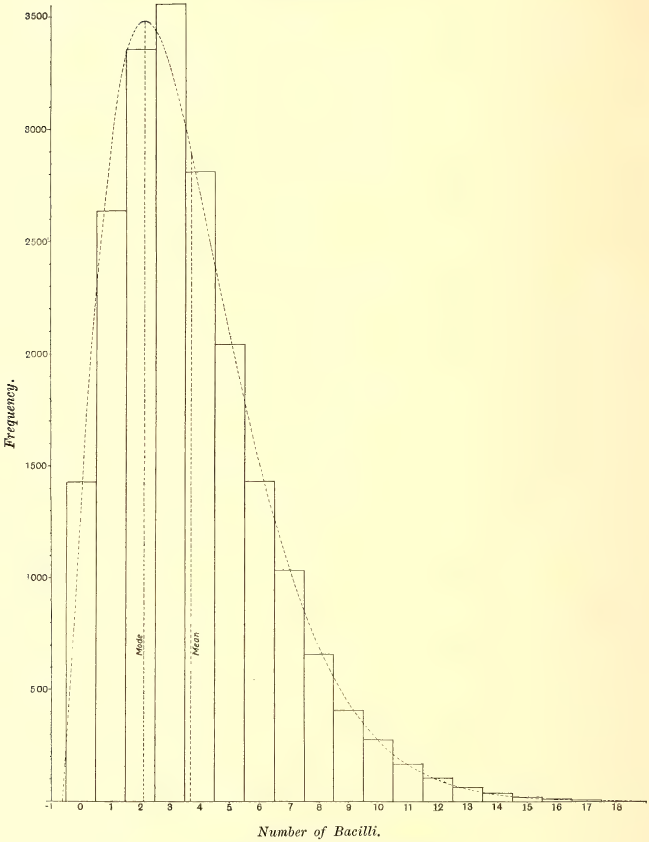
[*] 1428 cells with 0 bacilli per cell will occur not in 20,000, but in 20,360; hence in 20,000 we shall have not 1428 but 1402·75, the excess is distributed proportionally among the other groups: see p. 509.

[†] *Biometrika*, Vol. VII. p. 64.

TABLE II.



Graph 1. Curve of 20,000 cells.

Constants of Fitted Curve.

$\sigma=2{\cdot}6031$, $\beta_1=1{\cdot}0365$, $\beta_2=4{\cdot}2622$, $\kappa_2=-1{\cdot}6752$, $r=22{\cdot}8160$, $b=41{\cdot}5558$, $m_1=1{\cdot}3806$, $m_2=19{\cdot}4354$, $a_1=2{\cdot}7561$, $a_2=38{\cdot}7997$, $y_0=3478{\cdot}81$, Skewness $={\cdot}6069$, Mode at $2{\cdot}1000$, Mean at $3{\cdot}6797$, Start of curve at $-{\cdot}6562$, End at $40{\cdot}8996$, $\chi^2=86{\cdot}2$, P very small;

$$y=3478{\cdot}81\left(1+\frac{x}{2{\cdot}7561}\right)^{1{\cdot}3806}\left(1-\frac{x}{38{\cdot}7997}\right)^{19{\cdot}4354}.$$

Recognising the force of this objection, it was decided to examine the effect of including clumps in the count. To this end, another 2000 cells were counted on the slides already used and clumped cells were included. Of the 2000 cells counted in this way 36, or 1·8 per cent., contained clumped bacteria. Now if we allow, as seems reasonable, that this latter enumeration was a fair sample of the population, then the frequencies in our 20,000 cells would be changed in the manner shown in the second column of Table I. We have distributed the clumped cells among the other frequencies above zero in accordance with the proportions of those frequencies in the original count, since the small number of clumped cells which were present in the 2000 did not allow us to determine the true proportion of clumps, whether mainly of two, three, or more bacilli*.

The alteration which is effected by this correction is so trifling, that our erroneous method of counting, if it be considered an erroneous method, can hardly have been the cause of the poor fit which resulted, and we have not thought ourselves justified in discarding the data originally collected. Some other source of the poor fit must be looked for.

Other possible explanations are:

(1) The existence of heterogeneity in the material dependent on the fact that cells from different parts of the films are not, or may not be, strictly comparable with each other.

(2) An artificial heterogeneity dependent on the process of counting, other than that already discussed.

It has been noticed before that a large sample of material, even when adequately described by a frequency curve from the diagrammatic point of view, fails to satisfy the approved test. Indeed, many statisticians have been, as Pearson has remarked, far too easily satisfied with the test of mere inspection.

Elderton writes: "I have found in applying the test, that when numbers dealt with are very large, the probability is often small, even though the curve appears to fit the statistics very closely. The explanation is that the statistics with which we deal in practice nearly always contain a certain amount of extraneous matter, and heterogeneity is concealed in a small experience by the roughness of the data. The increase in the number of cases observed removes the roughness, but the heterogeneity remains. The meaning, from the curve-fitting point of view, is that the experience is really made up of more than one frequency curve, but a certain curve, approximating to the one calculated, predominates†."

It will have been noticed that the poverty of fit is mainly due to the cells containing one bacillus being in defect and those containing three in excess; these two groups have added nearly 58 to the value of χ^2. Now the work of counting is excessively monotonous, and after going through some hundreds, it seems impossible to escape an impression that a certain measurement, say 3 bacilli per cell, is modal. Hence a tendency will arise to place any doubtfuls within that particular class. If the count be limited to one or two thousands, the error so introduced may not appreciably affect the result, but it will do so if the data run to many thousands, since the same percentage deviation in a large as in a small experience has naturally a much greater influence on the fit.

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* The 36 cells which contained "clumps" were actually distributed as follows:

Number of Bacilli	Number of Cells	Number of Bacilli	Number of Cells	Number of Bacilli	Number of Cells
1	2	4	4	7	5
2	6	5	8	8	1
3	8	6	0	9	2

† *Frequency Curves and Correlation*, by W. Palin Elderton, p. 142.

The only prejudice to which the operator seems to have been—and thinks he was—subject was the idea which impressed itself on his mind during the count that the number of 3 bacilli per cell occurred with the greatest frequency; this idea may have inclined him to welcome threes with undue alacrity. It is possible that this may have helped in some degree towards producing an excess of threes in the final result. The operator believes that, at least in his case, the auditory sense is more impressed with the sound of numbers containing *e*'s and *i*'s, 3, 5, 9, than with others; he thus tended to believe that these numbers occurred more frequently than was actually the case. Evidently the truth of the matter cannot be exactly known until several independent observers have undertaken the same large count.

We have already mentioned the difficulty regarding clumps; another point is as to whether some gross heterogeneity were produced either by notable differences in various slides or by variations in the method of counting on different occasions, differences of which the operator was unconscious. In making the frequency tables just considered, there emerged necessarily a rough representation of a frequency curve for each five hundred cells. All these curves showed a rough similarity and each one a rough regularity. In some cases, however, the regularity was appreciably less than in others. On going through the data a second time, the groups of 500 which showed the greatest irregularities were excluded, on the supposition that they might represent a somewhat lower grade of accuracy in counting and might have introduced a gross heterogeneity into the total count. The application of this criterion led to the exclusion of 5000 cells. The remaining 15,000 cells were then analysed as before.

Tables III, III A and Graph 2 indicate the results of this process. There is a slight improvement in the fit, but the change is not pronounced. It can hardly be said that the results justify the exclusion.

TABLE III.

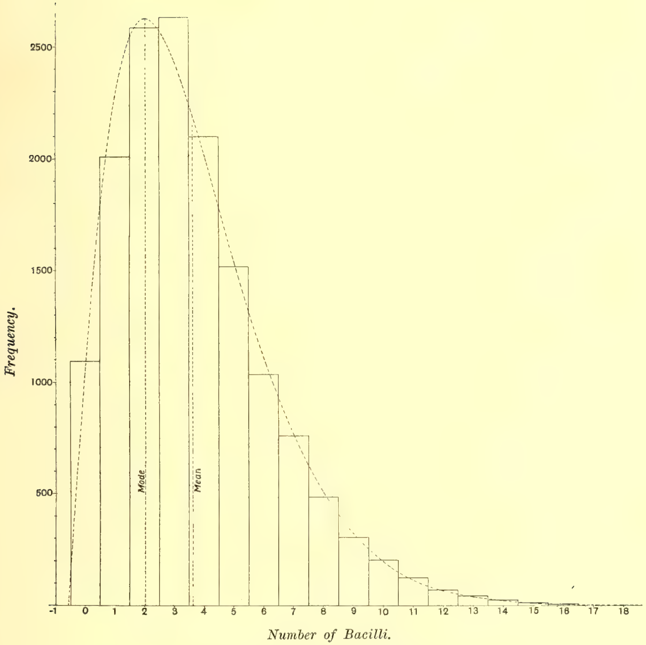
Actual Distribution of 15,000 *Cells.*

Bacilli per Cell	Number of Cells	Bacilli per Cell	Number of Cells
0	1092	*9*	306
1	2008	*10*	203
2	2585	*11*	126
3	2632	*12*	69
4	2096	*13*	44
5	1518	*14*	24
6	1035	*15*	12
7	760	*16*	6
8	484		

Although we should naturally have preferred to obtain closer fits, the result does not seriously affect the value of our material for the purposes of the present investigation. It is clear that the heterogeneity is not caused by a compounding of Gaussian distributions, and that we have to deal with markedly skew variation. We sincerely trust that some other workers may be tempted to repeat the experiment. We now turn to our main inquiry, viz. the problem of sampling.

The 20,000 cells were arranged in *consecutive* 25's, 50's and 100's, and grouped as shown in Table IV. In each case the group is an inclusive one, thus 76—83

TABLE III A



GRAPH 2. Curve of 15,000 cells.

Constants of Fitted Curve.

β_1	β_2	κ_2	r	b	m_1	m_2	a_1	a_2
1·0381	4·2346	−1·5233	20·4319	38·1013	1·2784	17·1535	2·6426	35·4587

y_0	Skewness	Mode	Mean	Start	End	P
2626·615	·61999	2·0360	3·6421	−·5533	37·4947	·00001

$$y = 2626{\cdot}615\left(1+\frac{x}{2{\cdot}6426}\right)^{1{\cdot}2784}\left(1-\frac{x}{35{\cdot}4587}\right)^{17{\cdot}1535}.$$

TABLE IV.

Actually Observed Frequencies of the Samples from 20,000 *Cells.*

SAMPLES OF 25		SAMPLES OF 50		SAMPLES OF 100	
Number of Bacilli per Sample	Number of Samples	Number of Bacilli per Sample	Number of Samples	Number of Bacilli per Sample	Number of Samples
44— 51	1	*112—123*	2	*267—282*	2
52— 59	14	*124—135*	9	*283—298*	9
60— 67	38	*136—147*	21	*299—314*	4
68— 75	74	*148—159*	30	*315—330*	13
76— 83	128	*160—171*	58	*331—346*	29
84— 91	145	*172—183*	83	*347—362*	36
92— 99	147	*184—195*	71	*363—378*	34
100—107	113	*196—207*	64	*379—394*	34
108—115	74	*208—219*	31	*395—410*	15
116—123	37	*220—231*	15	*411—426*	11
124—131	19	*232—243*	10	*427—442*	5
132—139	7	*244—255*	3	*443—458*	3
140—147	2	*256—267*	1	*459—474*	1
148—155	0	*268—279*	0	*475—490*	1
156—163	0	*280—291*	1	*491—506*	2
164—171	0	*292—303*	1	*507—522*	1
172—179	1				
Totals	800	—	400	—	200

includes samples with 76 and samples with 83 bacilli per 25 cells. For convenience of calculation, we took an arbitrary origin and unit of grouping in each case. In obtaining the moment coefficients Sheppard's corrections were used in all cases; Table V gives the constants and equations, Graphs 3, 4, 5 illustrate the curves.

TABLE V.

Curves Resulting from an Analysis of the 20,000 *Cells.*

Material	Number of Samples	Mean (in terms of Bacilli per Sample)	Mode (in terms of Bacilli per Sample)	σ (in terms of Bacilli)	β_1	β_2
Samples of 25	800	92·04	89·7488	16·7184	·1033	3·4321
Samples of 50	400	184·07	181·3964	25·7496	·1467	4·0907
Samples of 100	200	367·62	360·2632	39·9824	·3004	4·3500

Material	κ_2	r	m	ν	a	$\beta_2 - 3$	Skewness	P (Fit)
Samples of 25	·1436	25·2068	13·6033	− 10·3220	9·5149	·4321	·1371	·9765
Samples of 50	·0665	10·1436	6·0718	− 2·7070	6·2691	1·0907	·1066	·8673
Samples of 100	·1367	10·1723	6·0862	− 4·0483	7·0316	1·3500	·1840	·1727

Equations:
$$y=23{\cdot}428\left[1+\frac{x^2}{9{\cdot}5149^2}\right]^{-13{\cdot}6034}\times e^{10{\cdot}3220\tan^{-1}\frac{x}{9{\cdot}5149}}.$$

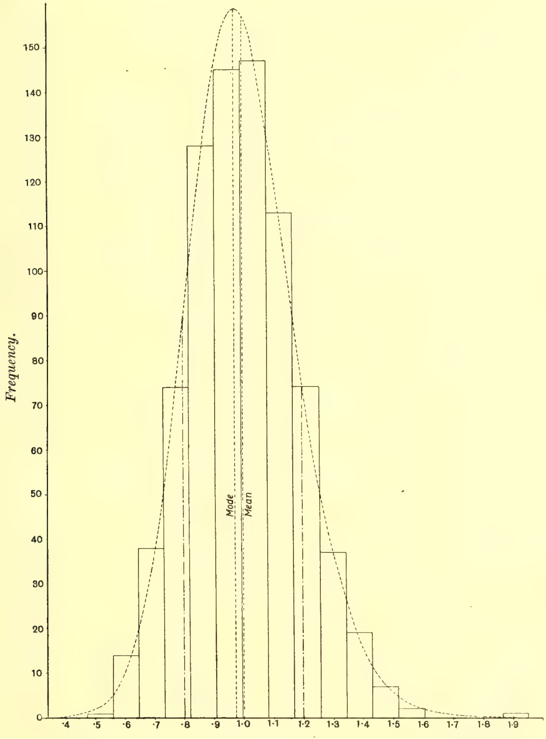
Origin at 2·6712, where 39·5 bacilli are taken as 0 and 8 bacilli per 25 cells form the statistical unit of grouping.

$$y=60{\cdot}047\left[1+\frac{x^2}{6{\cdot}2691^2}\right]^{-6{\cdot}0718}\times e^{2{\cdot}7070\tan^{-1}\frac{x}{6{\cdot}2691}}.$$

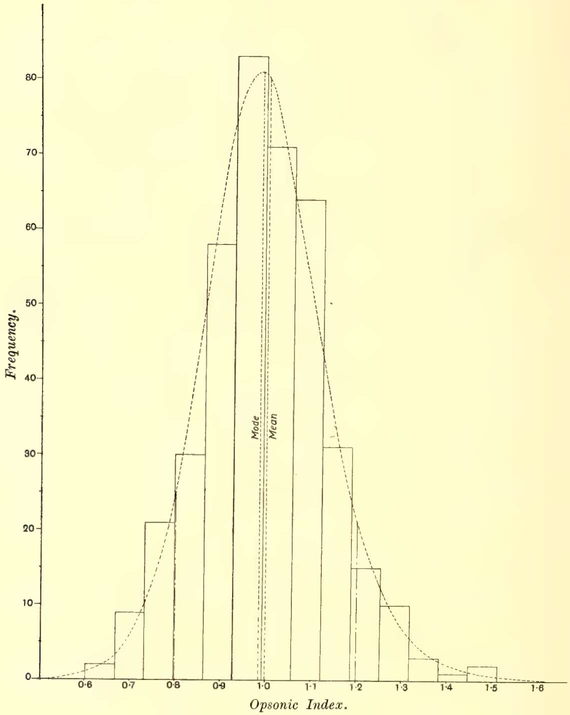
Origin at 4·87446, 105·5 bacilli taken as 0 and 12 bacilli per 50 cells as the statistical unit of grouping.

$$y=18{\cdot}117\left[1+\frac{x^2}{7{\cdot}0316^2}\right]^{-7{\cdot}0316}\times e^{4{\cdot}0483\tan^{-1}\frac{x}{7{\cdot}0316}}.$$

Origin at 4·021598, 258·5 bacilli taken as 0 and 16 bacilli per 100 cells as the statistical unit of grouping.



GRAPH 3. Curve of Samples of 25.



GRAPH 4. Curve of 400 Samples of 50.

The figures below the abscissa represent opsonic indices in terms of the mean of the whole series. That is to say, each group unit was translated back into physical terms and then divided by the mean number of bacilli per sample for the whole count. For instance, in Graph 3, the abscissa ·9 was really 5 in statistical units corresponding to 39·5 (the origin) + 5 × 8 bacilli per 25 cells. The mean is 92·04, therefore the opsonic index in terms of the mean corresponding to this abscissa is $\frac{79{\cdot}5}{92{\cdot}04} = {\cdot}86$ [*].

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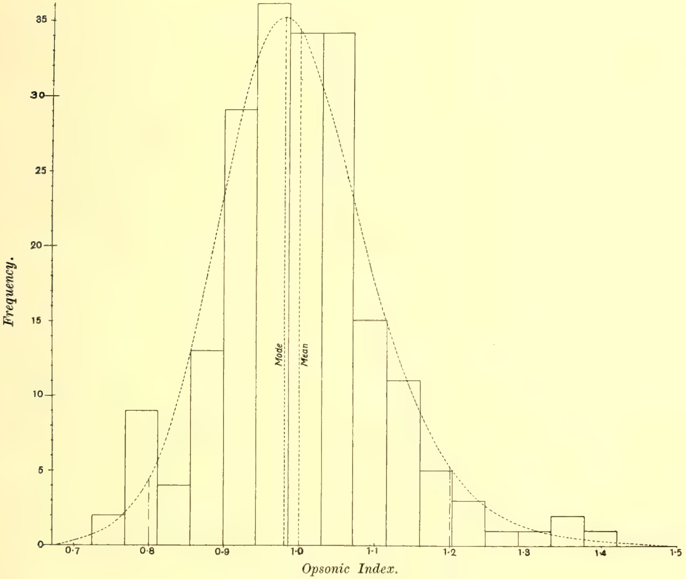
[*] The indices as plotted are only given to the nearest first decimal.

Objections may fairly be urged against this process. As we have previously remarked, the mean is theoretically not the best constant to use for descriptive purposes in the case of markedly skew frequencies and we suggested the mode as being more satisfactory. Harvey and McKendrick, in an important contribution to the subject*, have alluded to this point and remark with justice that the determination of the true mode requires calculation beyond the range of a laboratory worker. We have considered whether an approximation of sufficient accuracy might not be found. Pearson has shown† that in many cases a good enough value for the mode is given by:

Mode − Median = Twice the Distance from Mean to Median.

Unfortunately, however, the median cannot be determined with sufficient accuracy in the case of discontinuous variates such as phagocytic counts. Thus, we have experimentally no finer measurement than the integral number of bacilli per leucocyte; if we mass in a group all the m individuals containing the same number of bacilli and know that the median is the nth individual in that group, all we can do is to add $\frac{n}{m}$ of the unit of grouping to the next lower integral group measure. This is not nearly close enough in the case of such skew distributions; and we found that the value of the mode thus obtained differed materially from the real mode. It was not therefore possible to use this process with success in the case of our sample curves.



GRAPH 5. Curve of Samples of 100.

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* *Biometrika*, Vol. VII. p. 64.

† Pearson, *ibid.* Vol. I. p. 260.

We next treated, for purposes of comparison, the selected count of 15,000 cells in a similar fashion. Table VI contains the actual frequencies, Table VI A the constants and equations. Let us now consider in detail points suggested by these analytical results. The values of P show that in all cases the fit is close except in that of the sample hundreds. It appeared to us probable that in this case we had adopted too fine a unit of grouping, so that we re-worked, using a coarser unit which gave 11 groups, with the following result,

$$y = 29{\cdot}9734\left(1+\frac{x^2}{(4{\cdot}4732)^2}\right)\times e^{3{\cdot}7028\tan^{-1}(x/4{\cdot}4732)},$$

which gives $P = {\cdot}232$.

This finding confirms the suggestion to some extent, but it must be admitted that the 100's are still definitely less satisfactory than the smaller samples; the explanation of this is not clear. Table VI B, which gives all the actual and calculated frequencies reduced to a common scale, will show the reader how little change was effected by the rejection of 5000 cells.

TABLE VI.

Actually Observed Frequencies of the Samples from 15,000 *Cells.*

Samples of 25		Samples of 50		Samples of 100	
Number of Bacilli per Sample	Number of Samples	Number of Bacilli per Sample	Number of Samples	Number of Bacilli per Sample	Number of Samples
44— 51	1	*112—123*	1	*267—282*	2
52— 59	11	*124—135*	8	*283—298*	8
60— 67	29	*136—147*	15	*299—314*	3
68— 75	62	*148—159*	25	*315—330*	10
76— 83	98	*160—171*	48	*331—346*	21
84— 91	115	*172—183*	66	*347—362*	27
92— 99	107	*184—195*	50	*363—378*	30
100—107	78	*196—207*	47	*379—394*	27
108—115	56	*208—219*	22	*395—410*	10
116—123	23	*220—231*	9	*411—426*	6
124—131	15	*232—243*	6	*427—442*	3
132—139	4	*244—255*	1	*443—458*	1
140—147	1	*256—267*	1	*459—474*	0
		268—279	0	*475—490*	0
		280—291	1	*491—506*	2
Totals	600	—	300	—	150

TABLE VI A.

Curves Resulting from an Analysis of 15,000 *Selected Cells.*

Material	Number of Samples	Mean (in terms of Bacilli per Sample)	Mode (in terms of Bacilli per Sample)	σ (in terms of Bacilli)	β_1	β_2
Samples of 25	600	91·1133	89·0117	16·3123	·0457	2·8211
Samples of 50	300	182·2200	179·4383	24·5984	·0980	3·7994
Samples of 100	150	363·5667	360·1649	37·0043	·1019	4·5418

Material	κ_2	r	m	ν	a	β_2-3	Skewness	P(Fit)
Samples of 25	− ·0702	—	—	—	—	− ·1789	·1288	·9966
Samples of 50	+ ·0582	12·4217	7·2109	− 3·0885	6·7230	+ ·7994	·1131	·7365
Samples of 100	+ ·0292	7·4301	2·7151	− 1·2890	5·7783	+ 1·5418	·0920	·2164

Equations:

$$y=114{\cdot}724\left(1+\frac{x}{7{\cdot}1842}\right)^{7{\cdot}0048}\left(1-\frac{x}{12{\cdot}8362}\right)^{12{\cdot}5157}.$$

Unit of grouping 8 bacilli per 25 cells. 39·5 bacilli per 25 cells taken as zero.

$$y=45{\cdot}045\left[1+\frac{x^2}{6{\cdot}7230^2}\right]^{-7{\cdot}2109}\times e^{3{\cdot}0885\tan^{-1}\frac{x}{6{\cdot}7230}}.$$

Origin at 4·7217, where 12 bacilli per 50 cells form the unit of grouping and the zero is at 105·5.

$$y=26{\cdot}445\left[1+\frac{x^2}{5{\cdot}7783^2}\right]^{-4{\cdot}7151}\times e^{1{\cdot}2890\tan^{-1}\frac{x}{5{\cdot}7783}}.$$

Origin at 5·5642, where 16 bacilli per 100 cells are the unit and the zero is at 258·5.

Next as to the agreement between the theoretical constants of the curves of means and their empirical values as above determined. Table VII gives the calculated values of B_1 and B_2-3 [assuming that where n is the number of individuals in the sample and β_1, β_2 the constants of the original distribution $B_1=\frac{\beta_1}{n}$ and $B_2-3=\frac{\beta_2-3}{n}$] together with the actual values.

In only one case, that of the 25's from the selected 15,000, is the agreement at all possible. "Student," in a very interesting communication*, has suggested that the divergence noted by him in our previous curve of 80 means, depended on the existence of homotyposis in the samples. This suggestion no doubt contains some truth and could be satisfactorily tested by mixing up our individual records of leucocytes and drawing really random samples. We have not felt justified in delaying publication of our results until we could find leisure to undertake this very laborious task, particularly as other factors are involved. This is clear from the fact that the divergence is more marked in the case of the larger samples which must be less influenced by homotyposis.

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* *Biometrika*, Vol. VII. p. 210.

TABLE VI B.

Comparison of the Observed and Calculated Frequencies for Samples of 20,000 *and* 15,000 *Cells. All the Frequencies are reduced to* 100.

Twenty-Fives					Fifties					Hundreds				
Bacilli	20,000 Count		15,000 Count		Bacilli	20,000 Count		15,000 Count		Bacilli	20,000 Count		15,000 Count	
	Observed	Calculated	Observed	Calculated		Observed	Calculated	Observed	Calculated		Observed	Calculated	Observed	Calculated
44— 51	·13	·35	·17	·22	*112—123*	·5	·5	·33	·49	*267—282*	1	1·0	1·33	1·09
52— 59	1·75	1·40	1·83	1·51	*124—135*	2·25	1·54	2·67	1·61	*283—298*	4·5	1·86	5·33	1·95
60— 67	4·75	4·22	4·83	4·75	*136—147*	5·25	4·17	5·0	4·49	*299—314*	2	4·52	2·0	4·49
68— 75	9·25	9·47	10·33	10·74	*148—159*	7·5	9·24	8·33	9·99	*315—330*	6·5	8·97	6·67	8·91
76— 83	16·0	15·71	16·33	16·20	*160—171*	14·5	15·71	16·0	16·73	*331—346*	14·5	14·03	14·0	14·79
84— 91	18·12	19·44	19·17	18·89	*172—183*	20·75	19·80	22·0	20·52	*347—362*	18	17·14	18·0	18·72
92— 99	18·38	18·44	17·83	17·75	*184—195*	17·75	18·63	16·67	18·65	*363—378*	17	16·68	20·0	18·03
100—107	14·13	13·89	13·0	13·73	*196—207*	16·0	13·63	15·67	13·10	*379—394*	17	13·40	18·0	13·61
108—115	9·25	8·63	9·33	8·80	*208—219*	7·75	8·23	7·33	7·52	*395—410*	7·5	9·28	6·67	8·50
116—123	4·63	4·61	3·83	4·65	*220—231*	3·75	4·36	3·0	3·75	*411—426*	5·5	5·77	4·0	4·68
124—131	2·38	2·18	2·50	2·00	*232—243*	2·5	2·13	2·0	1·71	*427—442*	2·5	3·34	2·0	2·39
132—139	·88	·95	·67	·68	*244—255*	·75	1·00	·33	·74	*443—458*	1·5	1·85	·66	1·18
140—147	·25	·39	·17	·21	*256—267*	·25	·46	·33	·50	*459—474*	·5	1·00	—	1·13
148—155	0	·15			*268—279*	·5	·39	0		*475—490*	·5	·54	—	
156—163	0	·09			*280—291*			·33		*491—506*	·1	·59	1·33	
164—171	0				*292—303*					*507—522*	·5			
172—179	·13													
Totals	100·03	99·92	99·99	100·13	—	100·00	99·79	99·99	99·80	—	100·00	99·97	99·99	99·47

TABLE VII.

Number in Sample	B_1 (empirical)	B_1 (calculated)	B_2-3 (empirical)	B_2-3 (calculated)
(20,000 Count)				
25	·1033	·0415	·4321	·0505
50	·1467	·0208	1·0907	·0252
100 (1st curve) ...	·3005	·0104	1·3501	·0126
100 (2nd curve) ...	·3142	·0104	1·4601	·0126
(15,000 Count)				
25	·0457	·0415	−·1789	·0494
50	·0980	·0208	·7994	·0247
100	·1019	·0104	1·5418	·0124

We can now examine the accuracy of the opsonic method in the way attempted in our former paper, viz. we can inquire what the chances are of obtaining from a "population" of means, samples giving different indices in terms of the real mean of all such samples. In similar cases, it is usual to divide the frequency curve into a series of equal areas, setting up ordinates on either side of the mean or mode. Since this elaborate graphical method requires the services of a specially skilled draughtsman, and considerably increases the cost of production, we decided to adopt the plan followed in our last paper. A considerable number of additional ordinates were calculated for each sample curve and we determined the area from the beginning or end of the curve up to specified ordinates which corresponded to indices of ·6, ·7, ·8, etc. in terms of the mean. From these areas the Tables of Chances (VIII, IX, X) were deduced.

TABLE VIII.

Chances of Obtaining Certain Deviations in Index Values.

Opsonic Index in terms of the Mean	SAMPLES OF 25	
	Fraction of the Total Area bounded by the corresponding ordinate (Total Area = 1)	Odds against the occurrence of such a Deviation or a Greater
·6	·0089	111 to 1
·7	·0396	24·3 to 1
·8	·1276	6·8 to 1
·9	·2973	2·4 to 1
1·1	·2740	2·7 to 1
1·2	·1324	6·6 to 1
1·3	·0558	16·9 to 1
1·4	·0211	46·4 to 1
Beyond the limits *1·4—·6*	·0300	32·3 to 1
„ „ *1·3—·7*	·0954	9·5 to 1
„ „ *1·2—·8*	·2600	2·9 to 1
„ „ *1·1—·9*	·5713	·75 to 1

TABLE IX.

Chances of Obtaining Certain Deviations in Index Values.

Opsonic Index in terms of the Mean	Samples of 50	
	Fraction of the Total Area bounded by the corresponding ordinate (Total Area = 1)	Odds against the occurrence of such a Deviation or a Greater
·7	·0112	88·3 to 1
·8	·0618	15·2 to 1
·9	·2283	3·4 to 1
1·1	·2160	3·6 to 1
1·2	·0770	12 to 1
1·3	·0248	39·3 to 1
Beyond the limits *1·3—·7*	·0360	26·8 to 1
" " *1·2—·8*	·1388	6·2 to 1
" " *1·1—·9*	·4443	1·3 to 1

TABLE X.

Chances of Obtaining Certain Deviations in Index Values.

Opsonic Index in terms of the Mean	Samples of 100	
	Fraction of the Total Area bounded by the corresponding ordinate (Total Area = 1)	Odds against the occurrence of such a Deviation or a Greater
·7	·0006	1666 to 1
·8	·0216	45·4 to 1
·9	·1661	5·0 to 1
1·1	·1618	5·2 to 1
1·2	·0420	22·8 to 1
1·3	·0109	90·8 to 1
Beyond the limits *1·3—·7*	·0115	86·6 to 1
" " *1·2—·8*	·0636	14·7 to 1
" " *1·1—·9*	·3279	2·1 to 1

If we adopt about a ten to one chance as the limiting value for evidence of differentiation, then the limits, for single determinations of the index, are in the case of 25's, roughly 1·3—·7; for 50's, 1·25—·75; for 100's, 1·2—·85. Everyone has his own standard of accuracy or reliability; all we mean is that unless a given index is beyond these limits, the chances are less than ten to one against its being drawn from a normal population. We now come to a most important qualification of the value of these tables.

In the table as given, the measure is of the probable deviation of a sample from the real mean of the samples as determined from a long count. If the control mean, i.e. the denominator of the fraction which we call the opsonic index, has been determined from a large number of cells, say 1000, and the tested serum is measured on the basis of a small sample, then our results are directly applicable; but this supposition does not represent the usual state of affairs. In ordinary practice, two small samples, each of not more than 100 cells, serve as control and test, there is therefore, so to speak, a double source of variation; odds based on the results obtained above would over-estimate the reliability of any given determination. This is a point of considerable importance which has, we think, escaped the attention of writers on the subject; its elucidation would involve a careful examination of the whole theory of index distributions, and would

TABLE XI.

Constants of the Experimental Index Frequencies.

[Mean, Mode and σ are expressed in terms of the Index, i.e. in physical units.]

Material	Number of Samples	Mean	Mode	σ	β_1	β_2	κ_2	r
25's	800	1·0362	·9432	·2585	·7099	4·3914	·9619	—
50's*	400	1·0205	·9536	·2204	·6952	4·8945	·3635	11·2690
100's	200	1·0120	·9689	·1553	·3654	3·7138	·9032	—

Material	m	ν	a	$\beta_2 - 3$	Skewness	y	p	P (Fit)
25's	—	—	—	1·3914	·3600	326·1439	27·4967	·0853
50's	6·6345	$-8\cdot5166$	5·6343	1·8945	·2912	—	—	·8953
100's	—	—	—	·7138	·2780	491·3142	48·7621	·9355

Equations: $$y = \text{antilog } 43\cdot6116 \left(x^{-27\cdot4967} \times e^{-\frac{326\cdot1439}{x}} \right).$$

A change of ·1 in the index is taken as the statistical unit of grouping and index ·25 as zero.

$$y = 6\cdot2092 \left(1 + \frac{x^2}{5\cdot6343^2}\right)^{-6\cdot6345} \times e^{8\cdot5166 \tan^{-1} \frac{x}{5\cdot6343}}.$$

A change of ·1 in the index is taken as the statistical unit of grouping and index ·45 as zero. The origin is at 1·4468.

$$y = \text{antilog } 71\cdot8335 \left(x^{-48\cdot7621} \times e^{-\frac{491\cdot3142}{x}} \right).$$

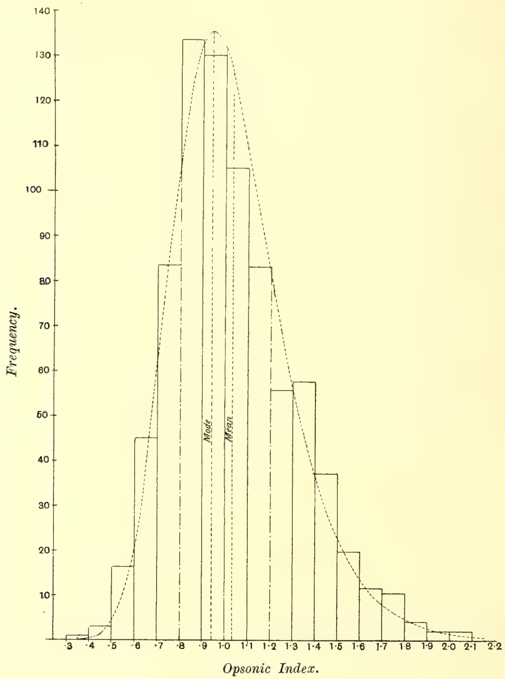
A change of ·1 in the index is taken as the statistical unit of grouping and index ·45 as zero.

---

* A slight slip has been found in the values of β_1 and β_2 for this series, they should be ·6836 and 4·9855. This error does not, however, visibly alter the form of the curve in Graph 7.

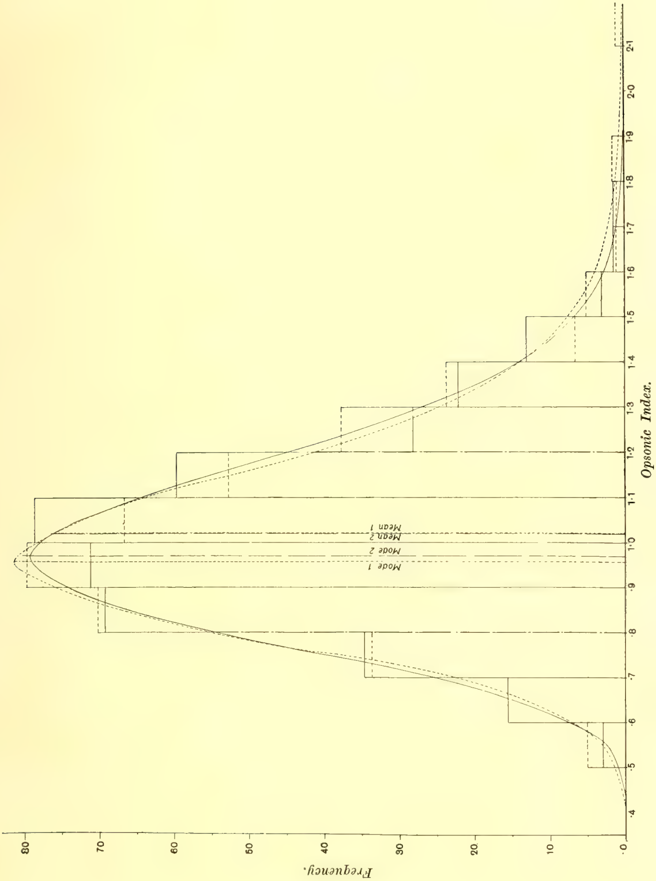
accordingly present features of interest quite apart from the particular case of the opsonic index. These remarks apply with special force to medical statistics which so frequently involve the use of ratios or indices.

The importance of the problem tempted us to devote some little time to a theoretical consideration of the subject and we arrived at a few analytical results which might, we thought, be of some value. It is, however, unnecessary for us to publish these incomplete investigations, since Professor Karl Pearson has succeeded in obtaining a full solution of the problem[*]. We therefore confine ourselves to an empirical solution which is sufficient for our immediate object.



GRAPH 6. Curve of 25's secondary without replacement.

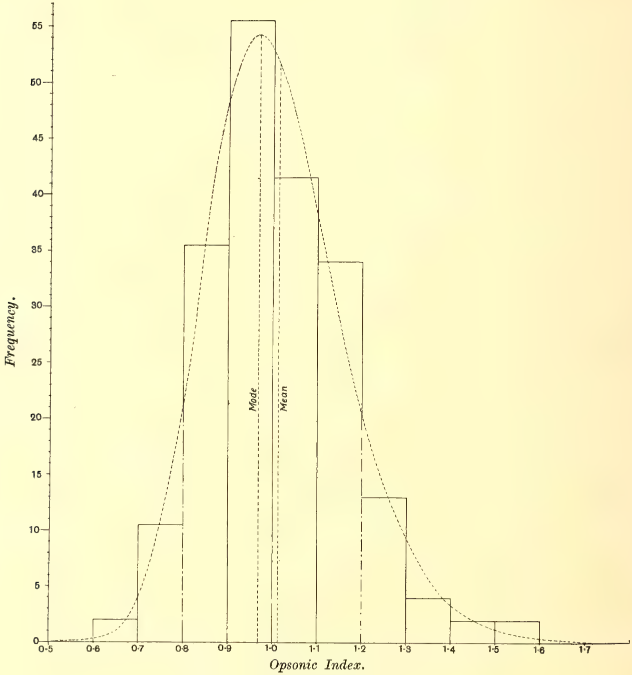
[*] See the following paper.



GRAPH 7. Curves of 50's with replacement—continuous: without replacement—dotted.

The actual number of bacteria in each sample of 25, 50 or 100 cells was copied out on a separate slip of cardboard, so that we had 800, 400 and 200 cards corresponding to the samples of 25, 50 and 100 cells. The cards used were small pieces of correspondence cards which, being smooth, could be shuffled very easily.

With these cards the following experiments were performed. Taking the case of the 400 cards relating to the fifties, the cards were thoroughly shuffled and one drawn out. The number on the card was written down, the card *not* being replaced. The remainder were once more shuffled, another drawing was made and the process was repeated until the whole 400 had been drawn. The cards were then once more shuffled and the process repeated, the second set of numbers being written down one by one under the first set of values. Each number of the first set was then divided by the corresponding number in the second set and the 400



Graph 8. Curve of 100's secondary without replacement.

indices thus obtained were analysed as usual. The same experiment was carried out in the case of 25's and 100's. Tables XI, XI A and Graphs 6, 7 and 8, contain the constants and data of the distributions and Table XII the chances of obtaining indices within assigned limits deduced from the areas.

TABLE XI A.

Experimental Determination of Index Distributions.

Index	Samples of 25 (Drawings not Replaced)		Samples of 50 (Drawings not Replaced)		Samples of 50 (Drawings Replaced)		Samples of 100 (Drawings not Replaced)	
	Observed	Calculated	Observed	Calculated	Observed	Calculated	Observed	Calculated
·3— ·4	1	·073 } 1·527	—	—	—	—		
·4— ·5	3	1·454	—	—	—	—		
·5— ·6	16·5	11·243	5	3·989	3	3·007	—	
·6— ·7	45	40·468	15·5	13·990	15·5	15·308	2	1·500
·7— ·8	83·5	89·850	33·5	37·969	34·5	39·863	10·5	11·697
·8— ·9	133·5	121·939	70	66·729	69	65·497	35·5	36·009
·9—1·0	130	134·204	79·5	80·213	71	77·683	55·5	52·780
1·0—1·1	105	122·028	66·5	71·759	78·5	72·001	41·5	46·583
1·1—1·2	83	96·635	52·5	51·973	59·5	54·817	34	29·003
1·2—1·3	55·5	69·188	37·5	32·676	28	35·425	13	14·177
1·3—1·4	57·5	46·012	23·5	18·792	22	19·873	4	5·850
1·4—1·5	37	28·990	6·5	10·249	13	9·830	2	2·145
1·5—1·6	19·5	17·560	5	5·452	3	4·334	2	1·00
1·6—1·7	11·5	10·339	1	2·870	1·5	1·717		
1·7—1·8	10·5	5·967	1	1·512	1·5	·814		
1·8—1·9	4	3·397	2	·803	—			
1·9—2·0	2	1·918	0	·432 } 1·735	—			
2·0—2·1	2	2·0	0	·5	—			
2·1—2·2	—	—	1		—			
2·2—2·3	—	—	—	—	—			
Totals	800	803·265	400	399·908	400	400·169	200	203·744

A comparison of these tables with nos. VIII—X indicates how far the limits of reliability are narrowed by a more complete study of actual conditions (see also Table XV *infra*).

It will be noticed that these experiments do not assume any special distribution for the original variates. They merely afford a partial answer to this question—if we had two sets of 400—200, 800—samples exactly the same as the 400 (200, 800) actually counted and formed at random 400 (200, 800) indices from them, one set always furnishing the denominator and no sample being used more than once, how are the resulting indices likely to be distributed? In strictness, therefore, these results are only applicable to our particular case. If, however, we assume that our counts are numerous enough and accurate enough to give the distributions of sample means of the size used, and for the emulsion used, with fair exactitude, then we can perform another experiment.

TABLE XII.

Chances of Obtaining Certain Deviations on the Basis of Three Experimental Index Distributions. (Method of Non-Replacement.)

Opsonic Index	Samples of 25		Samples of 50		Samples of 100	
	Fraction of Total Area bounded by corresponding Ordinate	Odds against such a Deviation or a Greater	Fraction of Total Area bounded by corresponding Ordinate	Odds against such a Deviation or a Greater	Fraction of Total Area bounded by corresponding Ordinate	Odds against such a Deviation or a Greater
·6	·0160	61·5 to 1	·0098	101 to 1	—	—
·7	·0665	14·0 to 1	·0448	21·3 to 1	·0075	132 to 1
·8	·1727	4·79 to 1	·1397	6·16 to 1	·0659	14·2 to 1
·9	·3247	2·08 to 1	·3067	2·26 to 1	·2460	3·07 to 1
1·1	·3546	1·82 to 1	·3135	2·19 to 1	·2571	2·89 to 1
1·2	·2336	3·28 to 1	·1835	4·45 to 1	·1121	7·92 to 1
1·3	·1475	5·78 to 1	·1019	8·81 to 1	·0413	23·2 to 1
1·4	·0899	10·1 to 1	·0549	17·2 to 1	—	—
1·5	·0537	17·6 to 1	·0293	33·1 to 1	—	—
1·6	·0317	30·6 to 1	·0157	62·7 to 1	—	—
1·7	·0188	52·2 to 1	—	—	—	—
Beyond the Limits *1·4—·6*	·1059	8·44 to 1	·0647	14·5 to 1	—	—
" " " *1·3—·7*	·2140	3·67 to 1	·1467	5·82 to 1	·0488	19·5 to 1
" " " *1·2—·8*	·4063	1·46 to 1	·3232	2·09 to 1	·1780	4·62 to 1
" " " *1·1—·9*	·6793	·47 to 1	·6202	·61 to 1	·5031	·99 to 1

Let us suppose that the distribution of samples of 50 that we obtained is actually the distribution of samples of 50 in general, then the following experiment may be considered. The 400 cards were shuffled and drawn as before except that after each drawing the card was replaced, the process being carried on until 400 numbers had been written down, a second series of 400 numbers was then obtained in the same way and the indices were computed. Tables XIII, XIV and Graph 7

TABLE XIII.

Constants of the Experimental Frequency Distribution Obtained by the Method of Replacement. (Mean, Mode and σ, Expressed in Terms of the Opsonic Index.)

Mean	Mode	σ	β_1	β_2	r	b	m_1	m_2
1·0196	·9697	·2056	·2077	3·2155	62·7551	44·6934	9·0844	51·6706

a_1	a_2	κ_2	β_2-3	Skewness	P
6·7337	37·9597	– ·8536	·2155	·2428	·8988

Equation :
$$y = 80{\cdot}0808\left(1+\frac{x}{6{\cdot}7337}\right)^{9{\cdot}0844}\left(1-\frac{x}{37{\cdot}9597}\right)^{51{\cdot}6706}.$$

A change of ·1 in the index is taken as the statistical unit of grouping and index ·45 as zero.

TABLE XIV.

Chances of Obtaining Certain Deviations, on the Basis of an Experimental Curve from Samples of 50 (Method of Replacement).

Opsonic Index	Fraction of Total Area bounded by corresponding Ordinate	Odds against such a Deviation or a Greater
·6	·0075	132 to 1
·7	·0458	20·8 to 1
·8	·1454	5·88 to 1
·9	·3096	2·23 to 1
1·1	·3165	2·16 to 1
1·2	·1795	4·57 to 1
1·3	·0910	9·99 to 1
1·4	·0413	23·2 to 1
1·5	·0167	58·9 to 1
1·6	·0059	168 to 1

give the statistical constants and other information which resulted from an analysis of this experiment. It will be noticed that the table of chances does not differ appreciably from that due to the other experiment (XV). On this account, and because of the doubtfulness of the assumption in the case of 100's and the smaller practical interest in the case of 25's, the experiment was not repeated in these cases.

TABLE XV.

Comparison of Odds against Certain Index Deviations as Calculated on Different Assumptions for Samples of 50.

Index	Using True Mean	Experimental Curve without Replacement	Experimental Curve with Replacement
·7	88·3 to 1	21·3 to 1	20·8 to 1
·8	15·2 to 1	6·16 to 1	5·88 to 1
·9	3·38 to 1	2·26 to 1	2·23 to 1
1·1	3·63 to 1	2·19 to 1	2·16 to 1
1·2	12·0 to 1	4·45 to 1	4·57 to 1
1·3	39·3 to 1	8·81 to 1	9·99 to 1

Without in the least wishing to underestimate the shortcomings of both our material and method, it appears to us that Table XII can fairly be used as a rough practical test of the accuracy of the opsonic method in the case of the Tubercle Bacillus and that it confirms the opinions expressed in our former paper. A theoretical discussion of the index problem in general is reserved for another paper and more suitable material. It is now proper to consider very briefly some practical points.

In the first place, it has been asserted that the Tubercle Bacillus is an unfavourable organism to select for the purpose of studying the accuracy of the opsonic technic. While we cannot assert that no more favourable example could have been chosen, we are of opinion that the Tubercle Bacillus is certainly not the least satisfactory of the organisms commonly dealt with. For instance, Tubercle Bacilli act, or are acted on, almost entirely as single individuals, while Staphylococci or Streptococci occur in groups or chains respectively. These groups or chains are counted as so many individuals which, from the point of view of phagocytosis, may or may not be correct. The opsonic power required to attract a group of cocci to the leucocyte *may* not be greater than that associated with the ingestion of a single coccus. Under such conditions we have the difficulties as to the correct treatment of doublets and triplets—to which allusion has been made by Harvey and McKendrick—in an acute form.

Next as to the practical accuracy of the method for diagnostic purposes or in the control of dosage. The results here communicated, which are in agreement with our previous observations and also, *mutatis mutandis*, with those of Harvey and McKendrick, seem to demonstrate that single observations falling within the limits of 20 °/。 on either side of the mean cannot, in the case of Tubercle Bacilli, be regarded as strongly suggestive of abnormality. If it is possible, as will often be the case when it is a question of diagnosis alone, to repeat the observation several times under comparable conditions, the opsonic method may be expected to yield results of a certain value. When, however, the opsonic index is used to control injections of Tuberculin, so that no two determinations are made under the same conditions, we do not think the method has more than a very restricted field of usefulness. This leads us on to refer to a cognate matter.

Glynn and Cox use language* which seems to imply that we have attacked the opsonic index in the sense of denying that differences in phagocytic power as between normal and diseased persons occur. We have not asserted this nor does any statement of ours bear such an interpretation. We have merely pointed out, a contention not seriously challenged, that the claims advanced by certain enthusiasts to the effect that all variations of 10 °/₀ from the normal are probably or necessarily significant, must be entirely unjustifiable.

At the risk of being tedious, we desire to emphasise once more the distinction between vaccine therapy on the one hand and opsonic estimations on the other. If our work tends, as it certainly may tend, to unsettle the opinion of some readers as to the relative accuracy of the tuberculo-opsonic index, we wish in no way to be ranked among detractors of the claim of vaccine therapy to be regarded as a new and powerful weapon in the struggle against disease. On the contrary we are convinced of its present importance and look to the future for its still greater development.

In conclusion we would again remark that the results here published are in essential agreement with those communicated by Harvey and McKendrick and ourselves in recent numbers of *Biometrika*.

Conclusions.

1. Large phagocytic counts extending to 15,000 or 20,000 cells are markedly skew and exhibit signs of heterogeneity.

2. The exclusion of clumped bacilli has not been the main cause of the poor fit obtained.

3. The heterogeneity may be due to difficulties in counting.

4. The distributions of the means of samples of 25, 50 and 100 cells drawn from these populations are also markedly skew and excellently represented by Pearson's type curves.

5. This skewness cannot be due entirely to homotyposis.

6. The odds against a second sample of the same material giving an opsonic index, in terms of the true mean, outside of the limits 1·2 to ·8 are, in the case of samples of 25, about 3 to 1, in the case of 50's about 6 to 1 and for 100's about 15 to 1.

7. When the index is estimated on the basis of two small samples and without determining the mean of the normal series from a long count, the odds are reduced to 1·5 to 1, 2 to 1 and 4·6 to 1 respectively, for samples of 25, 50 or 100.

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* Glynn and Cox: *Biochemical Journal*, Vol. IV. p. 300.

8. Even in the case of samples of 100, it is doubtful whether single determinations which give values even beyond the limits 1·3 to ·7 can be regarded as satisfactory evidence of differentiation.

9. The two previous conclusions are based on the analysis of four experimental frequencies.

10. The Tubercle Bacillus is not necessarily an unfavourable test of the opsonic method.

11. The opsonic index is less useful as a means of controlling treatment than for diagnostic purposes, since in the latter case it may be possible to make several comparable determinations.

12. Differentiation in respect of phagocytic power (Tubercle Bacilli) can possibly be established by the opsonic index in some cases.

ON THE CONSTANTS OF INDEX-DISTRIBUTIONS AS DEDUCED FROM THE LIKE CONSTANTS FOR THE COMPONENTS OF THE RATIO, WITH SPECIAL REFERENCE TO THE OPSONIC INDEX.

BY KARL PEARSON, F.R.S.

(1) GIVEN x and y two variables, the frequency constants of both of which are known, we often require in statistics the frequency constants of their ratio or index: $i = x/y$.

If the coefficients of variation are small, we have with the usual notation for means, standard deviations, moment coefficients, etc.*:

$$\bar{\imath} = \left(\frac{\bar{x}}{\bar{y}}\right)\left\{1 + v_x^2 - r_{xy} v_x v_y - \frac{\mu_3'}{\bar{y}^3} + \frac{p_{12}}{\bar{x}\bar{y}} + \frac{\mu_4'}{\bar{y}^4} - \frac{p_{13}}{\bar{x}\bar{y}^3} + \text{etc.}\right\} \quad \text{..................(i)},$$

$$M_2 = \sigma_2^2 = \left(\frac{\bar{x}}{\bar{y}}\right)^2 \left\{v_x^2 + v_y^2 - 2 v_x v_y r_{xy} + \frac{4p_{12}}{\bar{x}\bar{y}^2} - \frac{2p_{21}}{\bar{x}^2\bar{y}} - \frac{2\mu_3'}{\bar{y}^3}\right.$$

$$\left. + \frac{3p_{22} - p_{11}^2}{\bar{x}^2\bar{y}^2} + \frac{3\mu_4' - 2\mu_2'^2}{\bar{y}^4} + \frac{2\mu_2' p_{11}}{\bar{x}\bar{y}^3} + \text{etc.}\right\} \text{..................(ii)},$$

$$M_3 = \left(\frac{x}{\bar{y}}\right)^3 \left\{\frac{\mu_3}{\bar{x}^3} - \frac{\mu_3'}{\bar{y}^3} - \frac{3p_{21}}{\bar{x}^2\bar{y}} + \frac{3p_{12}}{\bar{x}\bar{y}^2} + \frac{9p_{22} - 6p_{11}^2 - 3\mu_2\mu_2'}{\bar{x}^2\bar{y}^2}\right.$$

$$\left. - \frac{3(p_{31} - \mu_2 p_{11})}{\bar{x}^3\bar{y}} - \frac{9(p_{13} - \mu_2' p_{11})}{\bar{x}\bar{y}^3} + \frac{3(\mu_4' - \mu_2'^2)}{\bar{y}^4} + \text{etc.}\right\} \text{...(iii)},$$

$$M_4 = \left(\frac{\bar{x}}{\bar{y}}\right)^4 \left\{\frac{\mu_4}{\bar{x}^4} + \frac{\mu_4'}{\bar{y}^4} + \frac{6p_{22}}{\bar{x}^2\bar{y}^2} - \frac{4p_{31}}{\bar{x}^3\bar{y}} - \frac{4p_{13}}{\bar{x}\bar{y}^3} + \text{etc.}\right\} \quad \text{...............................(iv)}.$$

These formulae go to the order of the 4th power in the coefficients of variation, but of course this is not to the same order of approximation in M_2, M_3 and M_4.

(2) It will be seen at once that these approximate formulae would be practically unworkable if x and y were correlated, as we should have to find 3rd and 4th order product moments.

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* A rule denotes a mean value, σ a standard deviation, v, $=\sigma/\text{mean}$, is a coefficient of variation; μ_2, μ_3, μ_4 are the moment coefficients for x, μ_2', μ_3', μ_4' for y, M_2, M_3, M_4 for the index i, and $p_{uv} = S(x - \bar{x})^u (y - \bar{y})^v / N$, where N is the total number of pairs. Thus $p_{30} = \mu_3$, $p_{02} = \mu_2'$, etc.

If they be uncorrelated we have:

$$\bar{\imath} = \left(\frac{\bar{x}}{\bar{y}}\right)\left\{1 + v_x^2 - \frac{\mu_3'}{\bar{y}^3} + \frac{\mu_4'}{\bar{y}^4} - \text{etc.}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\text{i})^{\text{bis}},$$

$$M_2 = \sigma_2^2 = \left(\frac{\bar{x}}{\bar{y}}\right)^2\left\{v_x^2 + v_y^2 - \frac{2\mu_3'}{\bar{y}^3} + 3v_x^2v_y^2 + \frac{3\mu_4' - 2\mu_2'^2}{\bar{y}^4} + \text{etc.}\right\} \ldots\ldots\ldots(\text{ii})^{\text{bis}},$$

$$M_3 = \left(\frac{\bar{x}}{\bar{y}}\right)^3\left\{\frac{\mu_3}{\bar{x}^3} - \frac{\mu_3'}{\bar{y}^3} + \frac{6\mu_2\mu_2'}{\bar{x}^2\bar{y}^2} + \frac{3(\mu_4' - \mu_2'^2)}{\bar{y}^4} + \text{etc.}\right\} \quad \ldots\ldots\ldots\ldots\ldots(\text{iii})^{\text{bis}},$$

$$M_4 = \left(\frac{\bar{x}}{\bar{y}}\right)^4\left\{\frac{\mu_4}{\bar{x}^4} + \frac{\mu_4'}{\bar{y}^4} + \frac{6\mu_2\mu_2'}{\bar{x}^2\bar{y}^2} + \text{etc.}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\text{iv})^{\text{bis}}.$$

Formulae (i) to (iv) are extensions of formulae given by me in a paper on *spurious correlation**. Formulae $(\text{i})^{\text{bis}}$ to $(\text{iv})^{\text{bis}}$ are due to Dr M. Greenwood, Jun., who obtained them in dealing with the problem of the distribution of the opsonic index. They show at once two noteworthy but not yet noted points, namely, (*a*) if the distribution of both x and y be symmetrical, i.e. μ_3 and $\mu_3' = 0$, M_3 will not be zero or the distribution of indices must be skew; (*b*) the mean of the ratio of two numbers picked out of the same series is certainly greater than unity if the series be symmetrical, and will probably be always greater than unity even if it be not, i.e. $\bar{\imath} > \bar{x}/\bar{y}$, which is unity for the same x and y series†. Dr Greenwood found, however, that these formulae did not give with sufficient accuracy the constants of the index distribution. This was probably due to two causes: (*a*) clearly we ought only to keep to the square order in M_2 and the cubic order in M_3 if we retain only to the 4th order in M_4; or if we keep to the higher terms in M_2 and M_3, we must go further with M_4; and (*b*) the values of v_x or v_y are not so small, that the convergency is sufficient when we take these lowest terms of the expansions. It seemed accordingly desirable to find some other way of attacking the problem, and Dr Greenwood asked me for suggestions. The problem he had in view was the distribution of the opsonic index when the blood of the same individual taken in the same manner at the same time was treated as test and as normal. If a wide range of values of the opsonic index could thus be obtained, it would cast some light on what deviations from unity must be looked upon as significant, when test and normal were different individuals.

(3) The idea that suggested itself to me was a fairly simple one, namely to tabulate the y-frequencies to a variable $z = 1/y$. The units of the z-frequency groups will not be equal, but they will all be sufficiently small for us to concentrate their frequencies at their mid-points. We can then calculate their moments easily. Let ν_1, ν_2, ν_3, ν_4 be the moments of x about the zero value of x, and ν_1', ν_2', ν_3', ν_4' be the moments of z about the zero value of z. Then

$$i = z \times x,$$

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* *R. S. Proc.* Vol. LX. p. 492.

† Given two dice, it would be advantageous to bet that the ratio of the number of pips on the two at a cast will exceed unity.

and if m_1, m_2, m_3, m_4 be the moments of i about its zero value, we have:

$$m_1 = P_{11}, \quad m_2 = P_{22}, \quad m_3 = P_{33}, \quad m_4 = P_{44},$$

where P_{uv} is the uvth product moment of z and x about axes through their zero values. In the special case in which z and x are uncorrelated, as in the opsonic index,

$$m_1 = \nu_1\nu_1', \quad m_2 = \nu_2\nu_2', \quad m_3 = \nu_3\nu_3', \quad m_4 = \nu_4\nu_4',$$

or we can obtain any moment about the zero of i by multiplying the corresponding moments of x and z.

These moments are then transferred by the usual formulae

$$M_2 = m_2 - m_1^2, \quad M_3 = m_3 - 3m_2m_1 + 2m_1^3, \quad M_4 = m_4 - 4m_3m_1 + 6m_2m_1^2 - 3m_1^4$$

to the mean as origin and the type of frequency calculated in the usual way from the corresponding β_1 and β_2.

In Greenwood and White's data we have, for the three series discussed below, elementary subranges rising by ·32, ·24 and ·16 of a bacillus per leucocyte for the distribution of the means of counts of 25, 50 and 100. I find that for such distributions, the value of the variate z will only be affected by about a unit in the third place of decimals in the worst cases, i.e. the lowest values of y in samples of 25, whether we use for z (i) the mean of the inverses of the start and finish of the subrange, (ii) the mean of the inverses of all the 32, 24, or 16 hundredths in the subrange, or (iii) the inverse of the mid-point of the subrange. I have accordingly adopted the last as the simplest value of z for practical purposes.

(4) *Illustration of the method.* I. *Greenwood and White's* 200 *samples of* 100 *counts.*

The data are given in Table I. The moments of the frequency distributions for y and z as variates about the zero of those variates were then found by tables of powers of numbers and a calculating machine*.

I. *Distribution of* 40,000 *indices for* 200 *samples of* 100 *counts.*

For x: $\nu_1 = 3{\cdot}67620$, $\nu_2 = 13{\cdot}67643$, $\nu_3 = 51{\cdot}50333$, $\nu_4 = 196{\cdot}40357$.

For z: $\nu_1' = 0{\cdot}275165$, $\nu_2' = 0{\cdot}076603$, $\nu_3' = 0{\cdot}021577$, $\nu_4' = 0{\cdot}0061494$.

These give:

$$m_1 = 1{\cdot}01156, \quad m_2 = 1{\cdot}04766, \quad m_3 = 1{\cdot}11126, \quad m_4 = 1{\cdot}20776,$$

which transferred to the mean give for frequency constants of the 40,000 possible indices:

$\mu_2 = {\cdot}02440$, $\sigma = {\cdot}1562$, $\kappa = {\cdot}2651$,

$\mu_3 = {\cdot}00213$, $\beta_1 = {\cdot}3123$, Mean $= 1{\cdot}01156$,

$\mu_4 = {\cdot}00235$, $\beta_2 = 3{\cdot}9472$, Mode $= {\cdot}9774$.

The distribution is accordingly of Type IV.

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* I have cordially to acknowledge help from Alice Lee, D.Sc., Julia Bell, M.A., and Amy Barrington, who have each worked out nearly the whole of one distribution for me, and from H. Gertrude Jones, who has prepared the diagrams.

TABLE I. *Greenwood and White's distribution of* 200 *samples of* 100 *counts. Opsonic Index. Tuberculosis.*

Variate x ...	*2·745*	*2·905*	*3·065*	*3·225*	*3·385*	*3·545*	*3·705*	*3·865*	*4·025*	*4·185*	*4·345*	*4·505*	*4·665*	*4·825*	*4·985*	*5·145*	Total
Frequency ...	2	9	4	13	29	36	34	34	15	11	5	3	1	1	2	1	200
Variate z ...	·364	·344	·326	·310	·295	·282	·270	·259	·248	·239	·230	·222	·214	·207	·201	·194	—

TABLE II. *Greenwood and White's Distribution of* 400 *samples of* 50 *counts. Opsonic Index. Tuberculosis*

Variate x ...	*2·35*	*2·59*	*2·83*	*3·07*	*3·31*	*3·55*	*3·79*	*4·03*	*4·27*	*4·51*	*4·75*	*4·99*	*5·23*	*5·47*	*5·71*	*5·95*	Total
Frequency ...	2	9	21	30	58	83	71	64	31	15	10	3	1	—	1	1	400
Variate z ...	·426	·386	·353	·326	·302	·282	·264	·248	·234	·222	·211	·200	·191	·183	·175	·168	—

TABLE III. *Greenwood and White's Distribution of* 800 *samples of* 25 *counts. Opsonic Index Tuberculosis.*

Variate x ...	*1·90*	*2·22*	*2·54*	*2·86*	*3·18*	*3·50*	*3·82*	*4·14*	*4·46*	*4·78*	*5·10*	*5·42*	*5·74*	*6·06*	*6·38*	*6·70*	*7·02*	Total
Frequency ...	1	14	38	74	128	145	147	113	74	37	19	7	2	—	—	—	1	800
Variate z ...	·526	·450	·394	·350	·314	·286	·262	·242	·224	·209	·196	·185	·174	·165	·157	·149	·142	—

We find: $r = 16{\cdot}5111$, $\nu = -9{\cdot}9138$ (since μ_3 is positive),
$a = {\cdot}52743$, $y_0 = 8638{\cdot}6$,

and the distribution of the 40,000 indices possible is given by:

$$Y = 8638{\cdot}6\left(1 + \frac{X^2}{{\cdot}27818}\right)^{-9{\cdot}2556} e^{9{\cdot}9138 \tan^{-1}\frac{X}{{\cdot}52743}} \quad \ldots\ldots\ldots\ldots\ldots\text{(i)},$$

where Y is the frequency at distance X from the origin of the curve which is at the opsonic index ·6949, the unit of X being absolute opsonic index measurement.

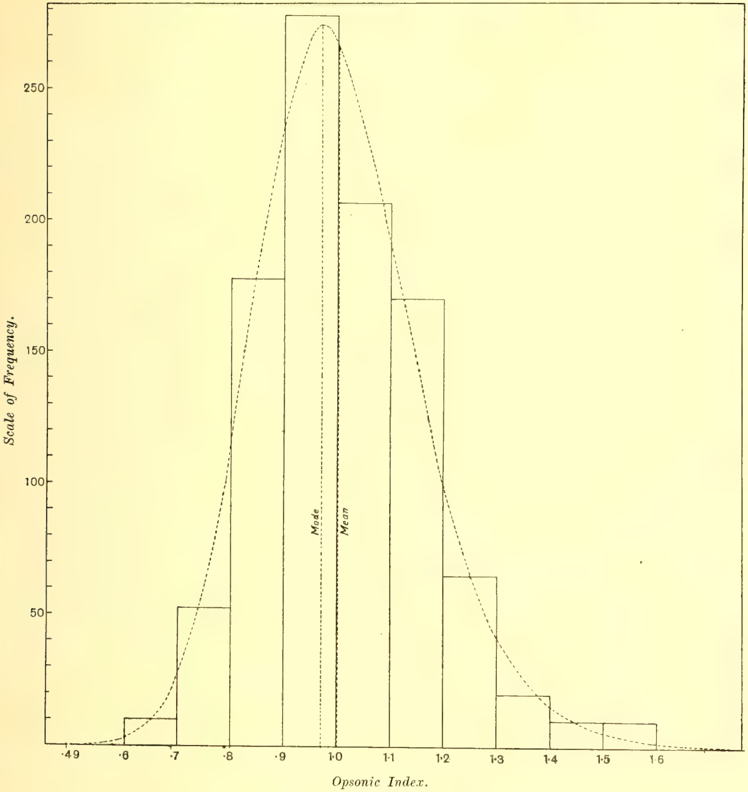


DIAGRAM I. Samples of 100 counted.

It will be seen, in accordance with the results drawn from the approximate formulae, that the mean value of the opsonic index is greater than unity and that there is marked skewness, i.e. no approach even with samples of 100 to a Gaussian distribution. Clearly the most probable value of the index is less than unity, and is rather farther from unity than the mean.

Greenwood and White have made an experimental determination by 200 random drawings. In Diagram I the frequencies of the above curve are reduced to 1000 total and plotted against their experimental data increased to 1000. This of course exaggerates the apparent deviations, but enables our three diagrams to be compared among themselves.

(5) *Further Illustrations.* The following frequencies are also deduced from Greenwood and White's results for 400 samples of 50 and 800 samples of 25.

With the same notation as before the constants of the distribution of the opsonic indices are as follows:

II. *Distribution of* 160,000 *indices for* 400 *samples of* 50 *counts.*

For x: $\nu_1 = 3{\cdot}68140$, $\nu_2 = 13{\cdot}82272$, $\nu_3 = 52{\cdot}92733$, $\nu_4 = 206{\cdot}69803$.

For z: $\nu_1' = {\cdot}277213$, $\nu_2' = {\cdot}078453$, $\nu_3' = {\cdot}022683$, $\nu_4' = {\cdot}006705$,

$m_1 = 1{\cdot}02053$, $m_2 = 1{\cdot}08443$, $m_3 = 1{\cdot}20055$, $m_4 = 1{\cdot}38591$,

$\mu_2 = {\cdot}042948$, $\sigma = {\cdot}2072$, $\kappa = {\cdot}5415$,

$\mu_3 = {\cdot}006191$, $\beta_1 = {\cdot}4837$, Mean $= 1{\cdot}0205$,

$\mu_4 = {\cdot}007567$, $\beta_2 = 4{\cdot}1022$, Mode $= {\cdot}9611$.

The distribution is of Type IV:

$r = 20{\cdot}86018$, $\nu = -22{\cdot}67044$,

$a = {\cdot}62536$, $y_0 = 16{\cdot}9740$,

and the equation to distribution of the 160,000 indices possible is given by:

$$Y = 16{\cdot}9740 \left(1 + \frac{X^2}{{\cdot}39107}\right)^{-11{\cdot}43009} e^{22{\cdot}67044 \tan^{-1} \frac{X}{{\cdot}62536}} \quad \text{............(ii).}$$

The origin of the curve is at the opsonic index ·3409.

Diagram II gives this theoretical distribution reduced to 1000 cases and compared with Greenwood and White's experimental data for 400 increased to 1000 also.

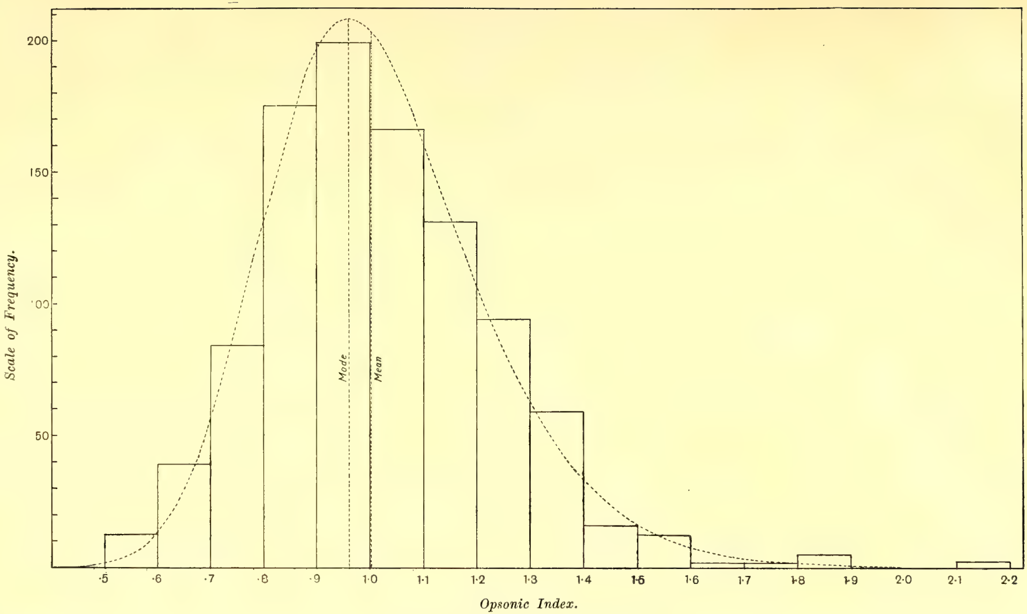
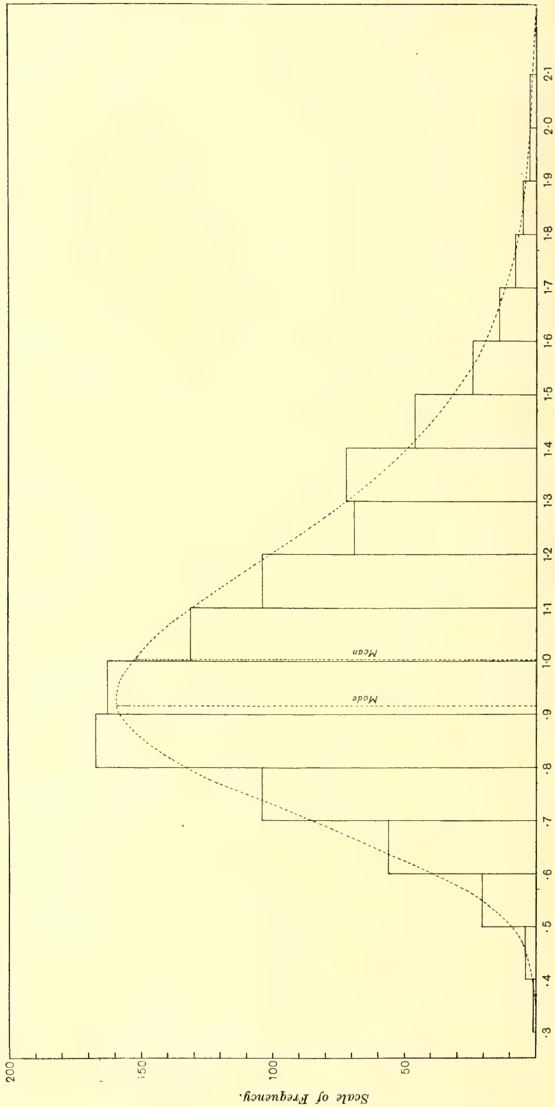


DIAGRAM II. Samples of 50 counted.



Opsonic Index.

DIAGRAM III. Samples of 25 counted.

III. *Distribution of* 640,000 *indices for* 800 *samples of* 25 *counts.*

For x: $\nu_1 = 3{\cdot}68160$, $\nu_2 = 14{\cdot}00990$, $\nu_3 = 55{\cdot}03056$, $\nu_4 = 222{\cdot}90230$,
For z: $\nu_1' = {\cdot}281298$, $\nu_2' = {\cdot}082055$, $\nu_3' = {\cdot}024861$, $\nu_4' = {\cdot}007834$,
$m_1 = 1{\cdot}03563$, $m_2 = 1{\cdot}14959$, $m_3 = 1{\cdot}36811$, $m_4 = 1{\cdot}74622$,
$\mu_2 = {\cdot}077063$, $\sigma = {\cdot}2776$, $\kappa = 1{\cdot}2480$,
$\mu_3 = {\cdot}017943$, $\beta_1 = {\cdot}7060$, Mean $= 1{\cdot}0356$,
$\mu_4 = {\cdot}025583$, $\beta_2 = 4{\cdot}3088$, Mode $= {\cdot}9330$.

The distribution is of Type VI*:

$r = -31{\cdot}25271$, $q_1 = 51{\cdot}67822$, $q_2 = 18{\cdot}42551$,
$a = 1{\cdot}52075$, $\log y_0 = 26{\cdot}6813462$,

and the equation to the distribution of the 640,000 indices possible is given by:

$$Y = \text{antilog}\ 26{\cdot}6813462 \times (X - 1{\cdot}52075)^{18{\cdot}42551}\, X^{-51{\cdot}67822} \quad \ldots\ldots\ldots\text{(iii)}.$$

The origin is at the opsonic index $-1{\cdot}4304$.

Diagram III gives this distribution reduced to a total of 1000 indices and set against Greenwood and White's experimental curve for 800 increased to 1000. This exaggerates the apparent deviations, but enables us to compare with the samples of 50 and 100. Considering the relative paucity of Greenwood and White's drawings, and the fact that we have fitted our curves to their non-replaced material, whereas a random sample would probably be better represented by the replaced material, the fits seem fairly close. The actual goodness of fit test was not applied, as at the time of writing the paper means of drawing the curves on a large scale and mechanically integrating them were not accessible.

(6) It is as well to look at a combined table of the frequency constants of the three distributions given above.

No. of Counts average based on	No. in Distribution of Means	Mean	Mode	σ	β_1	β_2
25	800	1·0356	·9330	·2776	·7060	4·3088
50	400	1·0205	·9611	·2072	·4837	4·1022
100	200	1·0116	·9774	·1562	·3123	3·9472

It will be seen that these values form a very consistent relatively smoothly altering system. But the approach to normality is very slow. Even with counts of 100 the distribution of the opsonic index is markedly skew and platykurtic, and it would not be safe to treat the distribution as a normal curve†. In all cases the modal or most probable value of the opsonic index of an individual tested by himself is less than unity and his average value greater than unity.

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* Type VI is adjacent to Type IV and in this case the curve is almost on the boundary line—i.e. Type V: see Rhind's Diagram, *Biometrika*, Vol. VII. p. 389.

† The reduction of β_1 and β_2 to the Gaussian values 0 and 3 is, of course, not at the same rate as if we averaged 25, 50 and 100 opsonic indices. What we are doing here is to average the number of bacilli in 25, 50 and 100 cells on which the two factors of the index are based—a very different process.

If we actually suppose the distributions normal and varying round the means with the standard deviations given above we have the following results:

Significance laid on an event which will not happen more frequently than	Number of Counts		
	25	50	100
Once in ten trials ...	·58—1·48	·68—1·36	·75—1·27
Once in eight trials ...	·61—1·46	·70—1·34	·77—1·25
Once in six trials ...	·65—1·42	·73—1·31	·80—1·23

This table is to be read in the following sense: If an opsonic index were based on a count of 50 cells, then once in eight trials an individual tested against himself would have an index lying outside the limits ·70 to 1·34. Or, again, with 100 counts once in six trials an individual tested against himself would have an index lying outside the limits ·80 to 1·23. It may be added that for most of the purposes of practical life or of exact science we should not consider an "improbability" which could happen once in ten trials as marking a significant differentiation. Much greater degrees of improbability would be required. In the case of medicine, however, much less certainty may be demanded of a judgment, and probably no weight would be given to an isolated opsonic determination in *diagnosis*. Still the matter gives ground for pause, the opsonic index of the same material tested by itself has a very wide range round unity.

The following table will show how closely Greenwood and White's experimental determinations agree with our theoretical evaluation of the constants:

Constant of Distribution	Samples of 25		Samples of 50		Samples of 100	
	G. + W.	P.	G. + W.	P.	G. + W.*	P.
Mean	1·0362	1·0356	1·0205	1·0205	1·0120	1·0116
Mode	·9432	·9330	·9536	·9611	·9689	·9774
Standard Deviation	·2585	·2776	·2204	·2072	·1553	·1562
β_1	·7099	·7060	·6952	·4837	·3654	·3132
β_2	4·3914	4·3088	4·8945	4·1022	3·7138	3·9472

The only substantial divergences are in Greenwood and White's values for β_1 and β_2 in the case of samples of 50. I have been through my results again and can find no error. Dr Greenwood has been through his figures and finds slight slips (see footnote, p. 521) in the values of β_1 and β_2. These are not sufficient to account for the divergence of their results from mine, and from the trend of their samples of 25 and 100. I am inclined to think the divergence is due to the presence of the outlying index 2·16 in their chance drawings—a result my curve shows to be exceedingly improbable not once in 3000 drawings, and the like of which does not occur in their samples of 50 (drawings replaced) (see p. 525). It would clearly be possible from their experimental method of drawing samples of 800, 400 and 200 indices to closely approach the corresponding theoretical distributions of 640,000, 160,000 and 40,000 represented in our curves. Our results then are in every way confirmatory of theirs, but place on a rather more satisfactory theoretical instead of experimental footing their deduction of index distributions. The general conclusion seems to be that except in the case of an extremely low or extremely high value of the opsonic index, little if any weight whatever ought to be placed on a *single* determination of this index. Hence the method would not be valid when applied to cases in which, owing to the evolution of a morbid process or the action of some drug, very few observations can be made under the same conditions, i.e. it must be of doubtful application in treatment. Further the concentration obtained by basing the index on a count of 100, rather than one of 50 or even 25, while sensible is not very rapid. It would require very large numbers—much beyond every-day practice—to reduce in a marked manner this variation of the opsonic index from unity, when an individual even is tested against himself. Generally the diagrams indicate that an extreme variation in excess is more likely to occur than an extreme variation in defect, but that the most probable index, when the individual is tested against himself, will be one somewhat less than unity.

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* They have used the boundary curve Type V in the case of samples of 100.

MISCELLANEA.

I. On the Value of the Teachers' Opinion of the General Intelligence of School Children.

Compiled from the Tables and Reductions of
H. GERTRUDE JONES, University College, London.

A great deal of random criticism has recently been made of school-teachers' estimates of general intelligence. We have been told that those estimates are absolutely without value, that any correlations obtained from them are idle and that the personal equation involved is so great that no use can legitimately be made of them. Good illustrations of this type of criticism will be found in the notices of Heron's memoir on *The Influence of Defective Physique and Unfavourable Home Environment on the Intelligence of School Children*, which Mr G. U. Yule has considered it desirable to publish in two separate journals*. As it does not appear that those who criticise the teacher's estimate of general intelligence in this way have made any experimental inquiry into the matter themselves, it may be of value to publish an investigation of a preliminary character made some time ago as an attempt to ascertain whether such classifications are really idle. More ample researches will shortly be published, but it may be said in advance here that they serve to confirm the present result.

The data are taken from schedules filled in by over 20 Aberdeen teachers†, who were not specially prepared for the task. 249 boys were taken from 4 different schools; their ages ran from 6 to 14 and they belonged to classes which were termed "Infants," I, II, III, IV, V, and VI. They were judged not only by different teachers, but in different schools, and their mental capacity was appreciated in four groups: (i) Excellent = E., (ii) Good = G., (iii) Moderate = M., (iv) Dull = D. This is a pure appreciation of general intelligence by a variety of teachers in a variety of schools. Next we have taken the individual boy's place in class as shewn by examination results and divided by the number of boys in the class. This may be adopted as a measure of the boy's examinational intelligence. If the teacher's estimate of general intelligence be of small value we should expect that when allowance is made for difference of class and age there would be little relation between examinational test and general intelligence.

The following characters were taken out and tabled, "Mental Capacity," Age, Position in Class (measured as above stated) and Class or Standard, and the following six tables obtained.

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* *Journal of the Statistical Society*, Vol. LXXIII. p. 547, and *School Hygiene*, Vol. I. p. 473.

† For the Royal Commission on Physical Training (Scotland). We have gratefully to acknowledge the loan of these schedules by Professor Matthew Hay.

TABLE I.

Mental Capacity.

Age	E	G	M	D	Totals
6	4	11	11	5	31
7	2	15	10	6	33
8	5	19	6	4	34
9	3	18	12	—	33
10	2	20	11	1	34
11	2	9	12	5	28
12	1	11	10	3	25
13	—	9	13	2	24
14	—	4	3	—	7
Totals	19	116	88	26	249

TABLE II.

Mental Capacity.

Standards	E and G	M and D	Totals
Infants, I, II, III	77	61	138
IV, V, VI ...	58	53	111
Totals	135	114	249

TABLE III.

Class or Standard.

Age	Infants	I	II	III	IV	V	VI	Totals
6	30	1	—	—	—	—	—	31
7	—	30	2	1	—	—	—	33
8	—	12	21	1	—	—	—	34
9	—	3	3	21	6	—	—	33
10	—	—	—	10	22	1	1	34
11	—	—	1	—	4	22	1	28
12	—	—	—	2	1	9	13	25
13	—	—	—	—	—	7	17	24
14	—	—	—	—	—	—	7	7
Totals	30	46	27	35	33	39	39	249

TABLE IV.

Mental Capacity.

Place in Class	E	G	M	D	Totals
·03	7	9	2	—	18
·08	3	8	—	—	11
·13	4	10	1	—	15
·18	3	12	2	—	17
·23	1	12	3	1	17
·28	—	11	1	—	12
·33	—	10	3	—	13
·38	—	7	4	—	11
·43	—	5	5	—	10
·48	—	11	9	—	20
·53	—	8	5	1	14
·58	1	5	8	1	15
·63	—	2	7	—	9
·68	—	4	6	—	10
·73	—	2	4	3	9
·78	—	—	8	1	9
·83	—	—	11	—	11
·88	—	—	5	5	10
·93	—	—	2	6	8
·98	—	—	2	8	10
Totals	19	116	88	26	249

TABLE V.

Age.

Place in Class	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	Totals
·03	2	3	—	5	3	2	3	—	—	18
·08	3	1	—	2	2	1	1	1	—	11
·13	3	3	1	3	1	—	1	2	1	15
·18	4	1	2	3	3	1	—	3	—	17
·23	1	3	6	1	1	2	—	3	—	17
·28	—	2	3	1	4	—	1	1	—	12
·33	—	—	4	3	4	—	2	—	—	13
·38	2	1	—	—	3	1	2	—	2	11
·43	—	2	1	1	1	2	2	1	—	10
·48	3	1	3	1	1	4	4	2	1	20
·53	1	3	3	3	—	3	—	—	1	14
·58	1	—	3	2	1	4	2	2	—	15
·63	1	2	1	1	4	—	—	—	—	9
·68	—	1	2	2	3	—	—	2	—	10
·73	1	3	2	—	—	2	1	—	—	9
·78	2	2	1	2	1	—	—	1	—	9
·83	1	1	—	2	1	2	1	2	1	11
·88	—	1	1	1	1	2	1	2	1	10
·93	2	1	1	—	—	1	2	1	—	8
·98	4	2	—	—	—	1	2	1	—	10
Totals	31	33	34	33	34	28	25	24	7	249

TABLE VI.

Class or Standard.

Place in Class	Infants	I	II	III	IV	V	VI	Totals
·03	2	3	—	5	5	2	1	18
·08	3	1	—	1	2	2	2	11
·13	3	4	1	2	1	2	2	15
·18	4	2	2	2	2	2	3	17
·23	1	3	4	3	—	3	3	17
·28	—	3	2	1	5	—	1	12
·33	—	1	4	5	1	1	1	13
·38	2	—	2	1	2	1	3	11
·43	—	2	1	1	1	3	2	10
·48	3	2	2	1	1	5	6	20
·53	1	6	—	3	1	2	1	14
·58	1	1	2	1	3	5	2	15
·63	1	1	2	2	3	—	—	9
·68	—	2	1	2	3	1	1	10
·73	1	5	—	—	1	2	—	9
·78	1	3	1	2	1	—	1	9
·83	1	3	—	1	—	2	4	11
·88	—	1	2	1	1	3	2	10
·93	2	1	1	1	—	1	2	8
·98	4	2	—	—	—	2	2	10
Totals	30	46	27	35	33	39	39	249

Let us look at the results of these Tables in succession :

Table I. Mental Capacity and Age. The mean ages are as follows :

$$\left.\begin{array}{lr} \mathrm{E}: & 8{\cdot}87 \\ \mathrm{G}: & 9{\cdot}89 \\ \mathrm{M}: & 10{\cdot}33 \\ \mathrm{D}: & 9{\cdot}38 \end{array}\right\} \text{All classes}: \ 9{\cdot}91.$$

There is thus a decrease of intelligence with age until we come to the worst class and here there is a drop in age. There are very certainly cross-currents at work, e.g. clever boys go young to school, and leave early, and extreme cases of dullness go to "special" schools at 9 to 11, and school lessens at any rate the outward appearance of stupidity. Worked out by the correlation ratio method the correlation is $\eta = {\cdot}1830$. This clearly should be considered as negative, but while the regression is not linear and therefore the correlation ratio sensibly greater than the correlation coefficient, we will take $r = -{\cdot}1830$ because we believe the "dullness" of the younger children to some extent wears off with continued school life or the dull are drafted elsewhere*.

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* The reader should examine the last column of Table I. We think several of the younger children would ultimately pass into "special schools" or be sharpened by school life. The same bi-modal character is obvious in column M, if to a lesser extent.

Table II. Mental Capacity and Standard. It is interesting to look at the data for this as a contingency table. This is given below as Table II *bis*:

TABLE II *bis.*

Class or Standard.

Mental Capacity.	Infants	I	II	III	IV	V	VI	Totals
E	4	5	3	2	1	3	1	19
G	11	19	15	18	21	10	22	116
M	10	14	7	15	10	18	14	88
D	5	8	2	—	1	8	2	26
Totals	30	46	27	35	33	39	39	249

The greatest contributions to the contingency arise from (i) the defect of Dulls in Standard III and (ii) the excess of Dulls of Standard V, accompanied as it is by a defect of Good mental capacities. These may be personal to the teachers of these standards, or to their selection of children, which was made for the purposes of physical measurement. But the corrected contingency coefficient which should be much emphasised by personal equation is only ·2123*. Now we shall see that the correlation between age and standard is very high, i.e. ·9366; hence the correlation between mental capacity and age for constant standard $= -\cdot 0461$, or is practically negligible. In other words, allowing for the personal equation of the master with regard to intelligence, age counts for very little in his estimate of general intelligence. If we even out the irregularities of the above table by forming out of it a mere four-fold table as in Table II we obtain $r = -\cdot 0558$. We expect that this is really the more correct measure of the true relationship between standard and general intelligence.

Table III. Age and Standard. The correlation here is clearly very high; determined as a correlation ratio it is ·9366. Practically children are moved up a class or standard a year.

Table IV. Mental Capacity and Place in Class. The correlation as found by correlation ratio is ·7246. The actual mean places of each category are:

E: ·122
G: ·312
M: ·500
D: ·851
All categories: ·456.

This list itself shews that the teacher's estimate of general intelligence represents very closely the examinational place which a student will take and for many inquiries may quite legitimately replace that estimate of ability.

Table V. Age and Place in Class. Found by the ordinary product moment method the correlation coefficient is ·0484, shewing that with increased age a *slightly worse* class position is taken, i.e. any advantage from age is counterbalanced by the clever elder boys leaving school earlier. The value, considering the numbers dealt with, is, however, hardly significant.

Table VI. Place in Class and Standard. The mean places are as follows:

Infants	·457
I	·494
II	·445
III	·393
IV	·388
V	·493
VI	·493

All classes: ·456.

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* This must be taken negative if treated as a correlation coefficient.

This series is very irregular. The η for the series is ·1553, but this is rather a measure of the irregularity than of anything else*. If we take the correlation coefficient for a four-fold table as follows

	Infants I, II, III	IV, V, VI	Totals
·05 to *·45*	71	53	124
·50 to *1·00*	67	58	125
Totals	138	111	249

we find $r = {\cdot}0580$, a number of scarcely any practical significance and shewing that really place in class is, as it should be, independent of standard.

We have then to sum up the following results:

Intelligence and Age	$r_{13} = -{\cdot}1830$
Intelligence and Position in Class .	$r_{12} = {\cdot}7246$
Intelligence and Standard . .	$r_{14} = -{\cdot}0558\ (-{\cdot}2123)$
Position in Class and Age . .	$r_{23} = -{\cdot}0484$
Position in Class and Standard . .	$r_{24} = {\cdot}0580\ ({\cdot}1553)$
Age and Standard	$r_{34} = {\cdot}9366$

In the case of two of these correlations for which some doubt might exist, we have placed in brackets the higher values found by different processes. The results for both values of the partial correlation coefficient have been worked out.

Correlation of "General Intelligence" and Examinational Place, for constant age and constant standard:

With lower values of r_{14} and r_{24}: $\rho_{12} = {\cdot}70 \pm {\cdot}02$.
With higher values of r_{14} and r_{24}: $\rho_{12} = {\cdot}99 \pm {\cdot}01$.

We may safely draw from these results the conclusion that the teacher's estimate of general intelligence is not a purely idle character, wholly valueless owing to the personal equation of the teacher. Whatever else it may be, it is highly correlated with the place which the student will take in an examination test and accordingly has at least one quite definite significance.

The first of these results shews that with the lesser and more probable values of the correlations we obtain, allowing for standard and age, a high correlation (·70) between the teacher's estimate of general intelligence and actual examination measure of capacity. There are, however, cross-currents at work in elementary schools; the one is the selection of the notably dull children about 10 who are drafted into "special schools." This causes rather a defect of dull children in the II, III and IV standards; the next is the selection of the more intelligent children in the highest standards (this is most obvious in V) to leave the elementary schools. The total effect is to make a somewhat high correlation ratio for standard and intelligence. If we took all the children in a class there ought to be sensibly no correlation between position in class and standard, but we actually find for classes II, III and IV excess of good places (averages ·445, ·393, ·388 instead of ·456) and for the high classes V and VI excess of bad places (averages ·493, ·493). This seems to be the result of the same cross-currents, the selection of the dull to go and of the dull to stay at different ages. If we now give the higher values to the correlations produced by these tendencies in elementary schools, we find on correction not that they have tended to disguise the real value of the "general intelligence" character by giving it a spurious high intensity, but that they have tended to *lower* its true importance. Those who complain of the different percentages of the divers grades of intelligence of the children in the different standards of the same elementary school often seem unaware that the same sample does not pass through all standards, at one stage there is a rejection of the very dull and at another the retention of the dull class. The main facts brought out are the substantial correlation between teacher's estimate of general capacity and examination test, and the fact that extreme allowance for age and standard tends to emphasise this relationship rather than to shew that the teacher's estimate is of little value. These observations were not made *ad hoc*, but they have been confirmed by other observations made on far larger numbers with additional safeguards for accuracy. The results of these will shortly be published, but it seems worth while indicating at a time when hasty criticisms are being made of the value of "general intelligence" estimates that the teacher's appreciation of mental capacity does mean something, and has a very direct and practical value, when properly registered and handled.

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* It must be remembered that not *every* child, but only 30 to 40 in each standard were physically examined, and the number so dealt with is not by any means the number in the class. This selection of children accounts for the mean place in class being ·456 and not ·500.

K. P.

II. Note on the Separate Inheritance of Quantity and Quality in Cows' Milk.

By KARL PEARSON, F.R.S.

Under the above title Professor James Wilson, of the Royal College of Science, Dublin, read a paper in May, 1910, which is published in Vol. XII. pp. 470—479, of the *Scientific Proceedings of the Royal Dublin Society*.

He states that: "It is a very general opinion that the milk of high-yielding cows is usually poorer and that of low-yielding cows richer in quality" (p. 470), and again he writes:

> "If we group together all the low-yielding cows, and find their milk *invariably* high in quality, we may infer that low yield and high quality are of the nature of concomitant variations. If we group the high-yielding cows together, and find their milk *invariably* of low quality, we may infer that high yield and low quality run together. But if we take these groups and any other groups we can form, and find that the quality *varies the same way in them all*—that is that there are low qualities, high qualities, and medium qualities in every one of them—then we are *justified in inferring* that the quantity and quality of the milk are independent of each other. And this is what we do find" (p. 471).

The italics are mine. It would be difficult to find a paragraph containing more fallacies in as many words. Two qualities may be associated together, even closely correlated, and yet one will not be *invariably* accompanied by the other. High, low and medium values of one character may occur with any given value of a second and yet there be high correlation between them. The whole problem turns on the *extent* of the variation of the first character for a given value of the second, and not the invariable appearance of one with the other. The author may have some conception of correlation other than absolute association, but neither this paragraph, nor his general treatment of the subject, shews any signs of it.

Professor Wilson's material is involved in the table below.

Now there can be no doubt of the value of such returns and of tabling them, although we regret the clubbing together of the tail frequencies, that common error of the non-statistical trained mind, which renders determination of the true correlation so difficult.

Percentages of Butter-Fat in Milk of Ayrshire Cattle[*].

Yield of milk	2·6	2·7	2·8	2·9	3·0	3·1	3·2	3·3	3·4	3·5	3·6	3·7	3·8	3·9	4·0	4·1	4·2	4·3	4·4	4·5	4·6	4·7	4·8	4·9	5·0	5·1	Totals
Under 500 gallons	1	—	4	10	7	16	17	26	56	66	52	67	69	61	36	30	23	16	9	6	7	3	1	—	1	1	585
From 500 to 600 gallons	—	—	4	7	6	21	41	63	99	103	107	125	105	89	67	51	25	18	4	3	7	1	1	1	—	—	948
From 600 to 700 gallons	1	—	3	4	13	17	35	63	64	103	106	99	86	62	54	45	27	10	5	2	7	2	—	1	—	—	809
Over 700 gallons	—	—	—	1	8	14	31	38	52	59	66	70	61	43	35	19	15	9	2	—	1	—	—	—	—	—	524†
Totals ...	2	—	11	22	34	68	124	190	271	331	331	361	321	255	192	145	90	53	20	11	22	6	2	2	1	1	2866

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[*] Data taken from *Report of Milk Records for Season* 1908, Ayrshire Cattle Milk Records Committee.

[†] Professor Wilson has 525, but only 524 cows appear in his printed table.

How now does Professor Wilson extract from these data the independence of quality and quantity of milk on which he insists, and which is chosen as the very title of his paper? He has prepared six elaborate diagrams, and he leaves the reader from mere inspection of these to come to the same conclusion as himself, i.e. that the variation is the same in all his groups and the quality independent of the quantity. Not being able myself to judge from such diagrams whether there is complete independence between quality and quantity, I thought it worth while to ascertain the mean percentages of fat and the variations in fat of his four groups. The results are as follows, the probable errors being added:

Quantity of Milk	Means	Standard Deviations
Under 500 gallons ...	$3{\cdot}710 \pm {\cdot}010$	${\cdot}372 \pm {\cdot}007$
From 500 to 600 gallons ...	$3{\cdot}673 \pm {\cdot}007$	${\cdot}319 \pm {\cdot}005$
From 600 to 700 gallons ...	$3{\cdot}651 \pm {\cdot}008$	${\cdot}321 \pm {\cdot}005$
Over 700 gallons	$3{\cdot}640 \pm {\cdot}008$	${\cdot}275 \pm {\cdot}006$

The means and variabilities are in percentage of butter-fat.

For the whole population of 2866 cows the mean is 3·672 and the variability is measured by 3·230. It will accordingly be clear that there is (i) a slight but perfectly definite decrease in quality with increase of quantity, the low-yielding cows have a higher, the high-yielding cows a lesser fat percentage than the average Ayrshire, although the increase and decrease only amount to about 1 °/₀ of the average yield, and (ii) the variation in quality is not the same for cows of each quantity category. Cows which give high quantity have far less variation in their yield of butter-fat than cows of low yield. The number of cows is so large relatively that the probable errors are small and there is no doubt about the results being significant. Owing to the manner in which the tails of the above frequency have been clubbed together it is not possible to work out the proper correlation by the ordinary product moment method. The means, however, suggest that the correlation cannot be very far from linear, and the correlation ratio η may be fairly taken to measure r. This gives us $r = -{\cdot}075$ with a probable error of $\pm{\cdot}013$, a result, if small, yet definitely significant.

It seemed worth while putting these numerical results on record, as a further protest against somewhat dogmatic inferences drawn from mere inspection of elaborate but unnecessary graphs.

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